Problem Solving with the Calculator

Peter Leesinsky

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ABSTRACT

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The processes employed by subjects solving four nonroutine mathematics problems with the use of a calculator were analyzed. The solution process was identified to consist of the general stages of reading, understanding, producing and evaluating. Each stage was analyzed in terms of specific processes and their related errors. A retrospective interview technique was used to discuss the printout calculated by each subject. Each solution transcription was then classified in terms of the processes and errors identified.

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CHAPTER I

RESEARCH IN MATHEMATICS EDUCATION

Problem Solving Research

This research is primarily investigating the ways in which students use a calculator to solve some specifically designed mathematical problems. A form of clinical interview was used to discuss in retrospect, with each student, the methods they employed on the problem. It is interesting, first of all, to note the features of what many prominent researchers consider the domain of problem solving. Kantowski states:

> An individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him.[1]

Problem solving for Kantowski is then composed of two aspects: ". . . the process, or set of behaviours or activities that direct the search for the solution, and the product or actual solution."[2] This research will be investigating the processes involved in the solution of the problems designed.

An interesting distinction is made by Kantowski between problems, as just defined and exercises in the following way:

> A problem differs from an exercise in that the problem solver does not have an algorithm that when applied will certainly lead to a solution.[3]

This distinction was immediately clear to me when I made the search for nonstandard calculator problems for this research. Most of what is offered as calculator problems would be defined as exercises using Kantowski's specification. Greeno takes a different view from Kantowski stating that "recent fundamental research in problem solving . . . has erod[ed] the distinction between knowledge-based performance and problem solving."[4] Rather than seeing a dichotomy between exercises (i.e., knowledge-based) and problems, Greeno thinks "it is seriously misleading to label performance in some situations as problem solving and in other situations in which the same kinds of cognitive processes occur as not involving problem solving."[5]

In commenting on Greeno's paper, Reif makes the following cogent criticism:

> . . . problems can differ widely in their complexity and their demands on flexibility for coping with diverse situations. Such differences in complexity and needed flexibility can imply qualitative rather than merely quantitative differences between types of problems.[6]

Reif's criticism is supported generally by the definitions of what constitutes a problem and what it means to solve a problem when one considers the work of Polya, Krutetskii and others. In fact, there is a commonality to much of what these researchers say.

Polya states:

> Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable.[7]

The idea of difficulty is a basic criterion for a problem to exist, as Polya clearly mentions: ". . . some degree of difficulty belongs to the very notion of a problem: where there is no difficulty, there is no problem."[8] Simon, though operating from a different perspective, also has a definition of a problem or problem situation that is elucidating:

> Problem solving is often described as a search through a vast maze of possibilities . . . . Successful problem solving involves searching the maze selectively and reducing it to manageable proportions.[9]

Simon sees a relationship between natural selection and problem solving in the following way:

> . . . human problem solving, from the most blundering to the most insightful, involves nothing more than varying mixtures of trial and error and selectivity.[10]

The operative term here is selectivity, and through this key word, Simon's definition can be linked to Polya as follows: "The selectivity derives from various rules of thumb or heuristics, that suggest which paths should be tried first and which leads are promising."[11]

What makes a problem into a problem is briefly put by Johnson-Laird and Wason: "There is no algorithm for choosing which method to adopt in trying to solve a problem."[12]

Mayer breaks the definition of a problem into three general parts:

> . . . any definition of "problem" should consist of the three ideas that (a) the problem is presently in some state, but (b) it is desired that it be in another state, and (c) there is no direct, obvious way to accomplish the change.[13]

For Mayer as a cognitive scientist, problem solving is synonymous with thinking and cognition.

Lester defines a problem as:

> . . . a situation in which an individual or group is called upon to perfrom a task for which there is no readily accessible algorithm which determines completely the method of solution.[14]

For the purposes of this research, it will be assumed that a problem has no immediately obvious method of solution and that for most subjects the problems used will require procedures that are not trivial because of the nonstandard characteristics of the problems designed.

The Processes Involved in Problem Solving

There are many aspects to consider in problem solving including task variables, subject variables, problem solving behaviour (i.e., process variables) and environmental variables (i.e., principally instruction variables). This research will focus primarily on an investigation of the process variables of problem solving. Lester lists the following characteristics of the individual as important subject variables:

> . . . conceptual style (e.g., does the individual tend to be analytically, spatially or intuitively oriented?); previous mathematical background; . . . reaction under stress . . . ; and . . . field independence (e.g., does the solver have the ability to ignore distracting information?).[15]

Polya describes some of the characteristics of a good problem solver: "It takes so much to succeed; formerly acquired knowledge, good mental habits, concentration upon the purpose, and one more thing: good luck."[16] A point well made by Simon is that problem solvers need good memories:

> Subjects get into trouble simply because they forget where they are, what assignments they have made previously, and what assumptions are implicit in assignments they have made conditionally.[17]

In this research no deliberate attempt was made to alter task variables or environmental variables, rather these were kept as constant as possible within the design of the investigation. As mentioned, the major consideration here will be given to analyzing process variables involved in the solution of the problems given. It would be difficult to isolate these processes from the individual and in fact it is assumed that there is, to a degree, an interaction between the subjective and process variables.

Tuma and Reif refer to this interplay of factors: "The kind of process used in solving a kind of problem depends both on the characteristics of the problem and on the knowledge of the problem solver."[18] There are very good reasons for studying the processes individuals use to solve problems. Polya focuses on the philosophical rationale when he quotes William James: "The solution of problems is the most characteristic and peculiar sort of voluntary thinking."[19]

Educators like Brownell were early proponents for an investigation of the process of solution as opposed to the product of solution. In discussing instructional weaknesses in mathematics he noted "Our attention as teachers is directed away from the processes by which children learn while we are over-concerned about the product of learning."[20] This emphasis on the result instead of the process was then reflected in the students attitudes as Brownell also suggests "the correct answer is their [i.e., the children's] sole consideration."[21] A clear illustration of how subject and trait variables interact. Brownell also draws our attention to a very significant aspect of the process of solution--the significance of errors: "most errors in mathematics are the result . . . of incomplete understandings, of inappropriate thought processes, and of faulty procedures."[22] This research will have a possible application in terms of remedial work by teachers and so error analysis of the problem solving process will be a component of the research.

Krutetskii also distinguishes between the different objectives research can have in describing what direction his own work followed:

> For the purposes of our study, it was necessary to establish not only and not so much the ultimate result of the examinees performance of a task but primarily the process used in that performance.[23]

If the focus of the research will be an investigation of the process of problem solving then questions or problems must be given that facilitate this.

Krutetskii clearly recognized this also in stating "Our problems were oriented not so much toward a quantitative expression of the phenomenon being studied as toward revealing its qualitative features."[24] LeBlanc too makes specific reference to the type of process problems required for research into how subjects solve problems. LeBlanc specifies the following definition: "A process problem . . . lends itself to exemplifying the procedures inherent in problem solving."[25] It is significant to note, in this regard, that much of the research done on problem solving that is interested in examining the processes involved often relies on nonroutine or nonstandard questions. The questions used in this research were developed with the objective of uncovering the processes subjects utilize with a calculator.

Kilpatrick suggests that the questions used are not creative enough:

> Anyone masochistic enough to sift through the pile of research literature on problem solving in mathematics will likely come away depressed . . . primarily because of the routine exercises used as problems in so many studies.[26]

Kilpatrick encourages the recent interest in research on processes of problem solving and recognizes the fact that:

> Such attempts have required the use of problems that are more complex and less obvious than those that the typical standardized achievement test or school mathematics textbook contains.[27]

A review of the exercises available as problems for the calculator tends to substantiate Kilpatrick's statement.

Johnson-Laird and Wason distinguish between two general methods of problem solving:

> . . . an algorithm is a mechanical procedure that always yields the answer to a problem in a finite number of steps, whereas a heuristic procedure is one which offers a useful shortcut but which does not guarantee a solution.[28]

A relation could be made between these two general processes and the previously distinguished exercises and problems. It is the general diversity of the problems used in studies on processes that concerns Lester in the following: "A primary reason it is difficult to draw any general conclusions about mathematical problems solving is that a wide variety of problems have been used in research."[29] Therein lies a dilemma in problem solving research: to investigate process, nonroutine problems are generally used; but nonroutine problems are by their very nature very different and so it becomes difficult to compare the research. Lester too sees a need for the study of the problem solving process when he states "knowledge of the processes used to solve a problem is more important than noting simply that the problem was or was not solved, how much time was taken, how many errors were made and so on."[30] An essential point is also made by Lester in the following statement: "Perhaps problem solving research gets its reputation for being a chaotic area of investigation because such a wide range of mental activities are involved."[31]

If the processes needed in problem solving generally are viewed as acquireable skills, and some educators take this view, Greeno derives the following conclusion based on his research, about these skills: "An analysis I carried out a few years ago (Greeno, 1978) led me to conclude that there is not a single homogeneous set of skills that we can identify as the important skills of problem solving."[32] Some research does try to make general statements about what abilities a good problem solver has, with Krutetskii's work noteworthy. Silver reports that "Krutetskii found substantial quantitative and qualitative differences between high and low-ability students in their perceptions of problem structure."[33] Silver's own research led him to conclude that "the significant positive relationship . . . between a student's abilty and the tendency to associate problems on the basis of mathematical structure was supported in this study."[34] There was also a "negative relationship between mathematical ability and the tendency to associate problems on the basis of contextual details."[35]

There also has been research into the characteristics of poor problem solvers. Anthony and Hudgins investigated the differences in processes used by better and poorer problem solvers. From the outcome of their research they reached the conclusion that there is "a pattern of differences in the problem solving protocols revealed between the better and poorer problem solvers, . . . [and that] poor problem solvers failed to recognize or act upon the demands of the problem, to differentiate between relevant and irrelevant data, or to deal with both parts of two-step problems."[36] In a second part of their investigation where they developed a remedial program for the poor problem solvers they found that "the absolute amount of gain achieved by the experimental pupils was small."[37] In regard to their last finding, if it is assumed that Polya is correct in saying "you learn to do problems by doing them.",[38] perhaps Anthony and Hudgin's results can be explained by the provision implied in Kilpatrick's statement that "Practice with lots of problems is clearly a necessary condition for problem solving proficiency--at least for most people--but it is hard to see why it should be sufficient."[39] Much of the research on problem solving seems to imply that the processes involved in problem solving can be learned, in essence that there is no "magic" involved in solving problems. And yet, what really makes a problem into a problem for an individual is that the processes he has learned to apply to problems are not obvious to him in their application. Gattegno et al. make this point also :

> If it is true that every problem--every real problem--challenges us differently, it is part of the advice we get from life that problem solving is not a discipline, like heuristics, but something requiring that we change ourselves in certain ways.[40]

Errors: The Analysis of Incorrect Procedures

If research is primarily interested in the processes used to solve a problem then unsuccessful solutions and errors generally may become significant. Piaget was interested in incorrect responses and "[carried] out cognitive studies in order to discover the underlying reasons for incorrect answers in younger children and correct ones in older children", according to Opper.[41] Research on the analysis of errors from a process perspective is not as common as research that addresses the errors made in the actual solutions, i.e., the products of the process. Gattegno et al. request an investigation of all problem solving processes: "All those paths which lead nowhere are as much a part of our experience as all those that go somewhere . . . ."[42] Englehardt notes this fact in his analysis of computational errors by children: "Although numerous studies have been conducted involving arithmetic achievement, little attention has been focused on the qualitative aspects of children's errors, i.e., the types of errors they exhibit."[43] It should be noted that in his own research, Englehardt uses a "test consisting of 84 arithmetic computation items."[44] The rationale behind investigating errors is outlined by Matz: "Naive problem solvers tend to make the same mistakes, and even the occasional mistakes made by adept problem solvers are typically uniform."[45]

It is interesting to note that the people who observe problem solving on a regular basis--teachers--do not notice the significance of student errors. Brown observed this in his investigation of "procedural bugs" subjects make:

> A common assumption among teachers is that students do not follow procedures very well and that erratic behaviour is the primary cause of a student's inability to perform each step correctly. Our experience has been that students are remarkably competent procedure followers, but that they often follow the wrong procedures.[46]

A possible danger in analyzing incorrect responses is that it is tempting to attribute randomness to procedures that defy classification because of deficiences in apparent planning. For example, this is done by Watson who lists one of Newman's subcategories under process skills as "random response".[47] Brown reflects on this problem: "All too often, behaviour that appears to be random has a simple, intelligent and complete underlying explanation."[48] From another perspective, some researchers link randomness with trial-and-error strategies where Mayer's definition of trial-and-error as "respond[ing] in an almost random fashion without much evidence at all of any 'thinking'."[49] as typical. A caution on viewing trial-and-error in this way is stated by Simon: ". . . trial and error is not completely random or blind; it is in fact rather highly selective."[50]

In a general review of error research, Radatz makes much the same statement:

> According to the present state of error research, student errors: are causally determined and very often systematic; . . . can be analyzed and described as error techinques; . . . ."[51]

Radatz then goes on to explain the reasons for making an error analysis:

> Analyzing student errors may reveal the faulty problem-solving process and provide information on the understanding of and the attitudes toward mathematical problems.[52]

He also warns against an oversimplification in error analysis, noting that:

> In classifying errors according to pupils' individual difficulties one should . . . acknowledge that errors are also a function of other variables. . . . Errors in the learning of mathematics are the result of very complex processes.[53]

In Watson's work on investigating errors in beginning mathematicians he makes reference to Newman's classification scheme: "The model postulates a sequence of steps, and failures at different stages are shown as different errors."[54] Newman's classification is described as a hierarchy, and errors are associated with: reading ability; comprehension; transformation (i.e., can the pupil select the mathematical processes needed); process skills (i.e., can the pupil perform the mathematical operations necessary for the task); and encoding (i.e., can the pupil write the answer in an acceptable form). Radatz himself associates errors with language difficulties, deficient mastery of prerequisite skills, facts and concepts and finally incorrect associations or rigidity of thinking. He sees the last cause as resulting when "some aspects of content or of solution process persists in the mind, inhibiting the processing of new information."[55]

The importance generally of errors is also specified by Radatz when he states: ". . . most errors derive from the pupils' use of very individual and sensible rules or strategies."[56] This research will focus on all processes subjects use to solve problems, with erroneous stategies viewed as just as characteristic and noteworthy as successful procedures.

The Methods of Obtaining Information on Problem Solving

The method for gathering the data in this research is through clinical interviews of subjects. The method used is described by Opper as originally developed by Piaget in his attempts "to [explore] the thought processes of children of different ages."[57] Essentially, in Opper's description, an "experiment" involving a concrete operation is presented to the subject, usually a child, by an interviewer. After the child has performed some designated manipulation the

> . . . interviewer asks a series of related questions which are aimed at leading the child to predict, observe and explain the results of the manipulations performed on the concrete objects.[58]

To systematize the process, a slightly different version of the clinical method uses "standard problems" and "identical questions" to form the experiment. Opper then clearly stipulates:

> . . . once having presented these identical situations and questions, the interviewer may then conduct the experiment as he deems appropriate.[59]

This description characterizes the inteviewer as quite active in the clinical process:

> The inteviewer can also pursue a line of response that appears to be interesting for his study as for instance, when a child provides an unusual but relevant answer.[60]

Opper feels that this technique "may lead to a better understanding of what the child really thinks about the problem under consideration."[61]

There is a difference between the clinical method of analyzing and what could be characterized as the statistical method. Some researchers distinguish these two approaches as nonexperimental and experimental. Swanson et al. discussed the validity, reliability and diagnosis of the clinical interview. Though they reported there was a heterogeneous collection of techniques they still found

> . . . a common dependence upon the verbal reflections of the subject, on contingent questioning, and on the creativity of the individual interviewer.[62]

A basis for viewing the clinical interview as nonexperimental was established by them "since reflection, contingency, and creativity are unorthodox from the perspective of the standardized, objective, replicable scientific method."[63] Lester refers to a possible cause of the heterogeneous nature of the clinical techniques used by different researchers:

> . . . in the absence of specific training in clinical data collection and analysis procedures, investigators employing such procedures develop their expertise on the job.[64]

Certainly, in the present study, this criticism is relevant.

Ginsberg casts some doubt on the nonexperimental label that clinical interviewing has when he notes that "the clinical method has received little critical examination ", and that "as a consequence of the lack of critical examination, we understand little of the clinical interview."[65] The method used by this investigator also has certain common traits with that employed by Krutetskii. Krutetskii outlined his general technique as follows:

> Thus we judged the actual process of thinking while experimental problems were being solved: by an objective record of the solution, the nature of the operations, diagrams, and sketches done by the examinee; by the record of the verbally revealed process of reflection during problem solving; by the nature of the answers to questions during the solution . . . .[66]

For the purposes of this research it is also important to note that Krutetskii states that his method makes use of "material from discussion about the solution after [my emphasis] its completion."[67] Such retrospection, which Lester defines as "analysis by the subject after the problem is solved . . . [involving] a discussion between subject and interviewer,"[68] will be an important aspect of this work on problem solving and requires further development.

In Polya's description of the steps in problem solving, he lists his fourth and final step as "looking back" which he sees as having the following value in the problem solving process:

> By looking back at the completed solution, by reconsidering and reexamining the result and the path that led to it, they [students] could consolidate their knowledge and develop their ability to solve problems.[69]

Retrospection, from this viewpoint, is actually a part of the problem solving process, and one that is helpful to the subject in developing his problem solving ability.

Krutetskii makes reference to work by L. Johannot where

> . . . a problem is given to an adolescent and he is left in private with it in order to search for a solution. Once a solution is obtained, the experimenter begins an open discussion with him, for the purpose of following his train of thought--of disclosing which methods he used and what hindered him from solving the problem correctly.[70]

There is a good degree of resemblance between this description of Johannot's work and the methods employed in this research. Krutetskii does make a specific criticism of this retrospective technique, claiming that "perspicacious children often guess what the experimenter wants of them and give the answer that is expected."[71] A criticism of the clinical method generally is made by Nisbett and Wilson. Based on their studies, they state that there is an indication "that the accuracy of subject reports about higher order mental processes may be very low."[72] Nisbett and Wilson go further in their analysis of the introspective technique, observing that "the evidence suggests that people's erroneous reports about their cognitive processes are not capricious or haphazard, but instead are regular and systematic."[73] The latter statement implies that there is probably some organizing principle behind erroneous reports.

The authors mentioned offer the following proposal to explain the observed behaviours:

> . . . when people are asked to report how a particular stimulus influenced a particular response, they do so not by consulting a memory of the mediating process, but by applying or generating causal theories about the effects of that type of stimulus on that type of response.[74]

The criticism just described must be carefully considered when analyzing the data of this research; though it will be assumed that, to a degree, and with certain reservations, subjects can give a reasonable representation of the processes they use to solve problems through their verbalizations. Swanson et al. outline the perspective that will be taken:

> . . . the fact remains that over a wide range of conditions and situations people are reasonably good at telling what they believe and think.[75]

This paper will be primarily concerned with the processes used to solve problems and Swanson et al. state the following in that regard:

> . . . if stages or steps taken in solving a problem are the aspects of processing that are of interest, verbal reports may be a valuable source of information.[76]

The reasoning behind the choice of the clinical method also requires some clarification. It is thought by some researchers that a "true" investigation of problem solving requires the clinical method. Lester notes the importance of Krutetskii's work generally and refers to his "nonexperimental" method specifically:

> Krutetskii's work is generally not quantitative and does not adhere to standard tenets of experimental design. However, his use of clinical interviews and nonexperimental procedures

> may prove to be the aspects of his investigations that will have the biggest impact on mathematical problem-solving research.[77]

Easley specifies some of the rationale behind the clinical method:

> In the light of the fragments of interviews we have analyzed, it seems absurd to pretend that one knows how to measure cognitive competences by administering standardized lists of questions when no validating clinical interviews . . . have been published.[78]

Zammarelli, in his research on the effects of play on concept formation, illustrates a supportive role the clinical interview can take:

> If the pupil succeeded in the test [of associating certain coloured blocks], he was then interviewed in order to confirm or contradict the available information as to the method he used to accomplish the task.[79]

It should be noted that Zammarelli is using a retrospective technique also. Thompson, in reviewing the work of Snow, reiterates the following point:

> Statistical techniques may be legitimately applied in areas such as agriculture where the subject is passive, but they are inappropriate to education where subjects are "active, flexible, adaptive processors of information available in a probabilistic, partially redundant environment".[80]

The link between clinical interviews and process analysis is established by Lester: "Our concern for the process used during problem solving necessitates the use of qualitative methods in addition to quantitative ones."[81] This research will analyze problem solving from a qualitative perspective.

Research on Problem Solving with Calculators

The research done on calculators generally is relevant to this research. In the Fifth Annual State-of-the-Art review on the use of calculators, Suydham states that research in calculators is "one of the largest bodies of research on any topic or material in mathematics education."[82] An extensive review of that research was made by Roberts in his study of the impact of electronic calculators on educational performance. He found that in his review of "thirty-four empiricle studies at the elementary, secondary, and college level", that "In the majority of studies (71 per cent) the basic research design utilized was the pretest-posttest arrangement."[83] In comparing elementary, secondary and college level studies he noted that computational advantages were found when the calculator was introduced at the elementary level, though "in only one study of the five investigating concepts were there conceptual benefits due to calculator usage "[84] and similar results were reported for the secondary level studies. At the college level, there was evidence [in seven of eight studies] that the calculator provided computational benefits and "There was some support [two of the three that studied concepts] for the conceptual impact of calculator use."[85] In categorizing the calculator research studies, the following criterion was used in distinguishing between a computational and a conceptual study:

> It must be emphasized that the line between Cm [i.e., computational] and Cn [i.e., conceptual] was somewhat arbitrary . . . . However, the essential difference was that if the posttest involved routine calculations as the primary task, it was considered to be Cm; if it emphasized interpretive exercises like word problems, it was considered to be Cn.[86]

Roberts makes two important criticisms of much of the research he reviewed:

> One problem encountered . . . . was the admission by some of the C [i.e., control] students of having used calculators outside the classrooms during the study.[87]

Bell states:

> . . . that by 1980 something like 80 million calculators may be in use in the United States, and that after 1980 perhaps 20 million calculators will be bought each year . . . .[88]

This seems to cast doubt on the results of the majority of studies listed by Roberts, which use an experimental group that receives a "calculator treatment" and a control group that receives none.

Another important characteristic of the majority of calculator research is a focus on achievement and attitude testing. A typical research item would be that by West[89] where the learning of multiplication facts is tested in a group that uses the calculator and in another group that uses paper and pencil. This research is typical in many significant ways; the nature of the problems used in the research which are basically computational exercises; the lack of any attempt to integrate the calculator meaningfully into the instruction of the calculator group; the use of statistical methods to test the hypotheses advanced; the lack of any testing of the conceptual ideas of the different groups; and the calculator group not being allowed the use of calculators on the posttest. These common features are all good bases on which to criticize the majority of calculator research. Linn makes an excellent point in research design using pre- and posttests:

> Even where the same test (or parallel forms) is used as the pre- and postmeasures, it is sometimes the case that different constructs are measured at the two points in time. For example, an item which measures problem-solving skill at one point in time may measure memory at a later point in time.[90]

Linn also notes that "Major disadvantages in the use of change scores are that they tend to conceal conceptual difficulties [i.e., in experimental design] and they can give misleading results."[91] A very significant comment about testing is made by Bell:

> Clearly most standarized tests (and much of classroom-based achievement testing) will be meaningless if children habitually use calculators in school and, as would seem natural, attack test problems with the same tools they use in learning.[92]

Bell makes a noteworthy comment generally about the educational use of calculators:

> Whatever schools do or do not do there will be increasing automation of calculation in society at large. Normal evolution of school practice could lead in the years . . . to sound and fully acceptable accomodation [sic] of the power presented by calculators. But this may not happen.[93]

This last comment could be linked to what could be described as the motivation behind the majority of testing and research done on the effects of calculators in schools, especially elementary classrooms.

Werner makes reference to the attitude that has forced calculator advocates into a defensive position:

> Perhaps the greatest resistance to using calculators in the elementary school comes from those adults, including some administrators and classroom teachers, who equate mathematical competence with computational skills.[94]

Yvon found very much the same results in his survey of "feelings of parents and elementary teachers toward calculator usage in schools."[95] A very interesting finding was that "Parents who had a member of the family using a calculator in his or her job disagreed with (the) statement: 'Skills with calculators will be essential to children's future success . . . .'"[96] It would seem logical that such parents should see a physical, applied use for the calculator and so its importance. A possible rationale for the resistance many teachers have to the use of calculators by students is offered by Reynolds:

> It is not always easy for teachers to stand aside and appreciate the abstract nature of numbers. Over the years, numbers become real and familiar, making it easy for the mature adult to forget the long process by which he learnt the concepts of place value and the meaning of the number symbols. Teachers who learnt about numbers without the aid of a calculator may find it difficult to appreciate the role that a calculator can have in forming a child's understanding.[97]

Werner does not see the school and the curriculum as it has developed as adapting to the fact that "automated calculation in society will continue to increase . . . [and] the rate traditionally associated with curriculum change is too slow to cope with the issues at hand."[98] When calculator research articles do include examples of the questions used in their research the exercise (as opposed to the problem) nature of the questions used calls the conclusions made into question. As examples of the computational nature of most research questions, consider the following:

> You need to buy 30 cans of soda for 29 cents each or a carton of six sodas for $1.35. How much do you save by buying the sodas for $1.35? How much do you save by buying the sodas by the carton?[99]

> Grade 7: A photograph of 50 spectators at a football game showed 30 men and 20 women. If there were 10,000 people at the game, about how many would be women?[100]

> Problem Solving: An aluminum recycling plant pays 15 cents for each pound of aluminum that you bring in for recycling. If you bring in 14.6 pounds of aluminum, how much should you receive?[101]

> . . . 4 items ask the student to change a fraction to higher terms or convert a mixed numeral to an improper fraction; 8 items ask questions about locating fraction points on a number line [as a description of the Stanford Diagnostic Test (SDAT) used in Hector].[102]

It is also important to note that in order to apply statistics to their research, many investigators approach their research quantitatively. Szetela tried to

> . . . determine if students who use calculators in story problems tend to try more problems, to use more correct operations, and to obtain more correct answers than students who use paper and pencil only.[103]

Cresswell[104] used achievement tests to test various null hypotheses. Leechford used the California Test of Basic Skills "as a standard tool to measure the students' achievement in computation and problem solving."[105] Wheatley used Days'[106] checklist coding system of processes to compare "the <u>total</u> [my emphasis] number of processes used."[107] Wheatley also noted the amount of time computing and the time spent analyzing the problem by the calculator and noncalculator groups. Roberts[108] reports a study by Laursen where, as well as applying statistics to achievement and attitude, he saw attendance (my emphasis) as a quantitative factor worth investigating.[109]

Bitter investigated teacher attitudes and though he found that in his workshop with elementary teachers that "it appears that teacher attitude toward the calculator in the classroom can be improved through the use of an inservice", he also noted initially that "teacher reluctance to allow (calculator) use in the classroom may negatively affect educational benefits desired from pocket calculators."[110] Shumway et al. investigated the possible negative effects calculators might have on students and made the following conclusion:

> Children grow significantly on basic fact and mathematics achievement tests taken without the use of calculators regardless of whether or not calculators were used during instruction.[111]

It should be noted that Shumway et al. were researching "the general public view that calculator use in schools would cause serious debilitation of students' mathematical achievement."[112]

Needed Research on Calculators: The Process Approach

A very significant criticism of calculator research is that it has avoided, generally, any analysis of the processes involved in problem solving.

The emphasis on standardized testing to the exclusion of any investigation of the processes is based on the rationale explained by Shumway et al.:

> Because current evaluation of school mathematics programs by policy making groups such as administrators, school boards, and parents is almost always made with standardized tests, standardized testing . . . needed to be a major component of the testing.[113]

Perhaps the demand for quantitative information has slanted the research because decisions based on quantity rather than quality seem more objective. The need for a qualitative investigation of problem solving has been recognized by researchers and reviewers of research. Moursund, in his review of research of calculator effects on mathematical abilities, states that "Few researchers have dealt specifically with the problem of examining calculator effects on the process dimension of problem solving."[114] Roberts notes the lack of studies on the calculator's impact on concept formation and remarks that "Until investigators see calculators as a strategy for solving problems, the concept-formation benefits of calculators will still be an unresolved issue."[115] Lowerre et al. make an association between standardized testing and the nature of problems solved with a calculator:

> Gains in areas such as understanding of negative numbers, comprehension of flow charts, real world problem solving skills, and actual ability to compute with the hand-held calculator were not measured at all. Nor will they be measured in other standardized tests generally in use.[116]

The limitations of research based on standardized testing is clearly recognized by the National Advisory Committee on Mathematics Education (NACOME).

In their list of recommendations for research and evaluation they state:

> There should be development and research focused on new approaches to achievement testing. Testing policy should be analyzed and appropriately changed in response to these results.[117]

The same committee also sees a need for clinical trials as one of the research methods. Mayer makes a similar criticism of calculator research:

> . . . the research community has been very slow in providing information that would be useful in this impending calculator-curriculum revolution. For example, most experimental studies have been concerned with whether using calculators in the classroom affects overall achievement and/or attitude in mathematics.[118]

A reason that investigators have not researched the process area of problem solving, using the calculator as a tool for the research, may be that they do not see past the calculator as a computational device. This restricted view of the calculator is mentioned in the work of Judd:

> The calculator can, however, make it possible for students who understand the numbers and relationships they're working with to focus on the real essence of problem solving, no longer overburdened by the tedium of laborious computations.[119]

Koop also, though mentioning calculators and problem solving together, clearly sees no direct relation between the two: "If calculators are used for the computational work, problem solving activities can receive more emphasis in the arithmetic course."[120]

NACOME also expresses the idea that calculations make problem solving easier, but again without being involved in the process of problem solving:

> . . . the committee expressed the belief that calculators would allow students to feel the power of mathematics and use time formerly spent on long, complicated computations to explore a greater variety of mathematical concepts.[121]

A similar function of the calculator is determined by Leechford when one of the conclusions of his research is that "The use of hand-held calculators in instruction allowed students to focus more on problem solving situations."[122] He also states that:

> . . . the use of the hand-held calculator allows students to focus on important mathematical concepts and processes, while minimizing the routine calculations which often interfere with the learning of mathematics.[123]

The research offered here will take a different view of the calculator's role in problem solving: the processes used by students solving problems <u>with</u> a calculator will be analyzed with the perspective that interaction with the calculator is a part of the solution process.

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CHAPTER II

THE CALCULATOR AND PROBLEM SOLVING

The Motivation

In the summer of 1982 I bought a printing scientific calculator and shortly after that a book by Wallace Judd.[1] The problem that follows is the first one that I found that was not of the simple computational nature:

> How to make your calculator count. Some people say that machines are dumb--but your calculator can count. To show these machine-snubbing snobs the potential of your mini-calculator simply: making sure your machine is on constant, putting in 10,000,000, multiplying by 1.0000001, pressing equals lots of times. Your machine will "count" with the numbers on the right-hand side!!! Can you figure out how to make your machine count by 5's? by 7's? by 3's? by any number? It can![2]

When this problem is tried on a printing calculator the actual counting is displayed in hard copy as a sequence of numbers. The really curious thing is to figure out why the calcalutor generates the arithmetic progression it does. From a problem solving point of view the significant aspect was that the calculator was central to the existence of the problem and the sequence was created because of "something" the calculator did to the factors involved. Different friends and relatives tried the problem given and the printouts were collected and analyzed. In a few cases there were definite similarities in the sequences of operations that people tried. An idea slowly began to emerge: were there certain common processes that people used to solve this problem and would mathematics students show similar results?

As a mathematics teacher I had experienced the "intrusion" of calculators into mathematics at different grade levels. Students were told not to bring calculators to class and that homework should not be attempted with calculators. There was something illegitimate about solutions based on calculator assistance. However, it soon became apparent that students on their own time were not following instructions and that good and poor students alike had ready access to calculators. Faced with a fait accompli situation I decided to not only allow calculators for homework, but encouraged students to bring their calculators to class. When solutions were given to students in class, I went through the calculator operations involved, developing a flow chart notation that was standard for all my classes. Once the use of calculators was "legal", students began to demonstrate their ideas on how to do problems with the calculator in home and class assignments. It soon became very clear that the calculator made computation easier but gave no indication of how the problem should be done. And yet, if students would look at answers generated by the calculator, even incorrect ones, it was clear to me that the calculator could be used as an instructional aid and not just as a calculater.

My classroom experience, coupled with the nonroutine problems of Judd's[3] book led to the design of a pilot study and a thesis proposal.

The Pilot Study

The pilot study required more nonstandard questions than I could find in Judd's[4] book so an extensive search was made through recreational mathematics texts, and the like, to find problems that either offered "puzzles" that utilized the calculator or could be adapted to be used with the calculator. This type of nonroutine calculator problem is not very common, particularly because I wanted problems that did not require an extensive prior mathematical knowledge. Another few questions were found in Rade and Kaufman's book Adventures With Your Pocket Calculator[5] and a few others were generated by just "playing" with the calculator and analyzing the printout obtained. Several hypotheses were tested: mainly whether the calculator printout supported by the taped protocol would indicate any of the processes the subject used to solve the problem; and whether the subjects would generate their own problems based on the questions they had done. A problem set of seven questions was tried on four subjects. At first subjects were asked to talk out loud while they worked with the calculator, but poor results led to a method where the subject worked initially alone and contacted the interviewer when the problem was solved or a definite, protracted block in the solution process was reached. A common introduction was given to each subject about the calculator and its functioning. A problem was noted very early in the research: if the subject entered a number it was possible to remove the entry from the display without having the printout record the removing process. Students were instructed to press the print key before they cleared a misentry. This was then entered on the printout by the calculator and indicated by a single asterisk beside the number on the printout, thus making misentries clearly identifiable. No preliminary set of exercises was given the subjects, though an example calculation was always done for the subject when he was using the calculator for the very first time. The calculator printout was then discussed with the subject in segments that the subject was previously shown to make on the printout by pressing the clear button of the calculator. The retrospective analysis was quite probing with supplementary (i.e., related) questions posed as the interview evolved. An attempt was made to use the printout as a basis for discussion with calculator work discussed in the same sequence as it was displayed.

Based on the results of the pilot study several changes were made. Three questions were dropped entirely and one question was revised. The questions did generate a good deal of calculator activity by the subjects and seemed to be accepted by the subjects as interesting and motivating. A protocol analysis sheet was developed that enabled a relating of printout to protocol to time. There was a clear indication that the thesis was feasible in that the printouts alone often gave a good indication of the processes the subject was attempting and, in some cases, the retrospective interview gave further clarification of the solution process illustrated by the printout. For example, consider the method Billy uses to solve problem 1 (note: each problem is described in chapter III). From the printout it is clear that he has reversed the operation from multiplication to division as well as changed the size of the second number. The printout clearly indicates the strategy (actually it appears that there are two) employed by the subject:

```
10000000. /
1.0000003 =
 9999997. **
 9999997. =
 9999994. **
 9999994. M+
```

It should also be noted that notation used by the calculator helps in spotting the actual sequence as the result found after a calculation, in Billy's case a series of quotients, is indicated by having double asterisks placed to the right of each term.

As another example, there is Tim's solution of the reducing sequence where he had to multiply two numbers to get the sequence given. From the printout by itself, it is obvious that Tim has been successful, but what led him to select the factors he did?

If we go to the clinical interview, his reasoning becomes clear as can be judged from the segment of transcript that follows where I precedes comments by the interviewer and T those by the subject:

I: You got the problem, didn't you? OK. So tell me how you figured it out. What did you do? I can see what you have there (i.e., on the printout). You got it. Let's get it out of there (advancing and removing the printout).

T: I just went, I put that many 9's in and I figured well, nothing can times it that, that I can think of, that did that, so I figured maybe that (i.e., all the 9's in the second factor) was behind (i.e., to the right of) the decimal point and you just times it by 10 million.

I: Say it again (i.e., trying to clarify ostensive language); I don't follow you.

T: OK. I put, I figured well, maybe those were all behind, like maybe that (i.e., the first term of the sequence-9999999) was point 999 . . .

I: Oh yeah, I see.

T: Times it (i.e., 9999999) by that 10 million, I knew that would bring it (i.e., the decimal point) out to the front, and then by fluke it might turn into . . .

I: OK. That's how you got the first one (i.e., the first starred number of the sequence, 9999999), but how did you figure out that that might give you the second one (i.e., the second term of the sequence, 9999998)?

I: I just chanced it.

A copy of the printout of Tim's work now follows:

```
10000000. x
0.9999999 =
 9999999. **
 9999999. =
 9999998. **
 9999998. =
 9999997. **
 9999997. =
 9999996. **
 9999996. =
 9999995. **
 9999995. =
 9999994. **
 9999994. =
 9999993. **
 9999993. =
 9999992. **
```

From the protocol it now becomes clear that Tim's strategy was to keep the same initial factor (i.e., 10 million) as in the previous examples and to make the second factor a decimal form of the first term of the sequence, that is he changed 9999999 to 0.9999999. It also seems highly likely that he worked to generate the first term of the wanted sequence and that his plan did not really extend to deriving the other terms of the seqence. It would be difficult to determine this from the printout alone, and the protocol here helps clarify the strategy used.

The proposal also served the purpose of helping the interviewer to develop a questioning method which was not as interjective and undirected as became apparent at the onset of the research. It was also noted that there were many examples of ostensive language in the protocols which made analysis after the interview quite difficult. The interviewer developed an awareness for ostensive language and learned to work on clarification by subjects.

Most subjects seemed quite at ease with the calculator even though the keyboard indicated functions they would definitely not know very much about. Subjects also demonstrated good ability to remember what they had done based on their review of the steps indicated on the calculator printout.

Related Calculator Research

From the review of calculator research previously mentioned there is a distinct difference between the methods and goals of this research and the general body of calculator research. However, there is some support of this research to be gained in the calculator research generally and from articles in the publications of mathematical education organizations, like the Mathematics Teacher, the Arithmetic Teacher, Mathematics in School and Mathematics Teaching. Schult, in an appendix to Moursund's book, describes research by McClintock where "a combined experiment and clinical-behavioural research design incorporated three treatment groups. Each received differential problem solving and calculator instruction."[6] The resemblance between McClintock's calculator method and the research here is noted in the following: "Protocols of individual preinterviews and postinterviews utilizing the Think-Aloud Technique were analyzed for process differences."[7] It should be noted, however, that McClintock was investigating problem solving instruction that incorporated the use of calculators as it facilitated achievement in algebra. Lowerre et al. refer to the method just described as "a guided discovery method [that] can be used because the children can quickly try many examples and detect patterns and algorithms."[8] Behr studied children's ability to abstract with a calculator and found that "there is evidence that the child has internalized the notion that the calculator will maintain a count of a set--even over a period of time, and while the child's attention was diverted elsewhere."[9] Research like this, which investigates the relationship between the calculator and mental constructs, illustrates a valuable direction of enquiry. Judd clearly indicated the different types of problems calculators could be applied to, as for example, the following where he describes the "Reversie" game:

> One person puts in a number that repeats two digits, such as 353535 (the smaller of the two digits comes first). The other person then tries to put in a number that can be added to 353535 to get 535353--in this case, 181818. . . . Older students might be able to figure out how to use the larger digit first, as in 626262.[10]

It was a consideration of problems like this that led to the research described here.

Henry[11] gives some examples where the results of calculations made on the calculator can be used to determine that a condition is sufficient in determining if a positive integer is prime. The results are illustrated in a table so that students can see certain patterns, thus illustrating the value of a printing calculator. Lichtenberg uses the calculator to develop mathematical concepts too but at a lower level when he asks "what number multiplied by itself gives 46?"[12] and then directs the student to use the calculator to find approximations. The same ideas could be extended in asking students to find the cube root of 1048.

Wheatley foresees the use of the calculator in problem solving when she notes that "different strategies might be feasible with a calculator, for example, iterative procedures or successive approximation."[13] Wheatley also made one of the few investigations that attempted to distinguish between calculator and pencil-and-paper groups in terms of the processes employed in problem solving. Though Wheatley offers the standard rationale that "if [this] computation could be performed on a calculator, then he or she [i.e., the student] might be able to focus more on strategies and less on computation", she also envisions a higher function for the calculator in stating "Furthermore, different strategies might be feasible with a calculator."[14] One of the questions investigated by Wheatley's research reflects her perspective, namely "When using a calculator, do elementary school pupils employ a wider range of problem solving processes?"[15] More will be made later of Wheatley's research as it was important in devising a method for analyzing processes used by subjects in this research.

Shult also researched the problem solving processes used by calculator and pencil and paper groups. He "used a group of 30 average sixth-grade arithmetic students to determine calculator effects on process and product dimensions of problem solving."[16] He concluded that "calculators do not appear to produce significant differences in the processes and strategies employed to solve word problems."[17] McClintock is another researcher who is reported "to have dealt specifically with the problem of examining calculator effects on the process dimension of problem solving." McClintock used the Thinking-Aloud technique and analyzed the protocols of individual preinterviews and postinterviews for process differences. Shult reports: "McClintock concluded that intensive problem-solving instruction which incorporates the use of calculators facilitates achievement in algebra."[18]

However, there is very little research that investigates the processes used by students when they attempt legitimate problems with a calculator. One of the few exceptions to this statement is the work of Mayer where he attempts to "describe users knowledge of how a four-function calculator operates" by searching for "differences among novices and experts in their conceptions of what goes on inside the calculator for various sequences of button presses."

He "provide[d] a framework for describing users' knowledge of calculator language, [i.e., users' intuitions concerning the underlying logic of a simple four-function calculator]."[19] Mayer's perspective of the calculator's function is considerably different from that of most researchers in saying "calculators provide the first step in the development of a user's computer literacy--the understanding of how to interact with computational machines."[20] It is interesting to conjecture what calculator would be available if more research were directed towards verifying the following conclusion made by Mayer:

> . . . the present study suggests that we cannot rely on choosing an "intuitive" calculator as the solution to all our problems, because even the best fitting calculator . . . is considerably different in performance from what our subjects expect.[21]

Most of the research available is limited in its application. Research that indicates students suffer no "damage" from calculator use is not as useful as the following observation by Mayer:

> The most obvious recommendation is to choose a calculator that works the way that the user thinks a calculator should work, i.e., match the characteristics of the machine to the intuitions of the users.[22]

Research that tests this recommendation would be invaluable to educators.

Teachers' Ideas: the Untapped Resource

Educators, principally mathematics teachers, have been forced to face the introduction of calculators into classrooms.

Several innovative teachers, consultants and advisers, have submitted suggested activities and problems that use the calculator in the problem solving process. It is unfortunate that more research does not use such questions as instruments to analyze the solution processes of calculator users. Some of the more innovative problems and suggestions will now be described.

Meyer describes a creative use of the calculator as a teaching aid:

> I began by using a series of activities designed to help students learn the function of each key. The + key was easy, so was the x key; but the tricky − key sometimes gave you a "−" in front of your answer. We explored to find why this happened and discovered that zero was not the beginning of the number line--there were negative numbers.[23]

The role of the calculator in this elementary school teacher's exercise is not that of a simple computational device. Wheatley et al. describe the following calculator activity:

> Place-value concepts can also be developed naturally with calculators. With the 387 entered, a child is asked "What can you subtract from 387 to give a zero where the 8 is?" Children gain knowledge of "tens place" when they find that 80 and not 8 will produce the desired result.[24]

Would it not be possible to use the question just given as a basis for investigating the child's concept of place value, by allowing the subject to experiment with a calculator under clinical interview conditions? Reynolds describes an iterative use of the calculator to help students understand the process of taking the square root:

> Calculators are good at interative processes--perhaps not so good as computers--but they do demonstrate the process very well . . . . For example, to find the square root of 26, we start with an approximation . . . . Divide 26 by 5. Since 5

> is too small, 26 / 5 = 5.2 is too large. Now repeat the process using the mean of 5 (on one side) and 5.2 (on the other). Hence the square root lies between 5.1 and 26 / 5.1.

This method can be continued and Reynolds makes the following noteworthy observation:

> . . . we could maintain that it [i.e., the calculator iterative process described] provides a much better understanding of how a square root is calculated than was ever the case when using 4 figure tables--or even with the older extraction process taught earlier this century.[25]

It is interesting to note that in the pilot study performed there was a question tried with several students using this iterative process to find the cube root of a number when the square root process was done as an example before the student. There were several difficulties in the structure of the problem that led to its exclusion from the research project.

Cox[26] suggests a question that has a basis in the fact that there is a limit to the number of digits the calculator will display. For example, he recommends students be directed to perform the calculation 12345 x 12345. Depending on the displayed answer (and he is correct in stating that calculators are programmed to handle overload differently) he suggests students be asked to devise a method for finding the solution to this problem using the calculator. Problems like this definitely go beyond the simple computation most calculator research uses for testing. Haylock describes some interesting problems he derived using a malfunctioning calculator, where pressing the zero button displayed pi (i.e., 3.14). He then "explore[d] with a group of students various ways of getting around this minor defect." What would he have discovered if he had made this exploration under clinical interview conditions? A few of the problems he investigated become legitimate problems in the light of the malfunction specified: "How would you display 380? 308 . . . ? Can you develop an algorithm for displaying any integer?"[27] One of the questions in the pilot study, and the main research, uses the same idea of a malfunctioning key to generate a nonroutine problem. Johnson criticizes the problem if it is left at this point with a simple question as to whether there is a pattern and it can be completed by the student. He notes:

> While this might be interesting, the situation as given does not go anywhere. The pattern is obvious . . . . When the patterns are of such an obvious nature, one needs to ask how the situation can be used to promote mathematical thinking.

If more researchers gave thought to the condition Johnson has made, perhaps the level of subjects' thinking in calculator research would be much higher. Johnson makes the following alteration in the problem just given:

> If the pattern lessons are to be primarily pattern search, then the pupil should be involved in a searching and testing situation, that is he should be required to study the various examples, to hypothesize a relationship; to test or verify the relationship, to modify on the basis of new information . . .[28]

There is a clear need for research generated by teachers themselves that utilizes innovative questions and a clinical method of analysis. This is a point made by Easley.

> It is extremely interesting to speculate what research in mathematics education in this country would become if the mathematical understandings of teachers and the learning problems they identify in their pupils were to be taken as the starting point for detailed clinical studies of teacher-pupil

interactions in order to arrive at helpful procedures, devices and materials.[29]

It is disappointing to note that there has been no concentrated attempt to use the wealth of calculator questions in the mathematics teacher literature as a basis for research that investigates how students solve such conceptually-oriented questions. In the most comprehesive review of calculator research found, Roberts specifies certain studies that he did not review:

> "It is important to emphasize that the present review does not take into account a variety of other empirical work such as surveys on calculator use and other less formal investigative activities."[30]

Perhaps by "less formal" Roberts means studies that use methods like the clinical interview. Roberts also raises an issue that has to do with the dissemination of research results generally: "Because most of the research in this area [i.e., calculator research] has been through the medium of doctoral dissertations (only a few of which have been published), the dissemination of findings has been very slow . . . ."[31]

The Nature of This Research

This chapter will close with a consideration of the general problem examined and the procedures and analysis that will be followed. From the ideas that generated the pilot study and the research itself, the conclusion was formed that there were patterns in the calculator printouts of subjects solving nonroutine calculator problems, both in terms of successful and unsuccessful strategies. Coupled with the results of the protocol, it was determined that the calculator operations gave a reasonably accurate representation of the problem solving processes of the individual.

The primary goal of the research will be to characterize each solution attempt based on the printout and protocol support. The second goal will be to determine general strategies, across subjects, applied to a problem in terms of the hard copy (i.e., printout) and protocol analysis. Patterns common in solutions and errors will be isolated in relation to each problem. A third goal will be to develop general categories of solution techniques (methods) that run across problems and are "general calculator methods". For this purpose, a modified form of the checklist coding system employed by Wheatley[32] will be used. Modifications will be made because Wheatley's method was introspective whereas this research uses retrospection and her work compared calculator and noncalculator groups while this research will analyze students using the calculator only. Although the method of analyzing students work certainly does not alter the processes they used, there are limitations as to what processes can be measured when different methods are employed. The research will also focus on some secondary concerns: the affective component of problem solving with a calculator as reflected in the protocols; the analysis of subject-generated problems; the effect of secondary questioning in the retrospective interview; and the general effect of interjective statements on the problem solving process.

It is important to clearly specify what will not be attempted by the research. There will be no comparison of calculator users and nonusers to demonstrate the effects of calculators as determined by achievement or attitude tests. Rather, it will be assumed that calculators are so pervasively used that it is now important to understand the processes used by problem solvers when they use their calculators. No attempt will be made to instruct calculator users on applying calculators to problem solving. Rather, it will be an objective of this research to determine how calculators are used by subjects based totally on what they currently know. No general heuristics or problem solving techniques will be explained to the subjects. No attempt will be made to use questions that are familiar to students because of relations to any curriculum they are using. This research is primarily exploratory, seeking to identify the ways students go about solving the problems given. It should also be added that no cognitive theory will be assumed or tested by this research which could generally be characterized as descriptive of the problem solving processes employed when a calculator is available.

End Notes

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CHAPTER III

THE DESIGN OF THE RESEARCH

Materials Used: The Calculator

The calculator chosen for the research was the Canon FP-10 printing calculator that had recharging capabilities. The reason for choosing this calculator was that it was the lowest-priced scientific (i.e., performed the more complicated operations like trigonometric ratios, finding roots or powers up to 99, etc.) calculator that used regular rolls of paper rather than the thermal paper many printing calculators use. The keyboard had the digits, common operations like addition, the memory functions and the more complex functions of the calculator nicely separated in regions. The following point discussed by Mitchell is relevant because three of the subjects used were grade 8 students, with grade 9 being the year algebra is formally presented in their education system. Mitchell noted: "Some preliminary studies suggest that prealgebra students are not distracted by the presence of trigonometric functions or other unfamiliar keys and notation."[1]

This conclusion is corroborated in research reported in Bell et al. in England where they found that "children have no difficulty in ignoring those keys on the calculator they are not supposed to use, including operations and functions that they do not understand."[2]

There were many design features of the calculator used that aided students. The clear key and clear entry key were in yellow, and the inverse key which gave most keys a second function (except for the digit, common operation, clear, advance paper and print keys) was in a lime green colour. The advance paper key, coloured white, was by itself in the upper left corner of the keyboard and the print key was in the upper right corner, unfortunately the same blue colour as neighbouring keys. A number or letter that indicated the function was engraved on each key. There were two black switches at the top middle and top right of the calculator. The middle switch determined the angle mode of operation: gradients, degrees or radians. The top right switch turned the calculator off and ~~on~~ and also switched the calculator into a printing or nonprinting mode. In all cases, the calculator was plugged in and switched to the printing mode. Subjects were specifically directed not to alter the printing mode of the calculator.

There are some peculiarities of this calculator (and most others have their idiosyncracies too) that need to be noted. First, it is possible to enter some number and then have that removed from the display by pressing the CI (i.e., clear entry) key. Such a change is not registered on the printout even though the calculator is in the print mode. However, if an entry is in error and the P (i.e., print) key is pressed before the CI key, then the error is printed before it is erased from the display. The printout also distinguishes such an entry as it places a single star beside the number as opposed to the usual double star indicating an operation has been performed. The printout below is an example of this feature. In the example, the wanted operation was 54 multiplied by 63 and 32 was mistakenly entered. Operating the CI key removed the 32 and indicated this by a single star. The check that follows shows that the desired operation was performed.

```
  54. x
  32. *
  63. =
3402. **

  54. x
  63. =
3402. **
```

Another feature is that a number is not printed by the calculator until an operation key is pressed. Thus the operator has to review the printout for all past entries but must look at the display for the current entry. Of particular importance is what could be described as a hidden function of the equal key. Whatever operation is pressed immediately before the equal key is pressed will be continued each time the equal key is operated. Consider the example below where the equal key is first used for repeated addition of 1 and then for repeated multiplication by 4.

Pressing the C (i.e., clear) key then equates the equal key with the number 0, as also shown in the example. The equal key can be depressed many times without waiting for the display or printout as the calculator is programmed to store the number of operations in a memory and then act on that memory at a slower rate than the numbers are entered. The calculator will do this twenty times before "error" is displayed and the equal key is no longer operational. Pressing the CI key or the C key will then clear the display and activate the functioning of the equal key again. Switching the calculator off and then back on is indicated by the word "clear" on the printout. This "clear" is not printed in the same vertical line as a "clear" printed because the C key has been depressed, but is slightly to the right. These features are also indicated in the example below.

```
 CLEAR
CLEAR
   1. +
   1. =
   2. **
   2. =
   3. **
   3. =
   4. **
   4. =
   5. **
   5. =
   6. **
   6. x
   4. =
  24. **
  24. =
  96. **
  96. =
 384. **
CLEAR
   0. =
   0. **
```

The memory of the calculator has a direct access feature through the M+ key. Suppose you want to calculate a product and then add a number to that later on. The proper sequence should be to press RM (i.e., recall memory) to make sure it is not holding a number other than 0 and then to press the SM (i.e., store memory) key to store the product in memory. Entering a number and pressing M+ will add that entry to the memory and place the new sum as the number in the memory as can be checked by pressing RM again. This is illustrated in the printout below where the product 6 x 4 is calculated, stored in memory and then 9 is added to the product in memory. Repeated pressing of RM does not affect what is in the memory, but repeated pressing of M+ acts as a constant operation of adding to the memory, again as illustrated in the calculation that follows.

```
  0. RM
  6. x
  4. =
 24. **
 24. SM
  9. +
 24. RM
 24. =
 33. **
 33. M+
 33. M+
 33. M+
123. RM
CLEAR
 24. +
 33. +
 33. +
 33. =
123. **
CLEAR
123. RM
```

It is noteworthy that like most calcalators, the FP-10 prints in blue ink, which is not as easy to read as black ink. The following specifications are important also and are derived from the manual included with the calculator:

Type: handheld, printing scientific calculator with display.
Calculation capacity: $+/- 1.0000000 \times 10^{-98} - 9.9999999 \times 10^{98}$.
Printer: microdot.
Display: fluorescent tube display.
Negative number: time value indication with minus sign.
Element: MOS - LSI.[3]

There is an operating system that the FP-10 uses, namely the Algebraic Operating System (i.e., AOS) which the manual describes in the following way:

The FP-10 uses the Algebraic Operating System. With this system, the calculator automatically judges the calculation priority in arithmetic (=, -, x, /) and power calculations as the numbers are entered, according to the written formula. The order of priority is: 1. a^x
2. x,/
3. +,-
Function calculations are carried out independently so that the priority order will not be affected.[4]

The example below is included to illustrate the order of operations:

5. / [note: the / will always indicate division]
4.
16. x
7. +
3. x
0.5 a^x
60. cos
0.5877852 =
4.1835907 **

In the example just done the following operations were carried out in the following sequence: four was squared; five was divided by the sixteen produced; that answer was multiplied by seven; the cosine of 60 was calculated; that answer was multiplied by one half; the answer to that product was multiplied by three; and that product was added to the product produced by multiplication by seven in step three.

Mayer investigated how subjects use their calculators and found that "the intuitions of any individual user are fairly constant."[5] He further recommended that "a calculator [be chosen] that works the way that the user thinks a calculator should work, i.e., match the characteristics of the machine to the intuitions of the user."[6] The question arises as to whether the algebraic operating system "inherited" when the FP-10 was selected will have a negative effect on subjects who are used to using a calculator that uses an arithmetic operating system (i.e., where the operations are immediately performed as soon as entered, without regard for the operational order of arithmetic)? Most of the cheap, four-function calculators are of this variety. Mitchell noted the following significant comment by a major calculator manufacturer: "One of the criticisms of this system [i.e., AOS or the algebraic operating system] reported by Hewlett-Packard is that some intermediate results are not displayed."[7] If we consider the results illustrated in the previous sample calculation, there now becomes a very important difference between what the calculator does, what is displayed, and what is printed. It is important to recognize that the problem solver must use the display to determine what intermediate results have been calculated and not the printout. For the purposes of this research, where much of the problem solving process is determined by retrospective analysis of the printout, this limitation of the FP-10 is a serious consideration. This problem would be removed according to Mitchell if more calculators were programmed in RPN (i.e., Reverse Polish Notation) as "the calculator user has control over the data after it has been keyed into the calculator."[8] In RPN, data is entered first before any calculations are performed and calculations can be performed one after another on the same data, thus demonstrating intermediate results.

The same question is addressed by Mayer in his study:

> . . . we found no relation between the conceptions of our subjects and the operating systems of their own calculators, nor between the conceptions of our subjects and the amount of time spent each week with a calculator.[9]

This conclusion is curious when another result of Mayer's work is considered:

> . . . most subjects opted for left-to-right evaluation of a chain, although substantial minority of experts [i.e., subjects who had computer language experience] assumed multiplications were carried out before additions.[10]

Mayer's study, however, is different in that subjects did not actually use a calculator, but rather were asked to predict what the calculator (i.e., a hypothetical one) would display given certain operations. Mayer linked these results to the type of calculators subjects said they used. In this study, the pattern question (i.e., problem 7) will explore subjects' conceptions of the order of operations. For the study, a calculator was selected that would follow the AOS order of operations so that students would be given the correct answer when experimenting with the calculator.

Materials Used: The Taperecorder

A tape recorder was selected that was compact and unobstrusive so as not to divert the subjects' attention during the interview. A battery-operated microphone that was very sensitive to quite low noises was selected. The Radio Shack Minisette-9 tape recorder met the replicating standards needed as well as having an inexpensive footpedal that acted as a pause switch. This last feature proved invaluable during transcription. Audiomagnetics 60 minute tape was chosen as it was very inexpensive and readily available. Though 60 minute tape meant that any taping longer than 30 minutes would require a reversing of the cassette and an interruption of the interview, the retrospective method meant that usually no more than 10 or 15 minutes of interviewing required taping. This presented no difficulty during the research.

Paper and Pencil

During the pilot study students were given the calculator and asked to try and solve the problem using the calculator. In a few instances, paper and pencil were made available should subjects wish to make use of them. Surprisingly, a few students did important parts of their work using the paper and pencil. Charlotte Wheatley noted the same phenomenon:

> Observational data revealed that some children chose to compute with paper and pencil when a calculator was available. This may have resulted from dependence on pencil-and-paper computation built up over the years of formal school education.[11]

It is this researcher's opinion that the fact that paper and pencil computations produce hard copy is also worthy of consideration, and was the driving force behind the decision to use a printing calculator in the research. For the purpose of the main research it was decided to remove any temptation to do calculations on paper by not making it available. If the subject made a point of definitely requesting paper and pencil then that was supplied. It is fair to state that for a very great majority of students, this restriction was not observed to be important.

The Problems Used in the Research

The First Problem.

From the pilot study a very good indication was given of the questions that would generate data that had enough "quality" to merit analysis. Four questions were selected and will be briefly reviewed.

Problem 1 is included below exactly as it was presented to subjects:

Problem 1:

Part a: use the calculator to multiply the numbers 10000000 and 1.0000001. Press the equal sign seven times and you will form a sequence of numbers with stars beside them. Describe this sequence to me.

Part b: suppose you wanted to get the pattern of numbers just below this paragraph. Tell me what two numbers you think you would multiply. Now use the calculator to test your ideas.

10000003. **
10000003. =
10000006. **
10000006. =
10000009. **
10000009. =
10000012. **

Part c: use two numbers to form the pattern of numbers below.

9999999. **
9999999. =
9999998. **
9999998. =
9999997. **
9999997. =
9999996. **
9999996. =
9999995. **

Part d: make up your own sequence of numbers.

The first problem consists of four parts which subjects were directed to do in the order specified. The popularity of decimals with the advent of calculators makes this a particularly good question type to research. Davies notes that "Children's reactions to strings of decimals are interesting, varying from 'The calculator has gone wrong' to ignoring the decimal places altogether."[12] Part a of the question just asked the subject to enter two factors and press the equal sign repeatedly. A sequence was produced where, in effect, an arithmetic progression with a common difference of 1 results. This progression is indicated by observing the starred numbers.

The solution to parts a and b are included below:

```
Part a: 10000000. x
        1.0000001 =
        10000001. **
        10000001. =
        10000002. **
        10000002. =
        10000003. **
        10000003. =
        10000004. **
        10000004. =
        10000005. **
        10000005. =
        10000006. **
        10000006. =
        10000007. **

Part b: 10000000. x
        1.0000003 =
        10000003. **
        10000003. =
        10000006. **
        10000006. =
        10000009. **
        10000009. =
        10000012. **
```

Part a of question 1 is to familiarize the subject with a way of generating a series of numbers by using the constant function of the equal key. For students who have not used this calculator before it also familiarizes them with the printout and how to find the results of operations (i.e., by looking for numbers with double stars beside them). This part is a restatement of one of the "tricks" in Judd's[13] book. In his solutions section of the book Judd offers the following: "To make your calculator count by 5's, just multiply 10000000 by 1.0000005. To make it count by any digit, just use that digit as the last one in the multiplier 1.000000x."[14]

It is important to note that eight digits must be used, that is, both factors must have the same number of digits. In this problem Judd is assuming that the calculator is an 8 digit one. Some of the earlier calculators are 6 digit and recently some of the newer ones will display 10 and 12 digits. The question does not change at all if the user tries the problem with any calculator other than an 8 digit display model. In fact, from the examples that follow it is clear that there is a specific relationship between the factors that creates the problem:

Example 1:

```
1000000. x
1.0000001 =
1000001. **
1000001. =
1000002. **
1000002. =
1000003. **
1000003. =
1000004. **
1000004. =
```

Example 2:

```
100000. x
1.00001 =
100001. **
100001. =
100002. **
100002. =
100003. **
100003. =
100004. **
100004. =
```

Example 3:

```
 100000. x
 1.00011 =
 100011. **
 100011. =
 100022. **
 100022. =
 100033. **
 100033. =
100044.01 **
100044.01 =
```

Example 4:

```
   1.0000001 x
   10000000. =
   10000001. **
   10000001. =
1.0000001 14 **
1.0000001 14 =
1.0000001 21 **
1.0000001 21 =
1.0000001 28 **
1.0000001 28 =
```

A review of these examples indicates that the first factor must be a power of 10 and the second factor must be 1 plus a power of 10 that is negative and one less than the first power of ten (i.e., the factors would be 10^{x} and $1 + 10^{-x}$). To create other series, for example adding by 5's, the 1 component of the second factor remains constant but the 10^{-x} is multiplied by 5. Generally then the factors would be 10^{x} and $1 + a10^{-x}$ where a is the common difference wanted. Is there any limit to the value a can assume? Here, with the use of example 3 above, it can be seen that there is a difference between what should happen in theory and what happens in "calculator" mathematics.

It is left to the reader, as a nonroutine problem, to discover what has happened to cause the pattern to break down.

An interesting point to note about part a of question 1 is that subjects may lose track, or not even notice, what the constant factor is because pressing the equal key each time performs repeated multiplication without printing the factors. The solution to part a previously included demonstrates this. It should also be observed that using the equal key to perform repeated multiplication destroys the commutative property for now the order the factors are entered is a determining aspect. Note the pattern produced is different in example 4 above where the order of factors is reversed.

Part b of the question asks the subject to generalize the example in part a and tests whether ~~the~~ subjects can find the iterations needed to produce a different series of numbers. It is significant that Krutetskii, in describing the "component mathematical abilities that arise from the basic characteristics of mathematical thought," the second is listed as

> An ability to generalize mathematical material, to detect what is of chief importance, abstracting oneself from the irrelevant, and to see what is common in what is externally different."[15] Part b is also a variation of Judd's original pattern question where he asks "Can you figure out how to make your machine count by 5's by 7's by 3's? by any number? It can![16]

This part asks for a prediction before the subject tries to generate the pattern to avoid dealing with lucky guesses and postrationalization.

More importantly this prediction can be viewed as a type of problem solving guess, using Polya's definition:

> Guesses of a certain kind deserve to be examined and taken seriously: those which occur to us after we have attentively considered and really understood a problem in which we are genuinely interested.[17]

It should be noted that part b purposely bears a close resemblance to the answer obtained in part a and in fact is an "auxillary" problem formed by changing the data, a form of recombination as specified by Polya.[18] The probable solution to part b would be 10000000 multiplied by 1.0000003 again as illustrated in the solution offered previously.

In part c the problem in part b has had the data changed and the pattern reversed so that it is now a decreasing rather than an increasing pattern. Krutetskii also lists such reversing as the sixth component of mathematical aptitude, describing it as "An ability to reverse a mental process (to transfer from a direct to a reverse train of thought)."[19] Krutetskii also refers to ideas Piaget had:

> He [Piaget] noted the reversibility of thought operations--a distinct mobility of mind in the forward and reverse directions--which related to the internal organization of operations. He [Piaget] indicated that for each thought operation there existed a reverse operation which, proceeding from result of the initial operation, can restore the initial data.[20]

Part c is more a reversal in the sense of process, from increasing to decreasing, and so bears more resemblance to Krutetskii's concept of reversal. That is to say, the reversal required does not request the production of the initial factors, i.e., 10000000 and 1.0000001. It should be noted that in the pilot study this part specified the operation should be multiplication, but based on results of the pilot, the wording was changed, and very much simplified, too, so that no operation was specified.

Part c can be solved using the generalized formula and substituting -1 for a in the factor $(1 + a10^{-x})$ to maintain the operation as multiplication, or, the operation could be changed. Part c also is derived from Judd's puzzle book where he shows the solver how to make the calculator count backwards by 2's.[21] This was converted into the question form as presented in part c. The pattern chosen, a decreasing-by-1 pattern, was chosen to bear a closer resemblance to part a, the pattern the subject had previously encountered.

Part d was a problem created with the idea that much of what a person observes or learns from a previous problem might be reflected in analyzing a problem they create themselves. Parts a through c restrict the solver in terms that the structure of the problem is predetermined. The open-ended aspect of problem generation was very tempting to the investigator. Mayer[22] sees a value in asking students to generated their own questions. It is relevant to state that Polya lists one of the steps in problem solving as a "looking back" stage initiated after the given problem has been solved. Polya gives suggestions of questions that can be asked after the problem has been solved. He states that subjects should be encouraged to use the result and the method of the problem they solved on problems that they essentially build out of the problem they already solved.[23] The calculator can lead to problem generation quite naturally. Johnson, in describing constructive curriculum uses of the calculator, describes an exploratory aspect of calculator activities: ". . . the pupil uses the calculator to generate output with the purpose that the output will demonstrate a concept or relationship . . . ." Johnson goes on further to illustrate how generating is important to this research: ". . . the output [may] assist in solving a mathematical problem."[24]

A difficulty with part d was in phrasing it so the students were given enough structure without being restricted. The following possibilities were considered as introductions to the question:

1. Can you make up a problem (exactly) like this one?
2. What information given in the problem could you change without changing the method of solution?
3. Can you change one of the conditions of the problem in order to make a new problem?
4. Invent a new problem suggested to you by this one.

Part d is a version of the fourth alternative in that students were directed to "make up" their own problem. Part d was then given to subjects with the investigator present and ready to explain the anticipated questions.

The Second Problem.

The second problem was originally problem 4 in the pilot study. Problem 4 was also an adaptation of a problem from Judd's book.[25] The adaptation of the problem as it was presented to subjects is presented below:

Problem 4:

> Sometimes even calculators make mistakes. But at least they are consistent, that is, the same mistake will be continued in different problems. Here are the results from a calculator that was brought into Mr. Joe's fixit shop, with one of the keys not working correctly. Which key isn't working?

Question 1: 41.7 + 5079 = 5120.7

Question 2:

```
 131
 267
 538
 934
 728
────
2358
```

Question 3:

```
   82.6
x  85.91
───────
7044.62
```

Question 4:

```
 307
x 64
────
1228
```

This problem presents some ambiguity for subjects, as indicated in the pilot study, in that they sometimes interpret the question as meaning that the calculator used in the research is not working correctly.

There is also a complication in that subjects need to realize that the malfunctioning key will prevent a certain digit or operation from being entered but that malfunction does not carry over into the display or printout. The error is restricted to the input function of the calculator.

To solve the problem it is first necessary to find what errors exist. One way of doing this is to do all the four problems over again with a calculator that is functioning correctly, like the printing calculator available to subjects. A test of each problem is included below, as an anticipated first procedure made by many subjects:

```
Question 1:   41.7 +
             5079. =
            5120.7 **

Question 2:   131. +
              267. +
              538. +
              934. +
              728. =
             2598. **

Question 3:    82.6 x
              85.91 =
           7096.166 **

Question 4:   307. x
               64. =
            19648. **
```

Assuming this method of solution is attempted the subject now has to make some sense out of the result that question 1 shows no effects of the malfunction while questions 2, 3 and 4 are affected by the malfunction. It should be noticed that the problem merely asks to find the "key that isn't working".

The malfunction can be of two general types: either the malfunctioning key is not operating at all which means depressing it enters no digit; or, its malfunction causes some other digit than the one pressed to be entered. This choice of interpretation will then determine whether the subject tests his solution once he has the malfunctioning key.

It is suggested by some calculator researchers that the calculator promotes certain types of checking procedures. Wheatley found in comparing the processes used in problem solving by calculator users and nonusers that "Subjects in the calculator group traced their problem solving steps three times as often as those in the noncalculator group."[26] Before generalizing this result, two other processes used in the comparison were "checks computation" and "checks conditions" and Wheatley found no significant differences between the two groups in the use of those processes. The type of checking involved in solving this problem will be related to the individual solution path chosen by the specific subject. McClintock found that "the calculator groups demonstrated more stability in the use of arithmetic algorithms and guessing-and-checking techniques than in the use of productive-inferences and algebraic equations processes [than the noncalculator group]."[27] In reviewing his own research, Shult states that he found, in comparing the processes used by calculator and paper and pencil groups, that "subjects who used calculators . . . were as inclined to check answers as subjects who used paper and pencils to solve problems."[28]

The Third Problem.

One of the more interesting ways of looking at an expression like 3x + 7 is to see it as generating many values by substituting in different values for x, multiplying by 3 and adding 7. If one value is given for x and then the value obtained for 3x + 7 is used as the next value for x, and this process continues, a pattern develops. In Krist's recommended uses for calculators in secondary mathematics she makes a suggestion that relates to the question designed here. Krist suggests that teachers "Use calculators as the ultimate function machine," and describes one possible application as follows:

> Program (or have the students program) 10 or 15 calculators so that a student need only enter [sic] a number (x) and initiate the program to evaluate f(x) . . . . The game is to determine the function . . the calculator is executing to produce the final result.[29]

In 3x + 7 if 2 is given as the initial value for x, the following sequence is generated: 2, 13, 46, 145 . . . . . The printout below illustrates what the generation of this pattern would look like:

```
  2. x
  3. +
  7. =
 13. **
 13. x
  3. +
  7. =
 46. **
 46. x
  3. +
  7. =
145. **
```

Problem 7 of the research was generated by a reversing of this thinking. In other words, suppose you were given the sequence of terms with stars beside them and told that there were two operations performed constantly and in the same order, you would then be asked to find the operations and the numbers involved. Continuing the example used, you would be given the numbers 1, 13, 46 and 145 and asked to find the constant operations and numbers, i.e., multiply by 3 as the first operation and add 7 as the second operation. The print feature, indicated by a single star to the right of the entry on the calculator, of the FP-10 calculator can be used to express this question in printout form, as indicated below:

Sequence without operations:

2. *

13. *

46.

145. *

As illustrated above, the printout without the indicated operations can then be used as the body of a question.

It was anticipated, based on the pilot study, that subjects would need to understand the printout and the significance of the spacing indicated between the numbers in the sequence. By looking at the calculation just illustrated, it is possible to tell how many operations are involved (i.e., 2) from the spacing between numbers in the sequence. A form of this question was piloted with Evan, a grade 8 subject of exceptional ability. The pilot question was found to be repetitive and with his assistance modifications were made to the question. In building the research question it was decided that there were too many possible combinations of operations and numbers so one of the operations was given to the subject in the body of the question, slightly masked by including it near the end of the question. The problem used in the research now follows:

Introduction:

below is an example of a printout of a sequence of numbers showing the operations used. The numbers with the stars beside them form the sequence. Beside that printout is the same sequence but with the constant operations missing:

Printout With Operations	Without Operations
8. x	8. *
3. +	
1. =	
25. **	25. *
25. x	
3. +	
1. =	
76. **	76. *
76. x	
3. +	
1. =	
229. **	229. *
229. x	
3. +	
1. =	
688. **	688. *

Question:

use the calculator to find the missing operations and numbers in the sequence below:

Printout With Operations	Without Operations
	140. *
	138. *
	136. *
	/
	3. =
	134. *
	132. *

To help students understand the question and the calculator form of its presentation, an introduction, including an example of a printout with operations and another printout without operations, was preliminarily discussed with subjects. Several of the terms were discussed, including what was meant by a sequence, constant operations and starred numbers. Students were shown a physical way of checking the sequence by sliding the printouts beside each other, and the solution was explained to be correct if the same numbers on the printouts lined up across from each other and they were separated by the same amount of space.

Although there would be very little investigation of the processes involved in solving this question if subjects all used the same methods, this problem was built with an anticipation of what subjects would do. It was reasoned that most subjects would work backwards in the problem, by starting with 134, reversing the division to find the previous term, and then find the constant operation that linked that term with 136. This hypothetical solution is included below:

Hypothetical Solution:

```
134. x
  3. =
402. **
```

The number generated, 402, then presented certain problems as, in designing the question, I had not taken into consideration the effect of the order of operations. The question "accidentally" had become much more challenging, and, since addition or subtraction (assuming they were one of the operations) had been performed last, they should be reversed first. The problem, hypothetically using the reversal strategy, could then be viewed as atypical and not the one most subjects would pursue. Trying to predict what most subjects would do led to the following line of reasoning: the numbers in the sequence all differ by 2, so what two operations taken together, where the second is division by 3, could produce this result? −6 divided by 3 is −2, so it was predicted that some line of reasoning like this would be employed. The printout detailed below is <u>a</u> solution (note: experience with patterns [illegible] one to be very cautious in stating that there is a unique solution) to the problem. However, further thought and my own experience with the way students use calculators, led to a prediction of trial-and-error approaches by most subjects. With the calculator's ease of computation it was felt that this question would investigate the tendency of the user to push buttons with no apparent reasoning or developing of a plan based on the generation of results. Briefly, it was believed that subjects would employ a random (undirected) trial and error. When using pencil and paper a problem solver who has no idea what to do may do nothing at all; whereas, with a calculator, it was hypothesized that the subject with no ideas might just press numbers to pass the time. It was thought that this problem might isolate such behaviour.

The Fourth Problem.

This problem, like problem 4, introduced an error or malfunction in the calculator so that the reasoning of the subject would be employed where ordinarily simple computation might be sufficient. It is interesting to explore the understandings subjects have of place value with a calculator because the ease of computation involved enables the subject to operate with decimals without any regard for place value.

The research made use of the fourth problem and to a good degree the first problem, to explore subjects notions of place value. The fourth problem forces subjects to recognize the place value of the 7's and 8's digits in each addend. Solving this problem involves replacing the 7 or 8 with other numbers that represent the same amount. A typical solution was predicted to be a representation of the 70 in 274 by perhaps 60 and 10 and the 800 and the 80 in 882 using a similar process. The problem itself, as well as a typical hypothetical solution is included below:

Problem: The 7 and 8 keys on my calculator do not work. How might I find the sum of 274 + 882 + 1,028?

Solution:

```
 264. +
  10. +
 662. +
 220. +
1027. +
   1. =
2184. **
```

Check:

```
 274. +
 882. +
1028. =
2184. **
```

It was also anticipated that subjects would "step out of the conditions" of the problem which specifies that the 7 and 8 keys don't work, and calculate what the answer should be using the 7 and 8 keys, a form of checking. The hypothetical check is also included with the problem in the section above. The nature of the decomposition chosen for each number was also seen as an indicator of the subjects' competence within the number system, as substantiated by further questioning in the retrospective analysis.

The Interview

It is important to note a primary shortcoming many researchers see in the clinical interview, perhaps indicating why often the pretest-posttest syndrome is the more popular method: it is not seen as systematic enough to permit any generalization of results. Opper addresses this issue:

> The interview method appears to be deficient since it is not applied in exactly the same way to all children. But this is no criticism: the logic of the clinical method demands an absence of standardization. Standardized procedures lack the flexibility required to uncover children's thought processes.[30]

The interview room was a large science lab. Subjects were seated on stools at the back of the room behind a large lab desk. The calculator was placed to their right and plugged into a power source using the FP-10's adapater. This prevented any problems with batteries running down and the like. At the front of the room, some 10 metres from the subject, was the researcher's desk. The times chosen for interviews was either over the lunch break (12:05 to 12:55) or after school (3:30 to 4:30). Only in a few cases did the lunch break provide insufficient time and result in a quick completion of the interview. This will be noted in the analysis of the protocols. All interviews were conducted in a uniform manner. If the subject was using the calculator for the very first time, some minutes were spent in demonstrating how the keys worked. Subjects were specifically instructed to use the calculator in their solution attempts. Several features of the calculator were explained explicitly: the clear function was used to separate the printout into segments of work; subjects were shown how to use the clear key and advance the paper key to divide the printout, The subject was specifically told to hit the print key if they changed their minds about an entry, before anything was erased using the clear entry key. This way misentries could be printed and become part of the retrospective discussion. Subjects were cautioned not to turn the calculator off or switch it into the nonprinting mode at any time. Both of these problems would, of course, cause the researcher serious problems. As mentioned earlier, if the subject switched the calculator off and on again, the calculator was programmed to print the word "clear". This "clear" would not be in the same vertical line as the "clear" printed when the C key was depressed. It was not possible to detect, from the printout, whether the calculator was in a print or nonprint mode as both modes were recognized as "equivalent" by the calculator.

The beginning of every problem-solving session always involved allowing the subject to quietly read the problem to himself. After the subject had read the problem the researcher always attempted to find out if the subject felt he understood what the problem was about, usually by a direct question. After any explanations or discussion, the subject was left by himself to work on the problem while the interviewer went to the front of the room and made certain he was occupied with some work. As much as possible, the researcher wanted to create the atmosphere where the subject felt he could work on the problem without interference, but, should he require any assistance, the interviewer could easily be summoned. As the session progressed, the interviewer would discreetly determine if the subject had reached a block; this only being done if a sufficient amount of time had elapsed.

If the subject did not approach the interviewer and there were signs of an impasse, the interviewer asked the subject, from a distance, if he required any help or explanation. Once the interviewer went to the back of the room to start the discussion, the taping was started. The interviewer tried to base the discussion on the record of the printout, using the divisions of the printout as an ordering theme for the discussion.

It is the researcher's feeling that the affective climate of the preliminary discussion and the retrospective discussion would be a crucial factor if subjects were going to discuss freely about what they had tried in solving the problem. The principles underlying the structure and climate of a clinical interview are very clearly stated by Opper and were guiding themes for much of the atmosphere the researcher tried to maintain. Opper describes the clinical method as a "diagnostic tool applied to reasoning in children,"[31] where the child's views of reality and thought processes are gathered from questions asked of the child. Opper sees the application of this method as an evolutionary spontaneous activity in the sense that the interviewer "should be prepared to devise rapidly situations and additional questions appropriate for the task at hand; he must have a mental set geared towards discovering how children think."[32] This active, interjecting role of the interviewer may be viewed by some as destroying the objectivity of the research, and having an effect on the conclusions that can then be reliably made. These critics see the researcher's role as a passive one. As can be judged by the protocols themselves, this researcher is very active during the interviews.

Opper refers to the pressure students feel whenever they are questioned about their actions. School, in that sense, may not necessarily be the best place to explore the reasoning of problem solvers. She specifies that "The child should not get the impression that he is going to be tested in anyway, since this might arouse feelings of anxiety and could possibly interfere with his spontaneous replies."[33] The general perspective of the interviewer is of paramount importance. If a legitimate investigation of process is undertaken, "The experimenter must refrain from trying to elicit what he believes to be the 'correct' or desirable answer." This is a consideration teachers, especially, need to make. Opper also suggests that there is a difference, generally, in the perspective of the adult interviewer and the subject: "Although the response may not be correct when viewed in terms of the performance of a mature person, it may give insight into the thinking processes of the child being interviewed . . . ."[34] Briefly, as much as possible, "The interviewer must attempt to take the child's point of view."[35]

A list of the interviews is now included with a cross-reference based on the date the interview was conducted:

Table 1.

Interview by date

Problem	Problem 1	Problem 4	Problem 7	Problem 8
Subject				
Mardi*	10/05		14/06	
Evan	11/05	27/04	25/05	
Sherri	11/05	13/04	01/06	15/03
Earl	16/05	14/04		24/03
Lisa	17/05	19/04	30/05	22/03
Shelley	17/05	25/04		
Bruce	24/05	25/04	02/06	
Ruth*		19/04	30/05	24/03
Clive*		26/04		25/05
Tonia*		20/05		
Ryan		22/05		
Shawn			15/06	
Wayne				15/03

Note: * indicates a grade 7 or grade 8 subject.

It should also be noted that a total of 9 protocols were deleted from the study and so not recorded in the table above. This decision was based on the methods of solution by the subject, the success of the interview following the problem solving, and the role of the interviewer in the problem solving situation.

The Subjects

The subjects were chosen from grades 7, 8 and 9 in Montrose Junior High School in Grande Prairie, Alberta, where the researcher was a grade 9 mathematics/science teacher. All of the grade 9 students were taught by the researcher. It is a fair generalization, based on their achievements on school tests, to say the subjects were above average in ability, especially the grade 7 and 8 subjects who could be classed as at the top of their year. From the above table it can be seen that a total of 18 subjects tried a total of 35 problems with two students trying all 4 problems, four students trying 3 problems, three students trying 2 problems and nine subjects trying only 1 problem. Of the 35 protocols, 17 were of girls and 18 of boys while 7 of the 35 were done by grade 7 or 8 students. It should be noted that subjects did the problems in the same order, namely, problem 8, problem 4, problem 1 and problem 7.

End Notes

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4. Ibid., p.38.
5. Richard E. Mayer and Piraye Bayman, "Psychology of Calculator Languages: A Framework for Describing Differences in Users' Knowledge," Communications of the Association for Computing Machinery 24 (August 1981): 520.
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13. Wallace Judd, Games Calculators Play, (New York: Warner Books, 1975), p.20.

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15. V. A. Krutetskii, The Psychology of Mathematical Abilities in School Children, eds., Jeremy Kilpatrick and Izaak Wirzup, trans., Joan Teller (Chicago: University of Chicago Press, 1976), pp. 87-8.

16. Judd, Games Calculators Play, p.112.

17. George Polya, How to Solve It, (Princeton: Princeton University Press, 1957), p.99.

18. ~~Ibid., p.77.~~

19. Krutetskii, Psychology of Math Abilities, p.88.

20. Ibid., p.44.

21. Judd, Games Calculators Play, p.21.

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23. Polya, How to Solve It, pp.14-15.

24. David C. Johnson, "Calculators: Abuses and Uses," Mathematics Teaching, no.85 (December 1978), p.51.

25. Judd, Games Calculators Play, p.62.

26. Wheatley, "Calculator Use and Problem Solving," p.330.

27. David G. Moursund, Calculators in the Classroom, cont. ed., Douglas L. Shults (New York: John Wiley, 1981), p.190.

28. Ibid., pp.190-1.

29. Betty J. Krist, "Uses of Calculators in Secondary Mathematics," Information Bulletin, no.8 (September 1980), Columbus: Calculator Information Center. p.3.

30. Sylvia Opper, "Piaget's Clinical Method," The Journal of Children's Mathematical Behaviour 1 (Spring 1977): 106.

31. Ibid., p.92.

32. Ibid., p.96.

33. Ibid.

34. Ibid., p.87.

35. Ibid., p.107.

CHAPTER IV

THE MATERIAL GENERATED BY THIS RESEARCH

Relevant Research Literature on Processes and Errors

Before progressing to the central theme of this chapter, the identification of the processes and errors made by subjects in this study, it is relevant to develop the context into which the processes fall: the problem-solving process generally. Although the literature is very sparse on developing common identifiable processes that generalize across studies, there is a commonality to what is perceived as the "stages" of problem solving. Krutetskii outlines "the three basic stages of mental activity in the process of solving mathematical problems"[1] as:

> . . . 1. Receiving information about the problem (related to an initial orientation toward its terms, an attempt to understand it); . . . 2. Processing (transforming) the obtained information for the purpose of solving the problem, and obtaining the desired result . . . . 3. Retaining information about the problem.

This is in very good agreement with the phases given to the entire problem-solving process by Polya:

> First, we have to understand the problem . . . . Second, we have to see how the various items are connected . . . in order to obtain the idea of the solution, to make a plan. Third, we carry out our plan. Four, we look back at the completed solution, we review and discuss it.[2]

The Newell and Simon model of problem solving as information processing also has "three distinct sorts, or states, of behaviour: understanding, method-finding, and coding [which] are involved in performing [the] . . . programming task," as described by Brooks in his research on the cognitive processes in computer programming. Coding is later defined as "designat[ing] sequences of computer instructions written in a programming language."[3]

Much will be made in this study concerning the error research on problem solving, and there is agreement on the general stages problem solvers go through in the error research too. It seems reasonable to suggest that any analysis of errors implies much about the problem solving process. Brownell concisely links errors with underlying processes in the following way:

> . . . most errors in mathematics are the result, not of imperfectly learned symbols, but of incomplete understandings, of inappropriate thought processes, and of faulty procedures.[4]

Casey also briefly states the idea developed here when suggesting that "The types of errors made by students point to the parts of the problem-solving process causing difficulties."[5] It can be speculated that if the actual problem solving process was unknown, and from the state of the research generally this is possible, then errors can indicate the unknown processes themselves.

Newman, in research on classifying errors, also presupposes certain levels to problem solving or as she specifies "there is a hierarchy of 'performance strategies' which need to be applied to solve written mathematical tasks successfully."[6]

She goes on to list these as abilites: to read; to comprehend the meaning of the question; to transform the question by selecting the needed mathematical processes; to perform those identified operations; and to encode the answer in an acceptable form.[7] It is fair to summarize all the research stated so far as dividing the general process of solving a problem into an input stage where the subject internalizes the problem, a transformation phase where the accepted problem is changed in such a way that the solution is developed and finally the solution produced as an output.

When the research literature was probed it was much more difficult to find material that identified definite processes used by subjects let alone any agreement on the selection of processes. Singular in this respect was the work of Kilpatrick[8] in developing a checklist coding system of processes used by subjects. A modification of this checklist was developed by Days[9] and used by Wheatley to evaluate the problem solving performance of 46 sixth graders divided into calculator and noncalculator groups.[10] Days' checklist organizes the processes selected under the general divisions of understanding, recall, production, evaluation and comments about solution. The first four headings closely parallel Polya's[11] descriptions of the problem solving process while the fifth comment category was developed to analyze the statements made by the subjects as then worked on the problem because Wheatley "encouraged [subjects] to think aloud as they worked through the problems."[12]

It is important to note the processes included in Wheatley's checklist, and this is included in the table that follows:

Table 2.

Wheatley's Checklist of Processes[13]

Understanding/Representational:
- Records problem.
- Restates problem in own words.
- Separates parts of the conditions.
- Performs exploratory manipulations.
- Uses mneumonic notation.
- Draws a diagram.
- Makes a list.

Recall:
- Recalls a related concept.
- Recalls a related problem.
- Uses method of related problem.

Production:
- Reasons deductively (if-then, since-then).
- Misinterprets problem.
- Selects solution on irrelevant basis.
- Has bright idea.*
- Makes successive approximations.*
- Guesses.*
- Uses unexpressed equations.*
- Estimates.*

Evaluation:
- Makes routine check.*
- Checks conditions.*
- Retraces steps.*
- Uses another method.*
- Questions reasonableness.*
- Compares solution with general known results.
- Changes approach.

Comments about Solution:
- Questions existence of solution.
- Questions uniqueness of solution.
- Questions necessity/relevance of information.
- Expresses uncertainty about final solution.
- Says he or she doesn't know how to solve problem.
- Says the problem is difficult.

It is significant that in Wheatley's research many of the listed processes were taught as problem solving techniques during a 6 week treatment period. The checklist was then used to evaluate the behaviour of the calculator and noncalculator groups on a problem solving test. Further, Wheatley targeted those processes given an asterisk in table 2 for analysis. Perhaps if Wheatley had discussed solutions with subjects after they had attempted solution, her targeting would reflect a different emphasis. Also, she notes specifically that the interviewer asked "nondirecting" questions.

Table 3.

Newman's Hierarchy for Error Causes[14]

<u>Reading Ability:</u>
- Word recognition.
- Symbol recognition.

<u>Comprehension:</u>
- Literal understanding.
- Specific terminology understanding.

<u>Process Skills:</u>
- Arithmetic: random response.
  - wrong operation.
  - faulty algorithm.
  - faulty computation.
  - no response.
- Spatial.
- Logic.

<u>Encoding Ability:</u>
- Words.
- Symbols.

<u>Careless Errors.</u>

<u>Motivation.</u>

There would appear to be a close relationship between Wheatley and Newman's work. While attempting to understand a problem it can be seen how reading and comprehension errors would result. In trying to solve a problem by recalling prior information the subject might not select the correct mathematical processes, which Newman classed as a transformation error. To continue the comparison, trying to produce the answer would result in the errors classed under process skills. The comparison terminates here because Newman is dealing with one-step arithmetic problems while Wheatley's work used many-step problems that required more interpretation. However, the similiarity between them led this researcher to focus as much on errors as processes in the research. In fact, they came to be viewed as aspects of the same measurable phenomenon. That is, for example, when a subject reads a problem and looks for the relevant information, if he is a good problem solver he will be successful at an aspect of problem solving that another subject who focuses on irrelevant data will not be. In fact, it may be easier to understand the mechanisms of problem solving through the use of error analysis. Davis makes a relevant comment on this:

> It is no accident that both Freud and Piaget gained their most penetrating insights into the nature of human thought by considering errors, wrong answers and "slips of the tongue".[15]

It is also noteworthy that Krutetskii compares very good and very poor problem solvers on certain common dimensions, for example memory, as in the following reference:

> Most capable pupils remember the type and the general character of the operations of a problem they have solved, but they do not remember a problem's specific data or numbers. Incapable pupils . . . usually recall only specific numerical data or specific facts about a problem.[16]

An indication of Krutetskii's recognition of the role of recall has already been mentioned where his third stage of problem solving is "retaining" information.

The use of the Newman error categories in this research is limited by the simplicity of the organization developed in Newman's work. When dealing with one-step arithmetic problems and "low achieving sixth-grade" pupils it is possible to achieve very simple solutions where subjects progress through a hierarchical system in one direction only once. However, there have been modifications to Newman's scheme to take into account many-step problems, notably a variation by Casey that was very useful and helped this investigator to develop his own form of error categorization. Casey used the categories developed by Newman with the condition that there was not an exact correspondence "between these error types and the order in which they would arise during the problem solving process," but rather that "students often go back and forth between stages while solving a problem."[17] Casey also modified Newman's transformation error category which was vaguely described as skill selection so that it became divided into strategy selection and skill selection parts.

A sequence was maintained as the following description indicates: "Having selected a plan of attack the student then selects the first appropriate skill."[18] Newman realized the nature of her problems (i.e., their simplicity, basically being computational) by stating that "At the time the study was conducted it was believed that not all items on the test involved the transformation step."[19] A major criticism of the Newman error analysis is also made by Clements when he notes that

> . . . although the Newman technique is invaluable for gross diagnostic purposes, more detailed probing is essential if the aim is to discover how a child thinks about a given area of mathematics.[20]

Another important aspect of the Newman technique is that motivation and careless errors are also recorded. However, the carelessness error category was not well defined as can be observed in Clements use of Newman's criterion for carelessness, where

> if, during the Newman interview, a student obtained the correct answer to a question [that had previously been wrong] without any assistance from the interviewer, then the error was classified as careless . . . .[21]

Grouping on this basis would include many potentially different errors as careless, in my opinion. There is also the significant matter of the effect of learning if the same question is repeated in two different tests separated by a period of time. All of these considerations led to a modification of the original Newman technique and the Casey form. The perspective that errors should be related to problem solving processes was a particularly important factor in the decision to make a modification.

Modifications to Produce a Scheme for Progress Analysis

What now follows is a list of the processes this researcher isolated from the printouts and protocol analysis. Coupled with the processes will be the error associated with a poor application of that process. Table 4 below will show the modifications made to the Days'[22] checklist while table 5 will pair the processes with their error counterparts.

Table 4.

Modifications to Days' Checklist of Processes

Category	Modification
Understanding/Represent	
Rereads problem.	Changed to "reads the problem".
Restates the problem.	Unchanged.
Separates conditions.	Deleted.
Performs exploration.	Unchanged.
Uses notation.	Deleted.
Draws diagram.	Deleted.
Makes a list.	Deleted.
Questions conditions.	Transferred to Evaluation.
Recall	
Recalls concept.	Unchanged.
Recalls problem.	Unchanged.
Recalls method of related problem.	Changed to "related method".
Recalls previous calculations.	Added.
Production	
Reasons deductively.	Unchanged.
Misinterprets problem.	Transferred to error category.
Selects solution on irrelevant basis.	Transferred to error category as "disregarding information".
Has bright idea.	Changed to "reasons intuitively".
Makes successive approximations.	Deleted as a process and treated as a strategy.

Guesses.	Transferred to error category as "random response".
Estimates.	Deleted.
Uses unexpressed equations.	Deleted.
Produces an algorithm.	Added.
Repeats algorithm with data change.	Added.
Modifies algorithm.	Added.
<u>Evaluation</u>	
Makes routine check.	Transferred to understanding as exploration.
Checks conditions.	Transferred to understanding section.
Retraces steps.	Included in "analyses result" process.
Uses another method.	Deleted.
Reasonableness query.	Deleted.
Compares solution with known results.	Changed to "analyses result".
Changes approach.	Unchanged.
Identifies error.	Added.
Discovers pattern.	Included in "analyses result" process.
<u>Comments about Solution</u>	
Questions existence.	Unchanged.
Questions uniqueness.	Unchanged.
Expresses uncertainty.	Transferred to error category.
Questions necessity of information.	Deleted.
Says can't solve it.	Unchanged.
Says problem is hard.	Deleted.
Says he got it.	Added.
Describes reasoning.	Added.

Table 5

Modified Coding System with Error Equivalents

Represent/Understand Code	Error Equivalent Code
Reads the problem.	81 Misreads.
21 Restates the problem.	
22 Performs exploration.	83 Misunderstands.
20 Questions conditions.	82 Disregards information.
24 Questions calculator operation.	84 Extraneous questioning.
Recall	
30 Recalls related problem.	85 Mathematics/Calculator error.
31 Recalls related concept.	" "
32 Recalls related method.	" "
33 Recalls previous calculations.	86 Repetition of previous work.
Production	
47 Produces an algorithm.	92 Transformation.
49 Modifies algorithm.	91 Incorrect modification.
48 Repeats algorithm with data change.	86 Repetition of previous work.
40 Reasons deductively.	93 Poor reasoning.
41 Reasons intuitively.	94 Random response (guesses).
Evaluation	
54 Identifies error/answer.	95 Recognition.
55 Analyzes result.	96 Nonevaluation of results.
53 Changes approach.	99 Inhibition.

The rationales behind the modifications to the checklist will now be explained with the overriding understanding that these must, by the very nature of the research, be highly subjective. To a degree, it can be argued that, assuming the concept of set, we see what we look for. This is not necessarily detrimental to the research if what we look for is actually there. Essentially, this list of processes was developed first by reading the protocols and looking for "things" the subjects did while pursuing the solution. Then the research on processes was surveyed, and the researcher attempted to combine what he had observed with the results of other researchers, principally those noted. Next, an analysis of the protocols for errors was made and the research on error analysis was gleaned, generally a richer area than process isolation. Finally the process categories and the error categories were combined and research for this union was reviewed.

The changes made to Days' "understanding" section of categories principally were determined by viewing understanding as a recursive phenomenon rather than just a starting point. The idea of certain processes as being pervasive is important and central to the idea presented here of problem solving. Clarkson, who also used Newman's categories, gives careless errors, such a "mobile" quality in stating "it is hypothesized that a 'careless' error may be made at any step in solving the problem"[23] as does Watson (another advocate of the Newman technique) in noting that "Motivation and carelessness could produce errors at any of the five stages outlined."[24] But Watson is working with grade two subjects, Clarkson with grade six students in Papua New Guinea and Casey with assisting teachers on analyzing students errors. This research deals with grade nine subjects (principally) of higher ability, generally, working on nonroutine problems. There is a good deal of merit in viewing the problem solutions analyzed here as being more "fluid" than that portrayed in much of the research perused. Based on the actual results, printouts and protocols of this study, the solving process involves an interplay between all the Newman stages, hence the idea of flow.

In referring to the modifications of table 4, a group of understanding processes were deleted that were not found to be significant in the research. It was felt that questioning the conditions of the problem was more properly an evaluation process. The deletions also make sense because Wheatley's research was principally to test the presence of pretaught heuristics that were then tested as possible processes in later tests. This research was principally exploratory without any preinstruction of techniques. Table 4 presents Days' checklist as it was listed in Wheatley.[25] In Wheatley's study the processes of "separating parts of the condition", "uses mneumonic notation", and "makes a list" were all "regularly taught techniques of problem solving as part of the daily schedule"[26] of the teacher in the pretesting period. Because of the use of a calculator, and no specific preinstruction on heuristics, these three processes were deleted from this study.

The recall section was mostly unchanged except that many subjects were observed to refer back to calculations they had made earlier in the session and this was perceived as a recall process. In several cases, not recalling previous calculations and repeating them over again was viewed as a type of error classified as repetition.

Many of the production processes were changed, deleted or moved to other sections. "Reasons deductively" was distinguished from "having a bright idea". Such inspirations were distinguished using Krutetskii's[27] idea of intuitive reasoning.

"Making successive approximations" was a highly common phenomenon but was viewed as more than a process and considered rather as a strategy, one that was selected for special emphasis by itself. "Guessing" was considered an error; an idea that will be developed later in this chapter. "Estimating" and using "unexpressed equations" were deleted though estimating will be included in the separate evaluation of the successive approximations strategy. Noteworthy additions were the three processes involving the origin, repetition (with data change) and modification of algorithms. The evidence for algorithms (i.e., procedures for solving problems in clearly outlined steps that are frequently repeated) was very strong in the research in terms of continuation within specific protocols as well as across protocols.

The evaluative processes were mostly left intact although, as analysis will show, certain processes were not as evident as first thought and probably should have been deleted. In designing a list of processes there would seem to be some that appear to be obvious candidates but actually are difficult to isolate within the body of the protocols. The criterion used in all cases was: "do I need this process to explain what the subjects are doing?" and "do the subjects actually demonstrate this process?". The "uses another method" process was included in a "change in approach" and the "production of a different algorithm" and so was deleted as a category. "Comparing the solution to a known result" was modified so that previous work on the calculator was viewed as "known results" and such reviewing was considered as the process of "analyzing results". "Identifying errors" was perceived as an important evaluation process as well as "discovering a pattern", which took on significance because of the structure of the first and third problems used in the research. To discover a pattern the subject must perform some form of analysis so this process was included under the previous process of "analyzing results". It was felt that clear descriptions by subjects of the reasoning that they employed was useful and so indicated by coding. However, as most explanations were the result of the interviewer's probing, these could hardly be considered as part of the solution process. That is not to say that some problem solving did not occur during the actual interview, as the protocols will demonstrate that this did happen. A final comment category added was any type of statement that indicated the subject felt that he had solved the problem. Such comments were used to confirm the process of "identification"--realizing that the answer had been obtained. Identifying errors was considered as the process of "recognition" and added to the process section.

"Comments about the solution" section was generalized to "comments about the solution process" to evaluate generally what ideas subjects had about whatever they had done, not just final solutions. "Expressing uncertainty" was treated as a type of error as will be explained later. "Questioning the necessity or relevance of information" was not perceived as an observed comment and was deleted. "Saying the problem was difficult" was included as a type of affective statement and such comments were included as a separate category for consideration and so removed from process treatment. Generally the comments of the subjects were considered as descriptive, indicating what processes were involved but not processes in and of themselves. A separate section of the research will deal with comments made by the subjects.

To organize the revised form of the Days' checklist as it was used in this research, the additions, deletions and transfers have all been effected and placed in table 5. As these processes will be used in analyzing the printouts and protocols of subjects, each process has been given an identifying code number on the left of the table. As has already been specified, most of the processes are linked to error equivalents and these have also been coded and listed to the right in table 5. It was a major hypothesis of the research that these two lists would be sufficient to properly analyze the problem-solving behaviour of the subjects involved.

Processes and Errors of This Research

Reading and Misreading.

The role of reading as a necessary prerequisite in problem solving is clearly outlined in the research literature generally. Corle developed a "reading efficiency" of problem solvers by "judging . . . the fluency of word pronunciation as the pupil read the problem aloud."[28]

Linville studied

> . . . whether the degree of syntax used in the sentences [of verbal arithmetic problems, where verbal actually means written] . . . and/or the level of vocabulary used in the statement of the problems are factors which contribute significantly to the degree of difficulty of the problems when the computational operations are held constant.[29]

He found that "Syntax and vocabulary level can both be determiners of difficulty in verbal arithmetic problems," and also that "Pupils who have scored high in reading achievement can be expected to experience greater success in solving verbal arithmetic problems than pupils who scored low in reading achievement."[30] Georges, as reported in Hollander, found that

> Of six classifications of reading difficulties, mathematical terminology problems were predominant, with 23.4 percent of all difficulties caused by mathematical vocabulary and 18.8 percent by symbols and notations."[31]

It is also important to note that Krutetskii used perception generally as a discriminator for good and poor problem solvers. It is not merely a matter of reading the words, but also what the subject "makes" of what he has read.

This point is made by Krutetskii:

> . . . we can state that under identical conditions for the perception of mathematical material, pupils with different mathematical abilities obtain (or more precisely, actively procure) different information.[32]

In this research, each subject was directed, at the beginning of each session, to read the problem to himself and determine if he understood the problem. As this process was always a directed activity it was not measured; however, evidence of misreading can be established in the protocols.

Consider the following excerpt from Earl's solution of problem 8 as an example of misreading, probably due to carelessness:

Printout		Line	Protocol Text
No.1.	274. + 882. = 1156. **	27 E:	Well, I thought that the second plus sign was an equals in there.
		29 I:	Oh, I see, you saw that as an equation.
No.2a.	1028. - 204. = 824. **	30 E:	Yeah.
		31 I:	Alright. So in the first one what did you do in No.1?
No.2b.	1028. - 294. = 734. **	32 E:	Well I thought that was 274 + 882 = 1028 so I was just checking to see if that was right or not. And ah . . .

At the grade 9 level, and considering the low reading level of the four problems generally, it was anticipated that there should be few reading errors. As established earlier, a reading error would result not only at the beginning of the problem solving process but also at any point where the subject misread a calculation and in that sense this error has a pervasive quality. Consider Sherri's reading error that occurs after her initial work in problem 1 as an illustration of this point:

Printout		Line	Protocol Text
No.2.	10000000. x 1.0000003 = 10000003. ** 10000003. = 10000006. ** 10000006. = 10000009. ** 10000009. =	27 S:	Well it said 10 million by, well 1 point 6 zeros and 1, and then when you look at the answer there, it adds 1 every time you press equals . . . when you asked me to get ah . . . 10 thousand 3, 10 thousand 6, 10 thousand 9 -
	10000012. **	32 I:	Actually 10 million. It's OK, anyways.

This reading error I would not classify as carelessness because when line 27 is compared to lines 30-31, we see she has trouble reading place value correctly. This error is actually present in the protocol of other grade 9 subjects tested in this research.

Other Understanding Processes and Their Attendant Errors

Once the subject has read the question we generally enter the area of understanding. The processes grouped in this section are all methods used by the subject to internalize the question in a form that makes sense to him, that compares to whatever he currently knows about the topic area of the problem. Mayer, in a review of what cognitive science has determined generally about algebra problem solving, makes reference to the understanding phase of solution:

> . . . two processes are involved in solving story problems: translation--understanding the problem, as manifested in translating the words of the problem into an internal representation in memory, and solution--applying the legal rules of algebra and arithmetic to this internal representation in order to deduce the answer.[33]

In this research the subject was asked directly, before the problem was attempted, whether he or she understood what the question actually demanded.

The role of understanding has an established significance to problem solving researchers. It could be stated that it is axiomatic that problem solvers require understanding to solve problems.

Krutetskii links perception and understanding if we "perceive" understanding as partly, at least, composed of an analysis followed by a synthesis, in the following way:

> . . . capable pupils perceive the mathematical material of a problem analytically (they isolate different elements in its structure, assess them differently, systematize them, determine their "hierarchy") and synthetically (then combine them into complexes, they seek out mathematical relationships and functional dependencies).[34]

It is suggested here that this "analytic-synthetic" process is part of a subject's attempt at understanding a problem.

Brownell, in his study of "levels of understanding" makes the important statement that "it is erroneous to conceive of understanding as if it were either totally present or totally absent. Instead, there are degrees or levels of understanding."[35] Donaldson, in her classification of errors arising from a set of reasoning tasks, grouped errors as:

> . . . structural (failure to appreciate relationships in the problem or to grasp some principle essential for its solution), arbitrary (lack of loyalty to the givens of the problem under the influence of previous experiences), or executive (errors that arise not from failure to understand how the problem should be tackled, but in some failure in actually carrying out the manipulations required).[36]

There is a close relationship between the process of "questioning the conditions of the problem" and the error category of "disregarding information". The error here classified as misunderstanding (i.e., inadequately developed understanding in the Brownell sense) is well defined by Donaldson's "structural" error category described. Errors classified as "arbitrary" are described in this study as "disregarding information".

The importance of recognizing those facts which are important in a question is also specified by Anthony and Hudgins in the following distinction between patterns of problem solving that distinguish problem solvers. They note:

> . . . poorer solvers failed to recognize or act upon the demands of the problem, to differentiate between relevant and irrelevant data, or to deal with both parts of a two-step problem.[37]

Krutetskii also refers to the role of perception:

> It is well known that in a number of cases the basic difficulties in mastering an intellectual skill or habit lie in the realm of perceiving the initial facts and not in the realm of operations that should follow this perception.[38]

From the research, misunderstanding errors become obvious when the subject expresses his difficulty openly, as in Wayne's difficulties with problem 8:

Printout	Line	Protocol Text
No.1. 1028. + 882. + 274. = 2184. **	01 I:	OK. So I want you to read the problem you're going to try on this (i.e., the calculator) and then any questions you have you ask me. OK?. I'm going to be up there. I want to leave you alone for a while to work it out. If you think you've got the problem, let me know. You'll know when you get it when you can work it all out on the calculator and show it to me.
		CALCULATOR OPERATED (No.1)
	08 W:	Mr. Leesinsky?
	09 I:	Have you got it?
	10 W:	Yeah. I don't understand what we're supposed to do?
	11 I:	You don't understand what you're supposed to do?
	12 W:	Find the easiest way to do it, right?

13 I: No, it's telling you the 7 and the 8 keys don't work. So you've got to figure out a way of finding the sum without using the 7's and 8's.

If the printout is compared with Wayne's admission in line 10 it is easy to see that he has not understood the problem. In other cases, the misunderstanding is not stated as openly and comes out in the course of discussion as in Bruce's misunderstandings of how the calculator operates:

Printout	Line	Protocol Text
No.5. 140. − 3. ÷ 3. 139. **	184 B:	Yeah. So I decided next one, No.5, to go to − to try it by a whole number . . . and I realized it was . . . just 1 point too high so I decided to move my point so I decided −
	188 I:	Hold on, we're going a little fast here. In No.5 you're subtracting 3 and −
	190 B:	Dividing by 3.
	191 I:	Can I ask you a question? What is − you got a 139 so what did the calculator really do? (Pause) If you went from 140 to 139, what did it do?
	194 B:	Took off 1.
	195 I:	OK. Can you see, looking at just this No.5 can you see why when you subtract 3 and you divide by 3 you'll be subtracting 1?
	198 B:	(Pause) Yeah.
	199 I:	Why?
	200 B:	Uhm . . . ah . . . it has something to do with the calculator itself, doesn't it?
	202 I:	That's right.
	203 B:	Well ah . . . I don't know . . .

204 I: Which one of these will it do first, subtract 3 or divide by 3?

206 B: Well it should - well, unless it doesn't have AOS it should subtract by 3?

208 I: It's got AOS (i.e., algebraic operating system: a registered trademark of Texas Instruments).

210 B: So subtract by 3.

If one were to evaluate Bruce by the recall he demonstrates concerning his calculator manual, as shown by his use of the term AOS, he would appear to know what he is doing. But Bruce clearly demonstrates that he does not understand the order in which the calculator is performing operations. Bruce took quite a while, until line 203, before he would openly admit that he did not know the order and still continued to cover his lack of understanding even after that.

An example of the error of disregarding information in a problem is Shelley's work with problem 4. The problem asks her to find which key (i.e., singular) is not working and yet Shelley makes a test where three different digits are treated as potentially malfunctioning keys and eliminated, as in the following excerpt from her protocol:

Printout	Line	Protocol Text
No.3, 131. + 267. + 538. + 934. + 1870. ** 1870. + 728. = 2598. **	98 S:	OK. In No.3 I was going through to see . . . like I was just adding them up and equalling them to see what numbers - I was just gonna - seeing if . . . if maybe they left out a few numbers to get it. To get the answers. Like maybe they just deleted one number and then I realized that they didn't.

No.9a. 82.6 x 0.91 = 75.166 **	104 I:	Oh, you thought maybe they had left out one of the numbers they were adding.
	106 S:	Yeah.
	107 I:	So what you did then is you left out which number to check it?
	109 S:	Well, I went through; I went down to 934 and then I equalled it.
	111 I:	So you're leaving out which number?
	112 S:	Ah . . . 728.

A good indication of the significance of an error can be its repetition. The fact that Shelley has missed a given condition in the problem is obvious again when her work on printout No.9a is discussed:

Printout	Line	Protocol Text
No.9a. 82.6 x 0.91 = 75.166 **	210 I:	That looks interesting. What have you done in No.9a? You've made a change. What did you change?
	212 S:	OK. Here I just -
	213 I:	In problem 3, heh?
	214 S:	I thought maybe they missed a number like in here maybe it wasn't in decimals. So I . . . I . . . I forgot about 85 and I put in the 0.91.
	217 I:	Left off the 85.

The processes of "performing an exploratory manipulation" is illustrated by most of the subjects attempting problem 4. Subjects performed exploratory checks to see which of the 4 questions were actually wrong. The results of such explorations were then used to attempt a solution of the problem.

The problem was more clearly defined once the manipulations had been made, as Sherri's work below on problem 4 illustrates:

Printout	Line	Protocol Text
No.1. 41.7 + 5079. = 5120.7 **	19 I:	You're going to try to tell me what you did. Let's call this part No.1.
No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	21 S:	OK. I looked at them and then I added them up. Like I did what it said. The first said 41.7 plus 5079, I added it. And it came to the right . . . answer.
	24 I:	So this is just the first problem.
No.3. 82.6 x 85.91 = 7096.166 **	25 S:	So that was right. So then I did the the second one.
		[a discussion followed on whether the correct answer was the printed answer given in question 2 or the answer the calculator determined.]
No.4. 307. x 64. = 19648. **	53 I:	Well, anyways, go ahead. So in No.2 you're just adding up what's there.
	55 S:	Yeah and seeing . . . got that. And in No.3 multiplied them and got that answer (i.e., 7096.166) which was different from this answer (i.e., 7044.62). And I multiplied problem 4 and got a different answer from what it says on the sheet.
	60 I:	OK. So in the first 4 you've redone the problems on a printing calculator.
	62 S:	So . . . what I did was I looked back on first problem which was correct and then I figured out which numbers weren't in the problem.

Note from her last statement that the first four checks were indeed explorations and that she analyzed the four results together after the initial operating.

"Questioning the conditions of the problem" is a process that is briefly but clearly indicated in Shawn's initial questioning before he attempts problem 7:

Line	Protocol Text
36 I:	Now here's the question I want you to do. Read it and see if you understand what I want you to try to do.
38 S:	You want me to find . . . ah . . . what constant operations you used.
40 I:	Right.
41 S:	To get these (i.e., the numbers in the sequence).
42 I:	Now, can I leave you to do it for awhile?
	[an explanation of the function of the P-key followed].
	PAUSE (30s)
	You have a little question for me. What's your question?
52 S:	Do I use the divided by 3 (i.e., an operation given in the question)?
54 I:	Yeah. All you got to be able to do is be able to see it.
55 S:	Does it have to times and divide first?
56 I:	It can be up to you. It can also be add or subtract.
57 S:	First. Oh. OK. And it was to have one of each. Add and subtract and . . .

Shawn goes beyond questioning the conditions of the problem. He finds the question too open-ended and tries to use his questions to determine limits that are not implied in the question. The latter part of his questioning would then be considered "extraneous questioning" and viewed as a deficiency on Shawn's part (i.e., not being able to accept open-ended conditions) and it can be considered an error.

Consider another example of the error of extraneous questioning, again indicating that the subject has difficulty when given too much "room" in interpreting the question. Bruce's final part of problem 1, where he is asked to generate any pattern he wants with two numbers will illustrate:

	Printout	Line	Protocol Text
No.9.	1000000. x 1.000009 = 1000009. ** 1000009. = 1000018. ** 1000027. **	290 I:	Ah . . . so that's pretty good. So what I'd like you to do - it'll just take a minute or two - I'd like you to make up any sequence you want to. You pick the two numbers - here - let's see what it says in here (investigator starts reading question out loud to the subject). How would you get that?
		296 B:	How would I do that?
		297 I:	You can make up anything you want.
		298 B:	Like something similar to this (referring to part a of the original question)?
		301 B:	And I could have it increasing by anything I want? CALCULATOR OPERATING (No.9) And I can make this any number I wanted there (referring to the second factor he is using). CALCULATOR OPERATING (No.9) And then it would keep doing it (presses equal key repeatedly).

If an error is simplistically viewed as doing something wrong, then Bruce is not committing an error. But if the view is that Bruce has been asked to produce this own algorithm and he has difficulty proceeding without more definition to the problem, then his hesitation, as indicated by the extraneous questioning, can be considered a type of executive error, in Donaldson's sense of not having the ability to operate.

The nature of the research required the addition of an understanding process where the subject asks about some aspect of the calculation (i.e., the process labelled as "questions calculator operation"). The question may then be used as an indication of a possible deficiency the subject has in his knowledge of the operation of the calculator, thus helping to identify a calculator recall error. Tonia's work on problem 4 is a case in point:

Printout	Line	Protocol Text
No.3. 82.6 x 85.91 = 7096.166 **	14 I:	So then I'll leave you with it . . . and if you have a lot of trouble - let me know. So just take your time.
		CALCULATOR OPERATED (Nos. 1 to 3).
		Tell me what you just asked me again . . . you don't know how to -
	20 T:	I can't get my decimals to line up - I don't know how I should punch it in to do that.

21 I: You don't have to get 'em to line up . . . . it'll (i.e., the calculator) just move it (i.e., the decimal) around depending on where it is. So they don't have to line up. You compare them - like there's one decimal (in 82.6) and there's two (in 85.91) there's three (in 7096.166) - you can tell how many places there are by how far to the left the decimal is -

30 T: Oh yeah, in multiplying they don't have to line up - I forgot.

Note that Tonia not only confuses calculator multiplication with normal computational multiplication but that she disregards the information given by the investigator and recalls the arithmetic of decimal addition and subtraction to enable her to continue.

Recall, Knowledge Errors and Repetition.

Once the subject has probed deeply enough into the problem to grasp what it is asking him to do, the solver is hypothesized to match these demands against the prior knowledge that he possesses. Krutetskii recognizes the role of retention as a problem solving component. He differentiates between the type of retention exhibited by competent and incompetent pupils as follows:

> Most capable pupils remember the type and the general character of the operations of a problem they have solved, but they do not remember a problem's specific data or numbers. Incapable pupils . . . usually recall only specific numerical data or specific facts about a problem.

This is consistent with Mayer's finding that "relational information is more difficult to mentally represent than other kinds of relevant information in a story,"[40] and that "subjects favor story problems that do not contain relational propositions [i.e., a single numerical relationship between two variables, like 'the length is 2.5 times the width']."[41]

The more that can be retained, the more information there will be to be recalled at a later time. A study by Hinsley et al.[42] is commonly referred to in the research on the role of recall. For example Mayer refers to the investigation in this way:

> . . . Hinsley et al. have found that subjects were able to sort story problems into consistent categories such as work, motion, distance-rate-time, triangle, current, and so on. Based on this research, Hinsley et al. detected 18 basic categories for story problems and suggested that people have schemas for each; that is, knowledge of the structure of each type of problem.[43]

Hinsley's work draws attention to the fact that this section is not only dealing with processes utilizing recall but also that the idea of a relatedness is involved. The subject solving the problem will try to remember those facts, concepts and methods that he perceives are "like" the problem at hand. Silver studied student perceptions of relatedness among mathematical verbal problems. He found a

> . . . significant positive relationship . . . between a student's mathematical ability and the tendency to associate problems on the basis of mathematical structure . . . [and a] . . . negative relationship between mathematical ability and the tendency to associate problems on the basis of contextual details.[44]

This is in agreement with the findings of Krutetskii as noted by Silver: "Krutetskii found substantial quantitative and qualitative differences between high and low ability students in their perceptions of problem structure."[45]

Though the role of recalled information as a determinant in problem solving success is not disputed by this investigator, it was found to be quite difficult to isolate the processes of recalling related problems, concepts and methods unless there was excellent substantiation in the protocols. As with many of the other processes, recall became more obvious when an error was made. Recall errors surfaced as mistaken facts and concepts though having no knowledge about a particular area also would mean that the subject had nothing to recall. The subject would indicate this by not being able to do anything. This brings out the role of prior knowledge in problem solving. Mayer refers to Gagne's work in this area:

> The importance of prerequisite knowledge is suggested by Gagne's concept of learning hierarchies and the idea that the acquistion of new information should follow a hierarchical sequence of material starting with what the learner presently knows and working up.[46]

The following conclusion by Larkin in relation to her study of expert and novice performance in solving physics problems concisely states the issue developed here: "In every domain that has been explored, considerable knowledge has been found to be an essential prerequisite to expert skill."[47]

Having no previous experience with related material is a problem recognized by Mayer: "a difficulty may arise when students are given problems for which they have no schema."[48] Davis also recognizes this problem in stating " Failure to retrieve any candidate frame whatsoever can occur if the individual has no appropriate frame stored in memory--surely one of the most common of all failures."[49] The terms schema and frame refer to the way information is postulated to be stored in the subject's mind. Davis defines a frame as "a specific information-representation structure that a person can build up in his or her memory and can subsequently retrieve from memory when it is needed."[50]

Without getting into the controversy concerning the existence of frames or schema, it seems reasonable to suppose that there is some mechanism like a frame or schema that is very important when considering recall of that which the subject sees as related to the problem given. Frames are also postulated to be deeply involved in translation (i.e., understanding) in that a particular frame, according to Davis "tells . . . [what] 'things to look for' in the input data and gives . . . [the] 'most likely' translations of ambiguous words."[51]

In this research, as well as classifying the recall of related problems, concepts and methods, the correctness of the needed information will be used to develop relevant error categories. For example, if the subject recalls a method, say subtraction for example, and performs the operation incorrectly, this will be classified as a mathematics knowledge error.

A final process has been added, that of "recalling information from calculations previously performed" by the subject on the problem. With the use of a printing calculator, the subject has hardcopy of all his previous work, and performing the same calculation again may mean the subject has failed to recall what he had previously done. This will be classified as a "repetition" error. The whole area of not using past calculations to help in solving the problem is directly addressed in the evaluation process of "analyzing results".

As an illustration of recalling a related concept, consider Shawn's reference to "flow charts", which he was taught in mathematics class:

Printout		Line	Protocol Text
No.4.	138. x [illegible]. = 414. **	110 I:	So you went back and tried 3 times 140. Why'd you do that?
	414. / 140. = 2.9571429 **	112 S:	Just to see what it would be . . . and then . . . . I timsed it bv 140 and then . . . divided it by . . . added -
		114 I:	OK. You're talking talking to me about No.4 now?
		115 S:	I was trying to do reverse in the steps. Trying to do the steps in reverse.
		117 I:	Explain that to me.
		118 S:	Like you know how - flow charts - how you can do them backwards.
		120 I:	Yes.

An example of recalling a related method is the use many subjects make of subtraction as soon as they faced the decreasing sequence of numbers in part c of problem 1. The work by Lisa exemplifies this:

Printout		Line	Protocol Text
No.5.	9999999. /	118 I:	You've got it decreasing by 1. What was your idea? This is now No.5. What was your idea?
	1.0000001 =		
	9999998. **		
	9999998. =		
	9999997. **	120 L:	To find something that . . . would get it to decrease by 1.
	9999997. =		
	9999996. **		
		122 I:	Cause that's the pattern, heh? So why did you think divide right away?
		124 L:	I don't know . . . easier to work with than subtract.

Bruce has related the decrease in the sequence of part c of problem 1 with multiplication by a negative number.

Printout		Line	Protocol Text
No.6.	9999999. x	203 I:	Just hold on a sec - I want you to tell me about it. So I'll just press the print button - that's what you were going to do - and we'll hold it that way.
	-1.999991 =		
	-19999908. **		
	-19999908. =		
	39999636. **		
		208 B:	I think I'd multiply by 9 million 9 hundred and 99 times . . . ah . . . I think it might have to be a negative number, I'm not sure. To get it to go down . . . it might have to be . . . say negative point - or negative 1 and that'd be 1 . . . (talking out loud to himself).
No.7.	10000000. x		
	-1.0000001 =		
	-10000001. **		
	-10000001. =		
	10000002. **		

An example of recalling calculations already made with the calculator is in the work of Evan on problem 4:

Printout		Line	Protocol Text
No.2.	131. +	120 I:	OK, you can tell me why you guessed it was the 6?
	267. +		
	538. +		
	934. +	121 E:	Because the 6 shows up in all the problems that weren't right.
	728. =		
	2598. **		
No.3.	82.6 x	123 I:	Is there any other one that shows up in all the problems that aren't right?
	85.91 =		
	7096.166 **	125 E:	No.
		126 I:	Well you told me it could have been 2. Is the 2 in all of them?
		128 E:	The 2 isn't in problem 4, though.
		129 I:	OK, what about the 3?
		130 E:	It isn't in problem 3.

The error classed as repetition of previous calculations is illustrated many times in Earl's attempts with problem 4:

Printout		Line	Protocol Text
No.8.	131. +	198 I:	So what'd you do in No.8 now?
	267. +		
	538. +	199 E:	No.8 . . . what did I do in No.8? I don't know what I did in No.8. . . . I got off the 7's for a second there. Oh yeah. I was just going to store it in memory.
	934. +		
	728. =		
	2598. **		
	2598. SM		
		203 I:	So you redid the whole problem (i.e., problem 2).
No.2.	131. +		
	267. +		
	538. +	204 E:	Right.
	934. +		
	728. =	205 I:	And you stored what the answer should be.
	2598. **		
		206 E:	In memory.
		207 I:	Why not look at No.2's answer and store it?
		208 E:	I never thought of that.

Sometimes the failure to recall previous calculations occurs during the interview when the subject, even with the aid of the printout, cannot recall what he has done. This will be classed as a recall error, but here repetition is not produced. Ryan's mental lapse during the discussion of his work on problem 4 is a case in point:

Printout	Line	Protocol Text
No. 2. 131. +	51	I: What did you do here where you've got M+?
267. +		
538.	52	R: . . . I don't know - I can't remember now . . . I was going to do something but then . . .
936. **		
930. +		
934. *		
728. =	54	I: M stands for memory.
2598. **		
2598. M+	55	R: Memory . . .
82.6 x		
85.91 =	56	I: So you were going to do something with the memory?
7096.166 **		
	57	R: . . . I don't know what it is . . . I was going to do something but . . . I couldn't . . . I was going to do something but then I couldn't remember . . .
	60	I: How to do it?
	61	R: Yeah . . . no . . . it wouldn't work out on it . . . I thought it through and it wouldn't work out - I can't remember what it was . . .
	64	I: So this is an idea that's lost now?
	65	R: Yeah.

Production Processes and Related Errors.

After the subject has internalized the problem and compared his understanding of the problem with his recollections we reach a crucial point, where the subject must start doing something that will eventually produce a solution. Because the subject is active at this point, the processes are classified as production. The production by subjects who solve problems was found to be a relatively "grey" area in the research literature. There is research on the use of heuristics or rules of thumb that the subject can perform but investigations of the processes used by subjects themselves should not be confused with this.

The production stage is commonly referred to as the problem solver "transforming" the problem. Krutetskii describes this as ". . . the solution proper, as a stage of obtaining the desired result."[52] Newman also referred to this phase as the "transformational" determined by whether "the pupil [could] select the mathematical processes which are needed to gain a solution." Newman interchanges the ideas of "mathematical processes" and "mathematical operations" defining the later as "includ[ing] numerical, spatial, and logical operations, as well as other kinds of operations, depending on the context of the task."[53] This definition is difficult to convert into actual processes that subjects use and that can be generalized to other research. Casey identifies this inadequacy and modifies Newman's transformation so that it includes "strategy" selection and "skills" selection.

Casey made the modification because his research used multi-step problems whereas Newman's studies incorporated one-step problems. Casey makes specific reference to this fact:

> When a student comes to solve a normal multi-step problem, he often first selects a strategy or method of solution . . . . The more involved the problem, the more this category assumes importance.[54]

Skill selection is then given a secondary importance: "Having selected a plan of attack the student then selects the first appropriate skill."[55] The use of strategies will be explored in the final chapter of this research; here, an attempt will be made to isolate processes used by subjects in their work.

The importance of the transformational phase is stated briefly by Clements when he concludes that "many children fail to solve word problems because they cannot decide how they should be tackled."[56] And it is not the general conclusion that subjects can be taught or shown some standard method for solving problems, for studies of the patterns of problem-solving thinking, such as that of Buswell and Kersh as reported in Hollander, state that "The major finding was the variety, rather than similarity, in patterns of thought."[57] It is, however, the finding of this research that there are certain general strategies used by subjects when employing the calculator--an idea that will now be explained and demonstrated through examples.

There is a reasoning component to the transformational phase. To discover the reasoning employed by a subject there are inherent difficulties, described well by Kilpatrick:

> Researchers attempting to study problem-solving processes have always had to contend with the difficulty of obtaining an external indicator of the inferred mental process.[58]

In this research, the subject's statements about what he has done will be matched against the printout to determine the reasoning employed. As well as deductive (i.e., formal) reasoning, it is generally recognized that insight or sudden bright ideas also play a role in solving problems. This process will be referred to as intuitive reasoning. The role of intuitive reasoning is clearly recognized by Krutetskii:

> It is well known that the solution of the most diverse kinds of problem tasks (including mathematical problems) does not always come as a result of clear and precise sequential trains of thought. In a whole series of cases, after unsuccessful, fruitless attempts at solution, a sudden inspiration comes, a guess-seemingly accidental and inexplicable idea for a solution arises in the mind . . . .[59]

A similar process is seen by Hollander as operating when she attributes successful problem solving to a factor described as "the strength of the student's ability to reason insightfully." Deductive reasoning is also a factor, described by Hollander as "the student's ability to employ abstract analytical reasoning."[60] Wheatley labelled such insight as "has a bright idea" which she described as "The student indicated that he or she had discovered a significant step of the solution.'"[61]

The term "discovered" means that there is not a logical, step-wise progression to the process. Krutetskii removes some of the magic surrounding insight when he states

> . . . if, in a reasoning process that is completely clear to everyone, all middle links are shortened, and only the first and last links remain, this often makes a stunning impression.[62]

In this research, though no proof of such "shortened middle links" will be offered, it will be assumed that "sudden flashes" experienced by subjects will be treated as reasoning (i.e., intuitive reasoning).

The process, "produces an algorithm" is quite general and so relatively common. This process is the first indication of what the individual will do to solve the problem and quite often will eventually lead to a solution through modification. Larkin refers to the difficulty of producing, or selecting an algorithm to use:

> . . . in solving physics problems, as in many puzzles and games, selecting what "moves" to make (what principles to apply) seems to be much more difficult than actually making the "move" (applying the principle).[63]

Consider the algorithm produced by Sherri to solve problem 7 in printout No.1. It can be seen that the algorithm produced is based on the example given in the introduction of the question; in fact, Sherri makes reference to that fact:

Printout	Line	Protocol Text
Ex. 8. x	115 I:	Oh you'll do a good job . So in this first part here what are you trying (i.e., No.1)?
3. -		
1. =		
23. **	117 S:	Over here.
23. x		
3. -	118 I:	You're pointing at the example, yeah.
1. =		
68. **	119 S:	It was times 3 subtract 1. So I tried plus 1 divided by 3, and it didn't work.
68. x		
3. -		
1. =	121 I:	Is there any reason why you picked 3 and 1, because they were here (i.e., in the example on the problem sheet)?
203. **		
203. x		
3. -		
1. =	123 S:	I'd just see if they'd work.
608. **		
	124 I:	But this 3 and 1, does it go with the 3 and 1 gave you, did that give you a clue?
No.1. 140. +		
1. /		
3. =	126 S:	Yeah, sort of.
140.33333 **		

If the subject has produced an algorithm that is the result of a "transformation" of the problem there should be some reasoning behind the process and usually, even if it is not immediately successful, the subject will modify the algorithm and make another attempt. Sherri's reasoning is very clear here, based on her explanation. She sees that one of the operations given in the example on the question paper has been reversed in the question: multiply by 3 is in the example and divide by 3 is in the problem. She speculates that the missing operation is the opposite of the other operation in the example--subtract 1. The ease of computation with the calculator may be a factor in the prevalence of this process in the research.

As an example of the process where the algorithm is modified, note Sherri's next calculation, i.e., No.2, and the description she gives of what her reasoning was:

Printout		Line	Protocol Text
No.2.	140. x 1. / 3. = 46.666667 **	139 I:	I think so. What's happening now? We'll call this No.2 here, after the first set of stars.
		141 S:	I times by 1, just trying different things.
		142 I:	So you . . . ah . . . this is different effort altogether separate from the first one.
		144 S:	Yeah.
		145 I:	It's similar though, 'cause it's using 1 and 3.
		146 S:	But I decided maybe it wasn't plus and subtract so I tried multiplying and dividing and . . . it got a lower answer but also decimals so I knew that –
		149 I:	Much lower key.
		150 S:	Yeah. 46 and 6 repeating.

The reasoning here is also clear. Sherri still thinks the sequence of operations in the example, multiply by 3 and subtract 1, will generate the missing operation. In No.1 she tried adding 1 and has modified that to multiply by 1 in No.2. Both of the processes illustrated, producing an algorithm and modifying it, are incorrect in that they cannot yield the answer to the question. The process in No.1 would be classed as a transformation error, meaning that the subject's error is not selecting the correct algorithm and the second error is considered as an incorrect modification.

Englehardt found "The error type which most dramatically distinguishes highly competent performance is the defective algorithm error type." He concluded that "Apparently one of the more difficult and critical aspects of computation is executing the correct procedure."[64] Rice, as reported in Hollander, found, in her study of problem solving with 95 pupils in grades 3 through 8, that "Choosing an incorrect fundamental operation was found to be the leading error of principal in all grades."[65]

Neither the transformation or incorrect modification error types is meant to indicate that the subject is wrong, rather that the algorithm chosen has not produced the answer but it may, with modification, do so. In Sherri's case, if she added negative 6 instead of adding 1 she would get the answer. In this research, it is often observed that subjects try algorithms that are not successful, but that they use the results of these transformation errors to find the solution. In that sense, what is seemingly clearly wrong can be viewed as a part of the process that led to an eventual solution.

An illustration of the process of "repeating an algorithm with data change" is then shown in Sherri's work on printouts 3 through 5:

Printout	Line	Protocol Text
No.3. 140. x 6. / 3. = 280. **	152 I:	. . . in this third one now (i.e., No.3) you got into multiply by 6 and divide by 3. Is there a reason why you picked those numbers?
No.4. 140. x 3. / 3. = 140. **	155 S:	I thought ah . . . you know like it was doubled . . . and you want it to go down 2.
	157 I:	Each time?
No.5. 140. x 2. / 3. = 93.333333 **	158 S:	Yeah, each time. So I thought if you doubled 3 . . . that you might get the answer.
		[Omitted an irrelevant section here.]
	184 I:	. . . OK, so did that (i.e., the operations in No.3) work?
	186 S:	You got a higher answer (i.e., 280) so it it couldn't be ah . . . multiply by 6, because that made it a lot higher, well it made it double.
	189 I:	OK. So in this No.4 now, you're trying still sticking with divide by 3 but now you're timesing by 3. And do you know what you're doing?
	192 S:	Getting 140 again.
	193 I:	Why?
	194 S:	Well, because you're multiplying ~~and~~ dividing by the same thing and they cancel each other out (laughing).
	196 I:	Yup (laughing), It's OK though, it's neat, because you see things happen. Alright, so in No.5 you're multiplying by 2 and dividing by 3. Is there a reason why you wanted to do that?
	200 S:	I just totally forgot about the 6, slipped my mind. And I thought, if that gives you 140 (i.e., in No.4) multiply by 3 and divide by 3, multiply by 2 and divide by 3.

The reasoning behind seeing the connection between the printouts grouped here is that they involve the same operations with changes in data. The algorithm remains multiplication followed by division with different numbers substituted to produce an answer that is closer to the wanted sequence.

If the algorithm is repeated without a data change, then this will be viewed as a repetition error, indicating that the subject has not recalled work that was previously done, as mentioned in the error discussion in the recall section just developed. The printouts do not have to be exactly identical for there to be a duplication of previous calculations. Bruce, in his work on problem 4, makes a calculator misentry (i.e., hits the add key instead of the equal key in the second-to-last step of No.8) but from the protocol he states himself that he has committed a repetition:

	Printout	Line	Protocol Text
No.3.	11. +	334 B:	No, I didn't really notice. No.8 is just a recheck of No.3. So I did it over.
	167. *		
	267. +		
	58. +	336 I:	No.3. Is there . . . if you did the same thing again, heh.
	94. +		
	728. =		
	1158. **	337 B:	Oh yeah.
No.8.	11. +		
	267. +		
	58. +		
	94. +		
	728. +		
	1158. =		
	2316. **		

Bruce refers to the repetition error as a recheck, but based on a review of his other protocols, and this one as a whole, it seems plausible that this is a legitimate postrationalization. It is possible to see what may have caused the error; basically, Bruce is eliminating the 3 key to see if it is malfunctioning, as illustrated in the printouts (i.e., Nos. 6 and 7). The 1 key was eliminated (No.6), the 2 key (No.7) and he just continues with this line of reasoning by eliminating the 3 key in No.8, forgetting that he had already tried this in No.3. When a subject gets into the "rhythm" of modifying an algorithm it is easy to see how a repetition error might result.

To reduce the reasoning involved in the solutions to the processes of deductive and intuitive reasoning is an admitted simplification. However, based on the printouts and protocols, there are definite instances when subjects do illustrate those processes. The caution must be stated that the isolation of certain processes does not imply that those are the only processes. In fact, it is reasonably clear that the reasoning processes that are identified in this research are not sufficient, generally, to explain the mental activity involved in the whole solution process.

As an example of deductive reasoning consider Mardi's work on problem 7, and the reasoning she explains behind her calculations:

Printout		Line	Protocol Text
No.2.	140. + 6. / 3. = 142. **	102 M:	Well . . . I thought . . . if that didn't work . . . maybe it's just, I want to add 6 and divide by 3. And that's . . . 6 divided by 3 is 2 so I thought . . . and then when I saw that answer -
No.3.	140. - 6 / 3. = 138. - 6. / 3. = 136. ** 136. - 6. / 3. = 134. ** 134. - 6. / 3. = 132. **	106 I:	OK. Hold on, you're racing into the answer . . . and it does too. OK. Can you tell me again in your own words why you added 6 and divided by 3 in No.2?
		109 M:	Well . . . I wanted to subtract 2. I had to find something that left 2 when divided by 3.
		111 I:	That left 2 when divided by 3.
		112 M:	Yeah. So something divided by 3 equals 2. And that's 6. So . . . I added 6 . . . then divided . . . switched the properties.
		115 I:	Would make it smaller. Is that what you mean?
		116 M:	Yeah.

Where deductive reasoning is indicated by a step-wise interconnection of printouts and ideas, intuitive reasoning is illustrated by an abrupt appearance of a path that leads to the solution. Sometimes the "flash" occurs during the interview and so becomes obvious in the protocol. Lisa gets the bright idea to reverse the operation given in example 3 of problem 4 so she can find what the malfunctioning calculator actually did:

Printout		Line	Protocol Text
No.3.	82.6 x 85.91 = 7096.166 **	91 I:	And in No.4 you?
		92 L:	Found 4 was wrong.
No.4.	307. x 64. = 19648. **	93 I:	OK. Alright. So you found No.4. You get any ideas from what answer you get in here (i.e., 19648 in No.4)?

No.5. 7044.62 / 85.6 * 85.91 = 82. **

95 L: I took this answer and divided it by that (starting to explain No.5).

97 I: OK. In No.5 . . . when you say and this answer divided by that . . . which numbers do you mean?

99 L: Ah . . . 7044.62 divided by 85.6 . . . no . . . 85.91.

100 I: OK. You've got a single star there . . . which means that you printed in the wrong number and . . . so instead of dividing by 85.6 you meant to divide by 85.91. Now when you divided, what did you get as an answer?

105 L: 82.

106 I: Did that tell you anything?

107 L: Uh huh.

108 I: What?

109 L: 6.

110 I: What about the 6?

111 L: If you punched it in it wouldn't have registered because it didn't work . . . cause you got . . . you got the answer you would have punched in 82.

This excerpt shows the sudden appearance of the idea to divide in No.5. There is no preliminary work that shows how the idea arose. The explanation of what the result of the operation in No.5 implies is classed as deductive reasoning because of the logical base that Lisa lays in her explanation.

Deductive reasoning is given in several of the protocols where it is clearly poor logic. This type of error will be classed as poor reasoning, which really only labels the failing on the part of the subject and would require in-depth analysis to determine the causation and remediation, neither of which are objectives here. Shelley's printout record of No.6, No.7 and No.8 as well as the protocol of her reasoning clearly indicates the error of poor reasoning:

Printout		Line	Protocol Text
No.6.	2598. - 2358. = 240. **	174	I: OK. So now we're going and you're interested in No.6 in finding out what?
No.7.	7096.166 - 7044.62 = 51.546 **	176	S: Ah, what the difference was between . . . . . . ah . . . the answer that I got and the answer that was on the paper.
No.8.	19648. - 1228. = 18420. **	178	I: You're talking about problem 2?
		179	S: Yeah.
		180	I: So this difference was how much?
		181	S: 240.
		182	I: Did that tell you anything?
		183	S: Ah . . . no I was just finding out what the difference would be. I didn't get it until -
		185	I: OK. But I was just wondering if there was a reason why you were looking for the difference.
		187	S: Ah, I was going down through these ones to see if maybe the difference would be the same.

Shelley's explanation in 187-8 is very weak, a simple look at the numbers involved, without any calculation, would show that the differences could never even be close. This raises an important issue. The method of probing employed in this research is to question the subject on parts of the printout and in particular to determine if the calculated result "told the subject anything". Clearly, there are calculations that are not purposive; they are performed without any justifying reason. Such a lack of reasoning will also be classed as poor reasoning. It is an objective of this research to isolate those calculations that do not have a rationale behind them or faulty rationale rather than to analyze the reasoning. Such an analysis might be a good subject for future research.

Charlotte Wheatley[66] classed "guessing" as a productive category. Certainly the person who guessed has done something (in the sense that production is an activity) but the implication behind guessing is that the person attempts to get the answer without any effort at reasoning. This lack of reasoning will mean that guessing here will be classed as an error of poor reasoning (i.e., no reasoning). The question naturally arises as to how to distinguish between insight and guessing. The context of the protocol is the reference point from which to make this distinction. Another response that will be classed with guessing because it involves no rationalizing by the subject is the random response. With guessing, the subject feels he may have the answer while with a random response the problem solver is just doing something without any plan or real hope of getting the answer. More will be made in chapter 5 of this nonreasoning response when evidence is considered for the successive approximations category. With the ease of computation the calculator offers, there is very little that stops a subject who has no idea of what to do from pressing buttons.

As an example of a random response that could be classed as a nonreasoning error consider Shawn's choice of his first factor and his first operation in trying to solve problem 7:

Printout	Line	Protocol Text
No.1. 140. x 2. / 3. = 93.333333 **	72	I: Alright, so in No.1 what are you doing?
	73	S: I was just testing out to see how close it was to . . .
No.2. 140. x 6. / 280. **	74	I: So you picked - did you pick numbers, any numbers?
	75	S: Just 2, anything smaller than 3.
	76	I: Why pick anything smaller than 3, any reason?
	77	S: Yeah, I don't know why it would have to . . . seemed like it . . . should work out . . . it had to be smaller than 3.
	80	I: Well, if it was 3, let's say you were timesing by 3 and dividing by 3. So you know what you would get?
	82	S: You'd, you'd get the same number.
	83	I: OK. So maybe . . .
	84	S: Well I figured the number would be adding instead of taking anything off if it was bigger than 3.
	86	I: That's right. Like you see the sequence is 2 less each time. So you tried 2 and you got 93.333333. Did that tell you anything?
	89	S: Yeah, it wasn't 2. It had to be bigger. Like 2.5 or something. And then . . .
	91	I: OK. So in No.2 you decided to times - you decided to multiply at first. Was there a particular reason for that?
	94	S: No, it just . . . seemed like it should work.

In the final line of the excerpt chosen, Shawn states openly that there is no reason for his selection of the process of multiplication. In fact, his initial attempt at solution could be characterised as "randomly selecting an operation to be performed and then modifying that random choice so that he gets closer to the answer". It is also interesting that Newman, and researchers using her hierarchy, when considering errors, use the following criterion:

> . . . if a pupil did not succeed during the interview session on one of the performance strategies for any given question, but succeeded at any of the subsequent strategies of that question, then the first strategy was not regarded as having caused the pupil to answer the task correctly.[67]

The end result will be that Newman will have the one error for each subject that caused the failure. This research is interested in all the errors that subjects make, and takes the view similar to Casey's modification of Newman's work. Casey found "using questions similar to the one used to demonstrate the interviewing instrument, . . . that students make up to 10 errors in one solution attempt."[68]

Evaluation Processes and Errors

After the solution has been completed, Polya[69] sees a value in "looking back" at the solution and "reviewing and discussing it", which is a form of evaluation. Krutetskii sees a mneumonic final step where "both the process and the result of the solution always leave some trace in the memory, somehow enriching a person's experience."[70]

At this point it is important to describe the model of problem solving that was gradually developed over the course of the research. Table 6 below will be used to help in the description:

Table 6.

Model of the Problem Solving Process

Represent/understand	
read	misread
restate	misunderstand
explore	disregards information
questions conditions	overquestions
questions calculator	
Produce	
develop algorithm	wrong algorithm
modify algorithm	wrong modification
repeat algorithm	repeat with same data
reason deductively	poor reasoning
reason intuitively	poor reasoning
Remember	
recall problem	math knowledge
recall concept	calculator knowledge
recall method	forget
recall previous work	repetition
Evaluate	
analyze result	nonevaluation
identify error/answer	inhibition (repetition)
change approach	recognition

Problem State	
initial	unsolved
middle	partial
final	solved

In this table, the list to the right represents the ongoing state of the problem. This condition of the problem can be evaluated by comments made about the problem in the protocol. Four basic types of processes can be applied to the problem: representing or trying to understand the problem; recalling past material or calculations that are related to the problem; acting on the problem (i.e., producing) to transform it; and evaluating the work done on the problem up to that point. Initially there is a sequence of representing, recalling, producing and evaluating, but the model visualized sees these phases as continuing in an infinite number of possible sequences to solution if there is no forthcoming solution. The model equates the application of a process with the act of moving one of the left lists to the right so that it represents the current state of the problem. For example, reading the problem would be represented by moving the first left-hand list to the right. Recalling a related method would then be placing the recall list over the problem state on the right. As the problem solving process continues more and more left to right acts are performed, always resulting in a different state of the problem. The idea that errors are merely "negative" sides of the "positive" processes is indicated by matching within each list on the left side. The advantage to this model is that problem solving is not viewed as a linear path but rather a constructive application of processes that can follow many different paths depending on the order in which the left hand lists are moved to the right.

The role of evaluation is central in this view of problem solving. It would be equivalent to moving a left list to the right that can analyze the effects of all the translations already effected. The analyzing processes are here viewed as deeply involved in the actual solution and seen as attempts to obtain direction as the solution attempt progresses. Polya introduces the idea of foreseeing, which has certain resemblances to evaluating as proposed here. He develops the "evolution" of the solution using the concept of foreseeing as follows:

> As our examination of the problem advances, we foresee more and more clearly what should be done for the solution and how it should be done.[71]

This presupposes that the solution is not immediately obvious in which case the solution path is linear, as in many one-step mathematical "exercises" researched. Krutetskii addresses this issue of "immediate" solutions. Distinguishing capable, average and incapable subjects in terms of a factor he isolated and identified as the "analytic-synthetic process of perceiving a problem's structure", Krutetskii notes:

> The analytic-synthetic orienting activity of average pupils in perceiving the terms of a problem is precisely an analytic-synthetic process, more or less drawn out in time. The analytic-synthetic orienting activity of capable pupils . . . is so "curtailed", so maximally limited in time, that the impression is sometimes created that it . . . is not procedural . . . .[72]

This evaluating process is similar to the role reflection, as described by Kilpatrick, plays in the solution process, although, according to Hofstadter and Dennett

> . . . reflection is not a rich enough metaphor to capture the active, organizing, evolving, self-updating qualities of a human representational system--the mind is not a mirror.[73]

The role given evaluation presupposes that there is a "structure" which could be functioning as an "evaluator". Kilpatrick, in describing the mind as an information-processing device, sees three types of possible processes, which can be compared to the understanding-producing-evaluating sequence.

Those processes are listed and described as:

> . . . assembly, performance and control. . . . assembly processes translate incoming information into a usable form, performance processes use that information together with information stored in memory to produce some outcome, and control processes manage the sequence and timing of the other processes.[74]

It is suggested here that the control processes achieve their goal by evaluating. The idea of an evaluator or control is present in other research also.

After the subject has performed some operation on the problem it is important that he recognize whether or not he has achieved the solution to the problem or that his solution is incomplete. In the event that he has committed an error he must be able to determine that his result is wrong. This is referred to here as "identification" and the associated error is classed as "recognition".

Brooks noted in his study of the error rates of novice and expert programmers that, surprisingly, "experienced programmers initially make about the same number of errors as beginning programmers, but are able to find their errors faster."[75] Another study, reported in Sheil, also found that novice and expert programmers "made about the same number of errors in their original programs, the experts removed their errors much more quickly."[76]

It may seem obvious that the subject should know when he has solved the problem and yet Casey recognizes that this is not always the case and divides Newman's encoding category into two subcategories, one of which was "the ability to recognize an 'answer' as <u>the</u> answer to the problem." He went on to comment that

> Sometimes students stumble through a problem and are unaware when they have the answer. They do not realize that they have actually found what they were looking for.[77]

As an example of the recognition process consider Sherri's work in her last calculations on problem 7:

Printout		Line	Protocol Text
No.10a	140. - 1. / 3. = 139.66667 **	343 I:	So now you've decided you're going to go to subtraction, and looking down ahead quickly you can see you've done a whole bunch of subtractions.
No.10b	140. - 2. / 3. = 139.33333 **	346 S:	Uh huh.
		347 I:	Alright -
		348 S:	See what I did was I did 1, like I -
No.11	140. - 3. / 3. = 139. **	350 I:	Yeah, yeah, right.
		351 S:	So I figured it had to be a higher number so I -
No.12a	140. - 6. / 3. = 138. **	352 I:	Higher than what?
		353 S:	Higher than 1. So I subtracted by 2 and divided by 3 (i.e., No.11).
No.12b	138. - 6. / 3. = 136. **	355 I:	Yes.
		356 S:	And I got 139 and 3 repeating -
	136. - 6. / 3. = 134. **	357 I:	Ah, does that tell you anything?
	134. - 6. / 3. = 132. **	358 S:	Yeah, like every time you go up one, you subtract by one more, it goes down 3 in the decimal place (i.e., 3 repeating). Right?
		361 I:	Yeah, I got you.

362 S: So when I subtracted 3 and divided by 3 I got 139 (i.e., No.12a).

364 I: So it stayed the same. Oh no! It went down by 1!

365 S: Yeah, went down by 1. So to do that –

366 I: Just a sec. So you're saying it's going down a third.

367 S: By a third. So what I did was when I got the answer 139, I doubled 3. Like I doubled 3 because I wanted to go three more down. So that's how I got the answer. And it's subtract 6 and divide by 3.

It should be noted that Sherri does not realize that she has the solution all at once, but rather gradually. She accurately describes the approach towards a solution and there is evidence that she realizes she is near a solution and that her algorithm will be successful. Polya addresses this idea by considering the concept of progress, which he describes as

> Advancing mobilization [i.e., extracting relevant elements from memory] and organization of our knowledge, evolution of our conception of the problem, increasing prevision of the steps which will constitute the final argument.[78]

In my view, the excitement Sherri conveys towards the end and her willingness to describe her reasoning all indicate that she did have some idea of the steps that would lead to a solution before she actually performed them.

A recognition error is evident in Ruth's work on problem 4 where her comment expressing uncertainty about the solution is a clear indicator that she doesn't "know" she has the answer:

Printout	Line	Protocol Text
No.1. 41.7 + 5079. = 5120.7 ** No.2. 131. + 267. + 538. +	17 I:	OK. So you've been working on it for about 15 minutes. I think what we'll do is we'll just go over what you have. OK. And then . . . ah . . . from that we'll get some ideas what's happening. Cause you think you have got the answer, do you?
934. + 728. =	22 R:	I'm not sure.
2598. **	23 I:	You're not sure, you may have it?
No.3. 82.6 x 85.91 =	24 R:	Yeah.
7096.166 ** No.4. 307. x 64. = 19648. **	25 I:	We'll go over it. And we'll see. I'll just take out what you've got in there. OK, just off hand, we'll go over this. What key do you think isn't working?

The protocol then establishes that Ruth checked all four examples in the first four printouts as can be seen in the printouts included with the text immediately above. The cause behind Ruth's doubts are then indicated in the protocol as follows:

Printout	Line	Protocol Text
No.5. 131. + 297. +	129 I:	And you got 19648 (i.e., in No.4). Did you conclude anything from that?
538. + 934. + 728. = 2628. **	131 R:	Well, that it had to be the 6. Cause that had a 6 and that had a 6 and that had a 6 (i.e., all three problems that were wrong).
	134 I:	Why couldn't it be a 2?
	135 R:	Because it (i.e., problem 4) didn't have a 2 in the equation.
	137 I:	In problem 4.

138 R: Yeah.

139 I: OK. So what change did you make then?

140 R: Well I . . . tried to make the 6 a 9 but it didn't work out. Like you know (see No.5 which was not discussed)?

Clearly, Ruth interpreted "malfunctioning", as presented in the text of the question, to mean that the calculator entered a 9 when the 6 was depressed. Her answer of 2628 did not duplicate the error in the question (i.e., 2358) and so she thought that there may have been an error in her work.

That analyzing the results is an important part of problem solving is indicated by several researchers. Kantowski not only sees analysis as important but also gives a good description of the evaluation role in problem solving. Kantowski, in a study that analyzed the sequences of processes used by high-ability ninth grade algebra students after they had received an "heuristic method of instruction" found

> Evidence of regular patterns of analysis and synthesis (deduction inferred from hypotheses followed by a synthesis, then by further inferences from the new synthesis and so forth) . . . .[79]

Larkin constructed a simulation program for solving physics problems that was designed to model memory. In her use of a worksheet where related information was written together and the information was sequential (with recent information towards the bottom) she saw the

> . . . problem-solver as "stimulus driven"--there [was] a constantly repeated cycle of interaction between thought processes and the gradually developing body of information on the worksheet.[80]

Larkin's worksheet is also thought to duplicate what subjects do here with the printing calculator. By entering calculations as they think of them and having the last calculation as the most visible (i.e., the paper advances, moving past calculations further from the subject), the conditions of Larkin's worksheet are reproduced.

Analyzing the results of calculations was found to be an important factor in problem solving according to Doty (as reported in Hollander) where, in attempting to related effective and ineffective procedures on verbal problems to "underlying causes in thinking" he found a "lack of critical evaluation of attempts of solution"[81] to be a primary cause of error. Matz describes the role played by evaluation of current productions in the following:

> When students do not assess whether a rule [i.e., those methods that the subjects know] gets them closer towards a goal, they execute steps that are obviously applicable, but not productive; their work has an aimless character.[82]

In this research it was found that subjects did not assess the results of their calculations and that this contributed to many protocols where the resulting printout was "aimless" and "unproductive" in character. Where students do analyze their calculations, the results do yield success. Consider the calculations and resulting analysis of Ruth, again, in problem 4:

	Printout	Line	Protocol Text
No.6.	131. +	163 I:	What have you got?
	267. *		
	207. +	164 R:	OK. For this . . . for the . . . third and fourth problem I worked it out with leaving the 6 out. And the second one it didn't.
	538. +		
	934. +		
	728. =		

2598. **

167 I: Let's just advance this for a minute. Cause you understand what you're doing. So in this one, No.6, what did you do there?

No.7. 82. x
85.91 =
7044.62 **

171 R: I took out the 6 . . . made it 207 -

No.8. 307. x
4. =
1228. **

172 I: But now, let's look at what you've got here. You get 131, 267?

174 R: But I took it out with the point (note the single star in the printout).

176 I: Oh I see. That's just an error and you printed it. And then you went 207. So you took it (i.e., the 6) out and replaced it with a 0.

179 R: Uh hum.

180 I: And it didn't work out.

181 R: Yeah.

182 I: OK. Now let's take a look . . . we'll skip ahead. And let's look at No.7. Now, here you got 82 point . . . 6 and you left the 6 off. Did you replace it with a 0 in there?

186 R: No cause it won't make a difference, the 0.

187 I: OK. And you got the answer they got, heh?

188 R: Uh huh.

189 I: Alright. So in No. 7 you were able to duplicate the error in the problem. No.8 . . . No.7 you duplicated the error in problem 3. In this one (i.e., No.8) you duplicated the problem in?

193 R: In No. -

194 I: 4. How did you do it?

195 R: Just took the 6 out. 4 and not 64.

196 I: OK. So you see what you're doing wrong in here? Or what you're not doing in here in in No.6 that you've done in No.7 and No.8?

199 R: Oh! Make it 27.

From the protocol it can be seen that Ruth has analyzed the printout. She can readily talk about the changes she has made in each calculation to produce the error given in the problem. She has clearly compared the given error with the calculated error and knows (i.e., identifies an error) that she has done something incorrect in No. 6. By running her through the analysis again she is able to determine the change that should be made in No.6 and finds the way to produce the final incorrect answers.

A common type of analysis to apply to the calculations is to look for a pattern. Sherri did this quite successfully in her work on problem 7:

Printout		Line	Protocol Text
No.11	140. - 3. / 3. = 139. **	358	S: Yeah, like each time you go up one, you subtract by one more, it goes down 3 in the decimal place (i.e., 3 repeating).
		361	I: Yeah, I got you.
No.12a	140. - 6. / 3. = 138. **	362	S: So when I subtracted 3 and divided by 3 I got 139 (i.e., No.12a).
		364	I: So it stayed the same. Oh no! It went down by 1!
No.12b	138. - 6. / 3. = 136. **	365	S: Yeah, went down by 1. So to do that -
	136. - 6. / 3. = 134. **	366	I: Just a sec. So you're saying it's going down a third -
	134. - 6. / 3. = 132. **	367	S: By a third. So what I did was when I got the answer 139, I doubled 3. Like I doubled 3 because I wanted to go three more down. So that's how I got the answer. And it's subtract 6 and divide by 3.
No.10a	140. - 1. / 3. = 139.66667 **		

```
No.10b    140. -
            2. /
            3. =
     139.33333 **
```

Sherri is analyzing the printout closely and solves the problem by discovering a pattern--that increasing the number she subtracts by 1 (i.e., from 1 in No.10a to 2 in No.10b to 3 in No.11) produces a pattern: a decrease of one third each time (i.e., from 139.66667 in No.10a to 139.33333 in No.10b to 139 in No.11).

Not analyzing, not evaluating the printout, produces the kind of protocol that Matz labels as "unproductive". This description is especially valid, for it removes the emphasis from the concept of being wrong to one of being efficient. In the protocols, there is ample evidence that subjects could have greatly reduced the length of calculations had they only looked at results they had previously done and assessed those more carefully. Consider Sherri's work on problem 7 again, to illustrate that the same subject who can make an initial error in one dimension of the process--not analyzing calculations here--can also be the subject who later successfully uses this process.

Printout	Line	Protocol Text
No.1. 140. +	131	S: Well you didn't get the right answer.
1. /		
3. =	132	I: Well you didn't get 140 did you? Or 138.
140.33333 **		
	133	S: I get higher than 140 and you want to get lower, want to get 130.
No.7. 140. +		
46. /		
3. =		
155.33333 **		

Clearly Sherri realizes that the operation pair she produces does not reduce the sequence. But based on the results from later in the protocol it is apparent that she does not analyze the calculations to see how the 140.33333 is generated.

Printout	Line	Protocol Text
	222	S: Well . . . I did something — I added, what did I add? I meant to add, ah . . .
	224	I: From the next step (i.e., No.7) you meant add 46, is that what you're talking about?
	226	S: Yeah. I added 46, now why do I do that? Adding it was changing from multiplying, right, and what I did was I divided 3 into 140 and I got 46 so I thought I add 46 to 140 and divide by 3, that number (i.e., 156.33333) but it got higher than 140—
No.8. 140. + 49. / 3. = 156.33333 **	231	I: Do you have any idea why?
	232	S: Ahm . . .
	233	I: This is No.7 now.
	234	S: Because . . .
	235	I: This is interesting Sherri, you see. Part of this problem you have to know the order of operations in which the calculator works.
	238	S: Yeah. And that I did wrong. Because I did adding first and it has to be dividing or multiplying before adding or subtracting.
	241	I: But did it add first?
	242	S: No.
	243	I: What did it do first?
	244	S: Divided.
	245	I: So what it will do first is 46 divided by 3 and then it will add that to 140.

247 S: 146, no that's 46.

248 I: Do you follow what I mean?

249 S: Yeah.

250 I: Like you see this - this 155.33333 is actually 15.33333 higher than 140. So that tells you that 46 divided by 3 is 15.33333. Do you follow?

253 S: Uh huh.

254 I: Did you realize that when you were doing it?

255 S: No. I just realized it now.

The issue raised here is that the subject did not try to analyze the printout to see how the numbers produced were derived. There was no attempt to do what was illustrated in lines 250-252. Another salient point is the difficulty of classifying errors when processes overlap. If the subject had recalled the general result in No. I, she might not have performed the operations in Nos. 7 and 8. However, if Nos. I, 7 and 8 are considered together it seems that she has had ample opportunity to determine how the addition and division produce the results generated and this analysis was not done. Errors of omission, by their very nature, are more difficult to detect for it is easy not to notice what is not there.

The process of making a routine check is considered an evaluation process but often it enters into the initial attempts a subject makes to understand the problem. In that sense it is really an "exploratory manipulation". The distinction between classifying the check as an evaluation or understanding must be made within the context of the protocol. When a check of each of the four parts of problem 4 is made, then those checks are considered part of the subject's attempt to understand what the problem is about. However, if the subject has determined the key that is malfunctioning and then checks each problem by duplicating the error, the check is part of evaluation--it is an attempt to verify the correctness of an hypothesized solution.

The work of Sherri on problem 8 illustrates the types of checking well:

Printout	Line	Protocol Text
No.1a 274. + 882. + 1028. = 2184. **	10 I:	No, then you'd be using the 7 button. So you can't do that. However, like what you did here . . . let's just stop for a minute.
	13 S:	Figuring out what the problem was like so I added them altogether and I divided by 7.
No.1b 2184. / 7. = 312. **	15 I:	OK. So in No. 1a you punched in the numbers that were there.
No.2. 2184. / 8. = 273. **	16 S:	Yeah and then I added them and divided the sum by seven.
	17 I:	OK. Let me ask you. Why did you punch the numbers in?
No.3. 312. + 312. + 312. + 312. + 312. + 312. + 312. = 2184. **	18 S:	I don't know. I just wanted to see what the sum would turn out to be.

Her last calculation (i.e., No.3) is also a check, but now, unlike her exploratory check in No.1a, it is to verify a result that she has determined as a possible solution in No.1b:

Line	Protocol Text
28 I:	Yes Sherri?
29 S:	Would that be possible? OK. I did use the seven over here (i.e., No.1b) to get that. But then what I do is add that number seven times. And you get the answer. But it's still using seven, isn't it?

It is interesting that as well as being a check, Sherri also solves the problem in the same step (i.e., No.3). Getting 2184 in No.1a and No.3 shows her that her answer is correct. Checking is purely a voluntary process initiated to test an answer; and though suggested as a valuable step, it is very common to note that subjects do not check their work generally. Not checking will not be considered as an error here.

An indication that there has been some evaluation is for the subject to "change her approach", to try a different method of attacking the problem. In order for such a change in direction the subject must evaluate the direction first followed and decide that this will not lead to a solution. Matz refers to a "Rule . . . applied repeatedly to an expression instead of once"[83] as linked to a phenomenon called "linearity" which "describes a way of working with a decomposable object [i.e., a problem in this research] by treating each of its parts independently."[84] Radatz refers to a similar error in describing errors linked to "rigidity of thinking".

Delving into possible causes he suggests that "Some aspect of content or of solution process persists in the mind, inhibiting the processing of new information."[85] Radatz is referring to the effect previous learning has on present tasks but in this research the "inhibition" error will be demonstrated without any suggestion as to the possible cause. Krutetskii asked subjects to solve a solved problem in a different way and by comparing very capable, capable and incapable pupils he was able to isolate a qualitative general factor he called "flexibility of mental process" that distinguished these groups. Krutetskii describes the factor in this way:

> Mathematically able pupils are distinguished by great flexibility, by mobility of their mental processes in solving mathematical problems. It is expressed in a free and easy switching from one mental operation to another qualitatively different one . . . . Incapable pupils are marked by inertness, sluggishness, and constraint in their thinking in the realm of mathematical relations and operations, by the settled, stereotyped character of their operations, and by the importunate retention in their minds of a previous principle of solution or of a method of operation that exerts an inhibiting influence when an operation needs to be reconstructed . . . .[86]

If a subject has produced (directly or from recall) an algorithm that is not immediately successful there comes a point in the solution process where he must evaluate it. Perhaps a modification will result in a solution. If not, then the subject should change his approach. To continue with an algorithm in a blind, repetitive manner will here be viewed as the error of "inhibition".

Polya alludes to this inhibition in the following way: "Human nature prompts us to repeat a procedure that has succeeded before in a similar situation."[87] He refers to the tendency as

> . . . an overdose of mental inertia: our mind persevered in the same course, although this course should have been changed by the influence of the circumstances.[88]

Sherri's work with problem 1 will illustrate both the positive condition of changing her approach and its negative counterpart of inhibition:

Printout	Line	Protocol Text
No.4a 9000000. x 1.0000001 = 9000000.9 **	100 I:	Let's take a look at No.4. What was your idea there?
No.4b 99999999. / 1.0000001 = 99999989. **	102 S:	I took the . . . I took a million off . . . that first number, like 10 million, I took a million off, to make 9 million, and then timesed it by 1 and . . . you know (i.e., 1 and ten millionths).
No.4c 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **	106 I: 108 S:	What was your idea there - what did you hope was going to happen? Well I thought maybe it might work out - it might come to . . . the answer but it didn't.
	110 I:	Now, from working with all these zeros and stuff, I noticed in the next step in No.4b, you're using a lot more 9's. Why?
	113 S:	Well, I noticed that in No.4a, when I timesed 1 and tenth of a million, that the 9 million point 9, so the 9 had to be there. And so I figured that if I put a whole bunch of 9's would come out in the answer.
	118 I:	OK. But you've got eight 9's in there.
	119 S:	Because it's eight digits in the first number -

120 I: But if you look at the pattern you want you have only got seven (i.e., 9's).

122 S: I didn't realize that.

123 I: Anyways, didn't you get any ideas from that though?

124 S: Yeah, that I had one too many because it came out . . .

125 I: One too many what?

126 S: One too many 9's.

127 I: So then what did you do?

128 S: I put seven 9's and timesed it by 1 -

129 I: Divided it.

130 S: Divided it. Yeah, I divided it before that too.

131 I: Why did you decide to divide instead of multiply?

132 S: Well, multiplying - I don't know - would just seemed to be getting me nowhere so I tried dividing, it was a different step - you know . . . to experiment sort of. And . . . I . . . timesed them, and then it worked.

137 I: Divided you mean.

138 S: Divided it. And it worked. But it came out with an 8 on the end of it (i.e., in 9999998).

All of Sherri's operations (Nos. 1 to 4a) were multiplications and after evaluation (with no supporting reasoning) she decides to change her approach as multiplying "seemed to be getting [her] nowhere".

She does not change her approach in Nos. 5a through 7b when she is "fixed" on the idea that the first term in her division must have 9's in it and cannot be larger than 99999999. Consider her calculations and the associated discussion.

Printout		Line	Protocol Text
No.5a	99999990. / 1.0000001 = 99999980. **	171	I: Oh, you divided by a bigger number. Why? Did you have any reason for trying that? OK, if you look at what you first did in No.5, heh?
No.5b	9999999. / 1.000001 = 9999989. ** 9999989. = 9999979. **	174	S: I had to get ah . . .
		175	I: The answer you're getting is very big (i.e., 99999980). So were you trying to make it small or something?
No.5c	99999999. / 1.000001 = 99999899. ** 99999899. = 99999799. **	177	S: Well yeah, and also . . . ah, I don't know.
No.6	999999. / 1.0000001 = 999998.9 **	178	I: Well you can make the answer smaller either by multiplying - by making the dividend smaller or the divisor larger. Now in most of these, in No.4, you kept the divisor constant (i.e., 1.0000001). Now in No.5 you started altering the divisor and sticking with the dividend (i.e., 999999) and it decreased by 10 each time.
No.7a	9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. **	185	S: And then I did it again. Only I took another zero out.
		186	I: To make it even larger, heh?
No.7b	9999991. / 1.0000001 = 9999990. **	187	S: And make a hundred - it decreased by a hundred.
		188	I: So that made sense, doesn't it? Like if you make the divisor larger by 10 then you will make the sequence of answers smaller by 10 times what it was.
		191	S: Yeah.
		192	I: Now in No.6?

193 S: I tried taking the 9 off and returning the second number to the full (i.e., 1.0000001 instead of 1.000001).

196 I: But you only went one step in No.6.

197 S: Well it came to a decimal . . . and . . .

198 I: You didn't like that.

199 S: You know it's not the answer so . . .

200 I: So now in No.7. Looks like that's like No.4.

201 S: I just wanted to get it on here so I could look at it and see how it sort-of worked out. Then I tried putting a 1 in the place of a 9.

204 I: Why?

205 S: I don't know . . . just seemed like something to do.

A good deal of the protocol is included because the subject explained very little and the researcher went off into a bit of a lesson. However, it is clear that Sherri has decided that only the dividend will be altered in Nos. 5 through 7. It is this "block" that stops her from making the small change she needs to make to No.4 or No.7a to produce the answer. When shown this change she realizes how close she was to the answer.

This chapter will now close with a list of the processes coded for each subject by problem. This is a compilation of the classification performed in the appendix and constitutes the main body of the research. From this list it is possible to obtain a general idea of the prevalence of each process.

It should be stressed that the numbers attached to each process are not to be used in any statistical way but rather as a general indication of the occurrence of a particular process.

Table 7.

Process Frequency by Problem and Subject.

Problem 1

Subject Process/Error	Ma	Ev	Sh	Ea	Li	Br
21/81	-/-	-/-	-/3	-/-	-/-	-/2
22/83	-/1	1/3	1/1	1/-	-/2	1/1
20/82	-/-	2/-	1/-	-/-	2/-	5/1
24/84	-/-	-/-	-/-	-/-	-/-	1/-
30/	-/-	1/-	-/-	-/-	-/-	-/-
31/85	1/-	-/1	-/1	-/-	-/2	1/3
32/87	-/-	-/-	-/-	-/-	-/-	-/-
33/86	4/-	2/-	2/1	3/-	2/-	5/-
47/92	1/-	1/1	1/1	-/2	-/1	-/4
49/91	1/-	5/1	3/-	-/-	1/-	2/-
48/86	-/-	-/-	4/-	4/-	2/-	1/-
40/93	1/-	1/-	2/-	1/-	2/-	4/-
41/94	-/1	1/1	-/3	1/-	1/-	1/-
54/95	-/-	2/2	2/2	2/-	3/1	4/1
55/96	1/-	2/-	10/-	2/-	4/1	8/1
53/97	-/-	-/-	1/-	-/1	-/-	-/-

Problem 4

Subject Process/Error	Ev	Sh	Ea	Li	Br	Ru
21/81	-/-	1/-	-/-	-/-	3/-	-/-
22/83	-/-	2/1	4/2	-/-	1/-	-/1
20/82	2/1	-/2	1/2	-/-	3/1	-/-
24/84	-/-	-/-	-/-	-/-	-/1	-/-
30/	-/	-/	-/	-/	1/	-/
31/85	-/1	-/-	-/1	-/-	1/1	-/-
32/87	-/-	-/-	1/-	-/-	-/-	-/-
33/86	2/-	1/-	4/2	-/-	-/2	-/-
47/92	3/2	1/3	5/11	3/1	4/6	3/1
49/91	-/-	1/-	3/-	-/-	-/-	1/-
48/86	6/1	8/1	17/4	5/-	9/-	5/-
40/93	12/-	8/-	12/-	4/-	13/2	9/-
41/94	1/-	1/-	1/3	2/-	-/2	2/-
54/95	2/-	5/1	5/1	3/1	6/2	3/1
55/96	10/1	5/1	18/2	8/-	5/3	9/-
53/97	2/-	-/-	1/-	-/-	-/-	-/-

Problem 4 (continued)

Subject Process/Error	Sl	Cl	To	Ry
21/81	1/-	-/-	-/-	-/-
22/83	-/1	1/2	-/-	-/-
20/82	-/3	3/6	-/1	-/1
24/84	-/-	-/-	-/-	-/-
30/	-/	-/	-/	-/
31/85	-/-	-/1	-/1	-/2
32/87	-/-	-/-	-/-	-/-
33/86	-/-	-/1	-/1	-/2
47/92	3/3	2/4	3/3	2/1
49/91	4/-	1/-	1/-	3/-
48/86	10/1	11/-	7/-	7/-
40/93	11/1	8/-	14/-	9/-
41/94	1/2	-/1	-/-	1/-
54/95	7/-	3/3	3/2	9/-
55/96	13/5	11/4	12/1	10/-
53/97	-/-	-/1	-/-	-/-

Problem 7

Subject Process/Error	Sh	Br	Ma	Sn
21/81	-/-	2/-	4/-	1/-
22/83	2/-	-/1	-/-	1/1
20/82	4/-	1/-	1/-	3/-
24/84	-/1	-/-	-/-	-/-
30/	-/	-/	-/	-/
31/85	-/1	-/-	-/2	-/1
32/87	1/-	-/-	-/-	1/-
33/86	2/-	-/1	-/-	-/-
47/92	1/4	1/-	1/-	3/2
49/91	2/-	-/-	1/-	1/-
48/86	9/-	8/1	1/-	7/-
40/93	12/-	5/-	5/-	8/-
41/94	-/2	-/4	-/1	-/3
54/95	5/1	5/-	1/-	4/1
55/96	13/3	10/3	2/1	11/3
53/97	-/-	-/-	-/-	-/-

Problem 8

Subject Process/Error	Sh	Wa	Ea	Ru	Li
34/81	-/-	-/-	-/1	-/-	-/-
27/83	1/-	1/1	1/-	-/-	-/-
20/82	4/-	-/2	-/1	-/-	1/-
24/84	-/-	-/-	-/-	-/-	-/-
30/[illegible]	-/-	-/-	-/-	-/-	-/-
31/85	-/-	-/1	-/-	-/-	-/-
32/87	-/-	-/-	-/-	-/-	-/-
33/86	2/1	1/-	-/-	-/-	1/-
47/92	2/-	2/1	3/-	3/-	2/-
49/91	1/-	2/-	2/-	2/-	1/-
48/86	2/-	2/-	2/-	1/-	3/-
40/93	1/-	2/-	1/-	2/-	3/-
41/94	1/-	-/-	-/-	-/-	-/-
54/95	2/-	2/1	6/1	3/-	-/4
55/96	4/-	-/2	2/-	2/-	5/-
53/97	-/-	-/-	-/-	-/-	-/-

Code Explanation

Subjects

Br = Bruce
Ea = Earl
Li = Lisa
Ru = Ruth
Sa = Shawn
Sl = Shelley
Wa = Wayne
Cl = Clive
Ev = Evan
Mg = Mardi
Ry = Ryan
Sh = Sherri
To = Tonia

Processes/Errors

21/81 = restates/misreads
22/83 = explores/misunderstands
20/82 = questions conditions/disregards information
24/84 = questions calculations/extraneous questioning
30/ = recalls problem
31/85 = recalls conditions/knowledge insufficient
32/87 = recalls method/forgets
33/86 = recalls calculation/repetition of previous work
47/92 = produces algorithm/transformation error
49/91 = modifies algorithm/incorrect modification
48/86 = repeats algorithm with data change/repetition of previous work
40/93 = reasons deductively/poor reasoning
41/94 = reasons intuitively/random response (guessing)
54/95 = identification/recognition error
55/96 = analyzes/nonevaluation
53/97 = changes approach/inhibition

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CHAPTER V

Successive Approximations as a Strategy

Analyzing the calculations of subjects by separate sections that are numbered is helpful for reference during and after discussion. However, it becomes apparent that there are relationships between blocks of calculations and that two adjacent calculations may actually reflect very different approaches to the problem at hand. The question, put simply, also arises as to whether the subject really "knows" what he is doing--in short, are the calculations performed part of a larger, global plan? What makes this question particularly challenging is that subjects can appear to know what they are doing with a printing calculator for all they have to do is press buttons and the calculator generates something.

A common strategy suggested for the solution of problems is that of successive approximations. Silver found, in examining the work of subjects solving word problems, that

> . . . their problem solutions revealed that successful problem solvers used a successive approximations approach on many of the problems.[1]

Polya describes the strategy as

> . . . a series of trials, each of which attempts to correct the error committed by the preceding and, on the whole, the

He equates trial and error with successive approximations stating that the latter is a term of "better characterization".

Birtwistle also sees the approach towards a goal as fundamental to successive approximations:

> . . . we must watch what is happening to the successive approximations. As we get each new approximation to the required answer, it should be approaching some particular value.[3]

Simon raises an important issue in his discussion on the trial and error technique:

> . . . trial and error is not completely random or blind; it is in fact rather highly selective. The new expressions that are obtained by transforming given ones are examined to see whether they represent progress toward the goal.[4]

It is suggested that Simon clearly indicates when trial and error can be viewed as successive approximations in stating that "Problem solving requires selective [this word is emphasized by Simon] trial and error."[5]

Hadass indicates a use for "true" trial and error in trying to construct equations from word problems. He describes the trial and test method as consisting of

> . . . several guesses of trial values for the unknown, each followed by testing its validity step-by-step throughtout the story. In this method, trial values are chosen freely and not [Hadass emphasizes this word] in order to hit the right solution.[6]

To determine the underlying equation the subject is directed to put "the data of several trials in a table, [and] the repeating pattern [and underlying equation] becomes apparent."[7]

It is suggested that the calculator is excellent for such "blind" substituting. In this research, subjects who proceed in this manner, and many do, will be viewed as operating without a strategy or plan. This method will also be contrasted with successive approximations, which does have a direction, as explained by Krutetskii. Krutetskii takes exception with those researchers who

> . . . regard learning as an activity that proceeds on the basis of trial and error, accidentally producing operations by guesswork, by the selection of felicitious operations (which are confirmed and therefore reinforced) and the gradual falling away of the infelicitous operations.[8]

It seems reasonable to suggest that Krutetskii would disagree with Simons' characterization of human problem solving as

> . . . nothing more than varying mixtures of trial and error and selectivity. The selectivity derives from various rules of thumb, or heuristics, that suggest which paths should be tried first and which leads are promising.[9]

Subjects who solve problems in this selective way will be viewed as controlled by the situation and not as adept as those solvers who have a preplanned course through the problem. The planning pupils would be similar to Krutetskii's capable subjects whose trials "were always purposeful, systematized attempts directed toward verifying the assumptions they had made."[10]

It is also indicated in the research that a subject may be initially unclear as to the nature of the question and many of the initial calculations are exploratory manipulations performed in an attempt to understand.

The assumption will not be made that the subject understands the question once he starts to perform calculator operations. Rather, the following observation by Krutetskii will be used as a guide in interpreting subjects' work:

> . . . when able pupils were solving hard problems, the trials were often not so much direct attempts at solving a problem as a means of thoroughly investigating it, with auxilary information being extracted from each trial.[11]

After these initial explorations, however, the subject should then take a direction in his calculations and a strategy should become obvious as he progresses.

Rice, as reported in Hollander, "administered a standardized test of problem solving to 95 pupils in grades three through eight" and found that

> A fifth of the subjects, interviewed individually, could give no reason why they had proceeded in their problem-solving attempts as they had.[12]

Lankford, in his research on computational strategies of seventh grade pupils was very general in his conception of a strategy, deciding that "A pupil's pattern of thinking as he computes may be called his 'computational strategy'."[13] Englehardt, distinguished between random response errors and defective algorithm errors (where the pupil responded by executing a "systematic", but erroneous procedure) by describing the latter as having "steps [that] were explainable, and responses to similar computational tasks were predictable."[14] Subjects then using algorithms, even defective ones, can still be perceived as using a strategy.

The model presented by Mayer of the programmer writing a program and its reliance on the idea of a plan may be relevant here. After understanding the program demanded and recalling the necessary information, Mayer sees "The second step [as] devising a general plan for writing the program that follows a pattern [of] stepwise refinement." Mayer describes the problem solution as

> . . . conceived of in general programming strategies. . . . this internal representation progresses from a very general, to a more specific plan, to a specific generation of code focusing on minute details.[15]

Mayer draws a comparison between this model and Polya's "working backwards" heuristic. In this research, a plan or strategy will be said to exist when there is such a "pattern of stepwise refinement". This is always assuming that the subject is indeed faced with a problem--a situation where the solution is not immediately obvious.

The strategy chosen here for investigation is successive approximations, principally because of it's prevalence in the research. An attempt will be made to distinguish this strategy from trial and error, the assumption being made that the two methods represent planned and unplanned approaches to problem solving respectively. Lankford, in research just referred to, detailed the type of randomness in problem solving behaviour that will here be classified as unplanned or nonstrategic when he notes the following characteristic of poor computers:

> . . . [they] often switched to something else that would produce an answer, when they encountered difficulty with an improvised algorithm, regardless of how remote from the proper procedure it might be.[16]

Now attention will be directed to work by subjects in this research, where trial and error is contrasted with successive approximations and the criteria for the distinction will be explained. It will be necessary to consider larger segments of the calculations, for strategies are dispersed over more of the subject's work; indeed, the strategy may not become apparent unless a good deal of the work is analyzed. Consider the calculations of Clive on problem 4 as an example of trial and error rather than planned operating (i.e., successive approximations). First, in No.9, Clive decides not to use the . key, in effect reducing the second factor by one one-hundredth. This produces a product of 7095.34. He is trying to produce the given erroneous answer of 7044.62. He notes that his answer is too large in the following:

	Printout	Line	Protocol Text
No.9.	82.6 x 85.6 * 85.9 = 7095.34 **	219 C:	Yeah, well it came . . . it came a bit closer.
		221 I:	. . . was there any reason why you picked the number 1?
		223 C:	. . . I was just sort of randomly choosing them.

Clive has admitted to making his choice by chance rather than design. His next change in No.10 is to eliminate the 8 key in the second factor, in effect reducing the second factor by 80 as the 8 is in the tens place. He produces an answer of 488.166 which is very much lower than the 7044.62 wanted.

The investigator then tries to see if he has analyzed this result:

Printout		Line	Protocol Text
No.10.	82.6 x 5.91 = 488.166 **	234 I:	And what are you doing now (i.e., No.10)?
		235 C:	I just timesed it out to see what it would come to . . .
No.11.	82.6 x 85.1 = 7029.26 **		[A discussion about an error that is not relevant here.]
No.12.	82.6 x 85. = 7021. **	247 I:	So you got 488.166 – did that tell you anything?
		248 C:	Well that – that was . . . ah . . .
No.13.	82. x 85.91 = 7044.62 **		PAUSE (10s)
		249 I:	Or didn't it (i.e., tell you anything)?
		250 C:	It didn't really say that much no . . . thought that was wrong.

Clive is not analyzing the result and is just seeing if the answer is right or not. From what he has done in No.9 and No.10 he now eliminates the 9, in effect reducing the second factor by nine-tenths. This produces an answer that is closer to the 7044.62 wanted but is lower. Based on this calculation, if he is using successive approximations as a strategy, then the second factor has to be increased over what it was in No.11. When he makes the second factor even smaller than it was in No.11, he is clearly choosing keys at random and is using trial and error to find the key, by a sort of exhaustion (i.e., eliminating every key until he finds the answer). He is successful in No.13 using this method. It is tempting to postulate that Clive does use successive approximations especially as he keeps stating that his answers are close to the wanted 7044.62 (in lines 256 and 276 of the protocol). However, aside from the random picking of a key to eliminate each time, there is no plan to get closer and closer to the answer by making proper modifications.

Analyzing Evan's work indicates a trial and error approach too, or at best a poor use of the strategy of successive approximations. Again the work is done on problem 4 and looking at Nos. 5 to 7 and No.9 it is easy to see that Evan has not used the results of his calculations to modify his algorithm correctly. He wants to produce the erroneous answer of 7044.62. By multiplying by 82.4 he produces a product of 7078.984. If he is using the strategy of successive approximations correctly and he wants to modify the factor 82.4, he should make it smaller. But he chooses to change 82.4 to 82.8, going in the direction of making the product larger, and indicating he is not modifying 82.4 according to a plan. His change of 82.8 to 82.2 is in the right direction (i.e., it will make the product smaller) but 7061.802 is still too large, and in No.9 when he makes 82.2 into 82.7 he indicates again that the changes are not being made under any strategy of successive approximations but rather by a random process of elimination.

Actually, Evan does not indicate in the discussion that he has not met the requirements of successive approximations and from the protocol, the natural assumption would be that he has been constantly analyzing printouts and repeating his algorithm with the correct data changes.

The most significant aspect of his work is [illegible] decision to change his approach and work with a different part of the problem. To illustrate all these points a rather long section of Evan's protocol is now included:

Printout	Line	Protocol Text
No.3. 82.6 x 85.91 = 7096.166 **	154 I:	OK. Let's take a look at this first part We're going to call it No.5. What were you trying to do in there? You're trying which problem?
No.5. 82.4 x 85.91 = 7078.984 **	157 E:	First of all I looked at problem 2 and thought they might be printing a 4 instead of 6, so in problem 3 I substituted a 6 for a 4. And tried that.
No.6. 82.8 x 85.91 = 7113.348 **	160 I:	Oh, I see. Did it work?
No.7. 82.2 x 85.91 = 7061.802 **	161 E:	Didn't work out.
	162 I:	Alright, when it says which key isn't working, like, when the key isn't working you think it might mean that it's putting in another number, right? Is that what you thought?
No.9. 82.7 x 85.91 = 7104.757 **		
	166 E:	Yeah.
	167 I:	So then you figure out what other numbers could go in besides 6, heh?
	169 E:	Yes.
	170 I:	And you tried 4 in there and now what did you do in No.6 here?
	172 E:	I tried the 8.
	173 I:	When you tried the 4 it didn't work - is there a reason why you tried 8?
	175 E:	Well, OK, I looked at this one and saw that they got -
	176 I:	You looked at which one?
	177 E:	At 9 in there.

I: Problem 2.

E: I saw they had a 9 so I -

I: Instead of the 5 on the paper, in problem 2.

E: Right. So I put an 8 in there . . . I added these up and found how I could get a 9 as a second digit instead of a 5.

I: Ah, OK. I think it's the other way around you mean. How to get a 5 instead of a 9.

E: Yeah.

I: Would 8 have done that? You're not sure, heh?

E: Not sure.

I: So really in No.6 like, you're using, you are checking problem 3, but you're using problem 2 to help you.

E: Uh hum.

I: Like what happens in problem 2 to find the numbers to check in 3. Is that right? Do you know what I mean?

E: Yeah.

I: OK. What are you doing in No. . . . OK, you didn't get the right answer.

E: No.

I: OK, from the answer that you got and the answer that they have, like they have 7044.62 and you have 7113.348. Did you get anything from that?

E: I thought the number should be lower than the one I was trying.

I: So what did you do to make it lower? You tried in No.7, 2.

E: Uh hum.

206 I: Instead of . . . ah, I see, this No.5, it's also higher than what you want, isn't it? 7078. So is that why you made this one lower than even 4? Did you look at those kinds of things?

210 E: Yes. That's what I did.

211 I: So 82.2 instead of 82.4, heh? What happened?

212 E: Didn't get the right answer yet.

Evan explains his error of choosing an 8 in No.6 instead of a digit smaller than 4 by saying he is using another problem besides the one he is working on. Perhaps, but he acknowledges using successive approximations after No.6 (i.e., see lines 701-2) and if the results of Nos. 5, 6 and 7 are used correctly, then in No.9 he should not substitute a value of 7 for the malfunctioning 6 key. It is reasonable to hypothesize, based on these results, that Evan is randomly selecting keys. There is no plan or strategy evident other than trying a key and selecting another that has not been tried if the first key doesn't work. Evan appears to be analyzing the calculated results for correctness (i.e., line 212) rather than trying to determine how the operations are working together to produce the generated results.

Tonia's work on problem 4 does not indicate a random choice of substitutions in Nos. 7, 8, 9 and 10. She already knows that the 6 is the malfunctioning key and is mistakenly trying to find what digit is replacing the 6. She is trying to produce the erroneous sum of 2358 by substituting in for the 6 in 267.

In No.7 she substitutes a 0 in for the 6 and obtains a sum of 2538 that is too large. However, she has no smaller digit to use in the place of the 6 and so uses a 7 to replace the 6 in No.8, producing a sum that is too large. Her next substitutions are successively smaller, and though it could be argued that she should see the futility of her work from a proper analysis of No.7, she is still using numbers in No.9 and No.10 that do get her closer to the answer (i.e., she is approximating). Tonia's discussion on these calculations is now included:

Printout	Line	Protocol Text
No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	250 I:	Now what did you do in there - what did you change?
	251 T:	I thought maybe the 6 wasn't working at all and that you were just getting a 0 . . . in its place.
No.7. 131. + 207. + 538. + 934. + 728. = 2538. **	253 I:	So instead of 267 . . . you put in 207 and then you were upset by the fact that even though you did that - what happened?
	256 T:	It got larger than the answer -
No.8. 131. + + 26. * 277. + 538. + 934. + 728. = 2608. **	257 I:	In the problem - you got 2538. So even though you made this smaller you still got a larger answer . . . now we talked a little bit about that . . . did you get any ideas like what you might do next?
	261 T:	. . . not really -
	262 I:	Oh you've got a nice . . . I wonder if that is - let's take a look at No.8 . . . and you tell me what you were doing in No.8.
No.9. 131. + 22. * 257. + 538. + 934. + 728. = 2588. **	265 T:	I tried to make the number bigger . . . make it 277 instead of 267.
	267 I:	So the 26 is actually a typing error . . . so you put a star (i.e., *) by that - and you put in - tried to make it bigger - was

No.10. 131. +
227. +
538. +
934. +
728. =
2558. **

there any particular reason why you tried that?

271 T: . . . I was confused that when I made it smaller that the number got larger . . . and so I tried to make the number larger to see if it would get smaller - I don't know.

275 I: It's alright . . . and then in - it still stayed bigger didn't it - 2608 . . . did you get any ideas from that? . . . like in No.9 now -

278 T: Just that the number was smaller than 6.

279 I: So what did you try - or another error in here - 22 heh - don't worry about it. I see 257 . . . is that the change you made?

282 T: Yeah.

283 I: What did you try to do in there?

284 T: I made the 6 a 5 . . . and my answer was still . . . quite a bit larger (i.e., than 2358) . . .

286 I: You get 2588 instead of 2358. Did you get any ideas in here from that?

288 T: I knew that it was something different going on . . . and I couldn't figure out what it was . . . yet.

290 I: In No.10 you've done what - what's the change you made in No.10?

292 T: I made it 227 instead of 267.

293 I: So still smaller . . . now this is the point at which you came up to see me. Why?

295 T: What I don't . . . I don't know how to show this . . . to make sure that I'm right, but I thought maybe instead of 6 adding that it was subtracting from the answer . . . like

The difference between Tonia's work and that of Clive and Evan is that Tonia is making systematic changes in her calculations and gradually getting closer to the answer. She is well aware of how close her approximation is to the answer and openly states that she makes changes to get closer. She also realizes that this approach will not produce the wanted 2358 and, in her own way, sees that "6 has to subtract", which, in a way, if 267 becomes 27, is what happens.

Another example of the use of successive approximations technique is Bruce's work in problem 7. From his calculations there is a clear direction to his work as he gets closer to the answer with each step. The record of relevant calculations is included below:

Printout

```
No.2a.      140. -
              1. /
              3. =
      139.66667 **

No.2b.      140. -
            0.8 /
              3. =
      139.73333 **

No.3.       140. -
              2. /
              3. =
      139.33333 **

No.4.       140. -
            2.1 /
              3. =
          139.3 **
```

```
No.5.      140. -
             3. /
             3. =
           139. **

No.6.      140. -
             4. /
             3. =
      138.66667 **

No.7.      140. -
            3.5 /
             3. =
      138.83333 **
```

It is clear that in No.2a Bruce starts an algorithm that he modifies until it is eventually successful. In No.2a he subtracts 1 and sees his answer is too large. In No.2b he changes his data incorrectly where decreasing the number he subtracts, from 1 in No.2a to 0.8 in No.2b, actually produced a larger answer. He realizes that this is the opposite direction he wants to go in and Nos. 3 through 6 reflect this lesson. Then in No.7 he "forgets" the lesson of No.2b and generates a number that is further from the 138 he wants. This series of calculations is considered successive approximations here rather than trial and error because of the order, direction and lack of random substituting.

It is this researcher's view that trial and error is employed by subject's who have little idea of what they are doing, that successive approximations is used by problem solvers who employ a limited amount of reasoning, but that a real understanding of the problem will not suffer from the tedious, exhaustive process of successive approximations. As an example, consider Lisa's work on problem 1 where she is attempting to generate the decreasing pattern of numbers in part c. Her work is included below:

Printout	Line	Protocol Text
No.4. 9999999. /	108 L:	Might not work (i.e., No.4).
1. =		
9999999. **	109 I:	What was your idea?
9999999. =		
9999999. **	110 L:	Divide by 1 but you just keep getting the same answer.
No.5. 9999999. /		
1.0000001 =	111 I:	Cause 1 is the identity element of division. OK, but maybe that's not such a bad idea.
9999998. **		
9999998. =		
9999997. **		
9999997. =		CALCULATOR OPERATING (50s) (No.5)
9999996. **		
		So what's happened? . . . you're getting it
No.6. 10000000. /		aren't you? Except you didn't get the -
1.0000001 =		
9999999. **	115 L:	Yeah.
9999999. =		
9999998. **	116 I:	First starred number did you? Did you notice that?
9999998. =		
9999997. **		
9999997. =	117 L:	Yeah.
9999996. **		
9999996. =	118 I:	You've got it decreasing by 1. What was your idea? This is now No.5. What was your idea?
9999995. **		
	120 L:	To find something that . . . would get it to decrease by 1.
	122 I:	Cause that's the pattern, heh? So why did you think divide right away?
		PAUSE (5s)
	124 L:	I don't know . . . easier to work with than subtract.
	125 I:	You could have subtracted too, heh . . . . Do you think of dividing as making things smaller?

127 L: Yeah.

128 I: OK, I was just wondering . . . so what's happened is you've got the pattern except you've started one too small. How could you fix that up?

PAUSE (17s)

131 L: You mean get it so you can find the first number?

132 I: Yeah.

133 L: You could try dividing 10 million by . . .

134 I: The number you've got there - the second. Why would your pick 10 million?

135 L: Cause it's 1 more than 9999999.

The puzzling aspect to Lisa's work is her "jump" from No.4 to No.5. She does this without any experimenting and seems to know what change to make in the divisor to produce the desired result. When this again needs modification, she makes the adaptation from No.5 to No.6 without intervening steps. When comparing this to the process of successive approximations it is possible to view the latter as representing less understanding. There is a tendency to suggest that the calculator is a useful tool for solving problems using successive approximations. This research indicates an abuse of the calculator ~~in that~~ subjects are not making an attempt to understand the operations used. It appears that subjects operate as if the calculator will eventually give them the answer if they merely persist. Successive approximations needs to be used in association with an analysis of the operations used.

Carelessness

Another consideration of this concluding chapter will be an exploration into an error category that the researcher did not think would be an important part of the research and so was initially ignored--the consideration of careless errors. Carelessness has been researched as an error category. Newman included a careless category where "The error was random, that is, it is not likely that the same error would occur if the pupil attempted the task again."[17] Casey modified this category and referred to it as an "unknown block" where the following criteria were set: "We call a mistake random, or sometimes careless, if no specific cause can be found for it."[18] This catch-all character of the careless error was criticized by Watson:

> . . . it did not seem justified to argue that a difference between the test and the interview [i.e., Newman classed those errors as careless that were not repeated in the interview after being performed on the original test] was due solely to carelessness or lack of motivation during the test. It seemed wiser to make no assumptions about reasons for errors in the original test when the child's thinking could not be assessed.[19]

Newman found a surprisingly large number of errors her sixth grade subjects committed were classified as careless, in fact, 649 of 3002 total errors (i.e., 22%) were classified as caused by carelessness. Watson specified that "If the Newman and Clements procedure is followed . . . thirty-two percent of errors in the original test would be classed as due to carelessness or motivation."[20] Such large numbers did not orient this research to a consideration of carelessness as it was presumed that the difference in questions and groups in this research and that mentioned in the research quoted would make carelessness too small a category to warrant attention.

However, in research quite different from the Newman type, there has been recognition of carelessness as a legitimate error category. Menchinskaya, as reported in Radatz, named four causal areas of errors which cannot be distinctly separated, one of these being "mechanical errors due to lack of interest or to diversion."[21] Knifong, in an analysis of written responses to word problems, recognizes a category of "clerical errors" stating these errors

> . . . are endemic to human activity and have little to do with understanding or word problem ability. To qualify as a clerical error, (a) the correct answer was worked out and copied correctly or not at all into the answer space; (b) the correct operation was used but the wrong numeral was inserted because a numeral was miscopied from the problem to the computation, or because a poorly formed numeral was misread; or (c) the work was correct but the student became confused, skipped the problem and copied the right answer in the wrong problem space.[22]

In stating the implications of his data, Knifong states:

> The major feature of the data was the frequency of clerical and computational errors positively identified as the sole source of difficulty.[23]

How sharply this contrasts with Ginsberg's perception of error generally, as reported in Radatz:

> For Erlwanger, Ginsberg and others, most student errors are not of an accidental character, but are due to very individual problem-solving strategies and rules which go back to earlier experience and understanding of mathematics.[24]

The perspective just stated was that of the researcher, and though the objective of this research is not a quantitative evaluation of error types, there is evidence of a disproportionately large number of what Faitong would class as clerical errors.

In research specific to calculators there has also been some recognition of clerical errors, with the work of Thompson deemed relevant. Thompson recognizes that

> A major difference between pencil and paper calculations and those involving a calculator concerns the use of an extra step--a step one could call "transcribing".[25]

Based on the results of this research, there is a good deal of support for Thompson's observation that

> All written calculations have to be fed into the calculator, and it is this process of transcribing the information which is likely to provide a major source of calculator error, for the number to be operated on has to be read correctly, remembered correctly, and entered into the calculator correctly.

In this research it quickly became apparent that the transcribing or clerical errors would be numerous and by their number, significant. Calculator misentry errors, where the subject in effect intends to enter one number but actually enters another, are a form of transcribing error that here will be generalized as a careless error. Because the calculator has a clear input function (i.e., a key labelled as CI) it would have been possible for subjects to eliminate a misentry without registering the change on the printout. To prevent this the subjects were all instructed on the use of the print key which functions to print anything currently displayed. By hitting this key first and the clear input key afterwards it was possible to record misentries on the printout. As an example, a section from Bruce's work on problem 4 now follows:

Printout	Line	Protocol Text
No.3. 11. +	192	I: . . . now what are you doing in No. 3?
167. *		
267. +	193	B: That one, I ran through problem 2 again.
58. +		
94. +	194	I: No you haven't.
728. =		
1158. **	195	B: OK. That one I was trying the process of elimination. Like, removing the 3 from it.
	197	I: OK, so let me just see what happens.
	198	B: I was saying, basing it that, if the 3 key didn't work and I typed 131 I'd get 11 instead of 131. And then 167 worked. 538-
	201	I: But what happened — oh! 167 is a misprint.
	202	B: Yeah.

Bruce has misentered 167 and remembered to have the entry printed before moving on. It is important to notice that there is no reasoning or explanation for the error based on the discussion. This will be the basis for classifying an error as careless—that there is no plausible reason and the subject offers no explanation. With the clinical interview technique it is realistic to expect to make such classifications accurately.

Another type of error that will be classified as carelessness is entering the incorrect number without the subject's awareness.

Evan's calculation below on problem 1 will be classified in this manner:

Printout		Line	Protocol Text
No. 2.	1.000001 x	91	I: And I'd like you to tell me – and in No. 2 – your idea was . . . did you want this to be six zeros?
	10000002. =		
	10000012. **		
	10000012. =		
	1.0000014 14 **	93	E: Yes I did.
	1.0000014 14 =		
	1.0000016 21 **	94	I: So that's an error.

This misentry had previously been discussed with the subject while he was performing the calculations and the interviewer called attention to this error; Evan was not aware that he had misentered the first factor. Errors of this type will also be classed as carelessness if the protocol can establish that the subject had meant to enter another number.

Another type of error that will be treated as carelessness is any "slip" made by the subject during the discussion that is not perceived as related to any problem with understanding or recall. These are really errors where the subject means to say one thing and says something else. The following passage from Sherri's protocol on problem 1 is typical of this type of carelessness:

Printout		Line	Protocol Text
No. 4c.	9999999. /	123	I: Anyways, didn't you get any ideas from that though?
	1.0000001 =		
	9999998. **		
	9999998. =	124	S: Yeah, that I had one too many because that came out . . .
	9999997. **		
	9999997. =		
	9999996. **	125	I: One too many what?
		126	S: One too many 9's.

127 I: So then what did you do?

128 S: I put seven 9's and timesed it by 4

129 I: Divided it.

130 S: Divided it. Yeah. I divided it before that too.

From the readiness with which Sherri agrees with the contradiction of line 129, and her statement concerning previous division in line 130, her verbal error is interpreted as carelessness.

As a final indication of the prevalence of the carelessness category of error, a separate table is now included. It is significant to note that the second problem, where subjects made more calculations, had a proportionally larger number of errors.

Analyzing the Activity of the Interviewer

In this research the interviewer played a very active role. This is not totally uncommon in the research surveyed. In supporting the need for diagnostic interviews Watson noted that "the interview technique reveals far more of mathematical thinking than does analysis of written responses to a test,"[27] where he includes "questioning the pupils" as part of the interview technique. Hollander, in her literature review of verbal arithmetic problems and the thought processes involved, describes several relevant researches. She refers to Uhl's work where students were "questioned . . . regarding approaches used."[28]

Table 8.

Careless Errors.

Problem-	First	Second	Third	Fourth
Subject				
Mardi	–		–	
Evan	1	–		
Sherri	1	2	1	2
Earl	–	–		–
Lisa	–	2		–
Bruce	1	4	2	
Ruth		1		1
Shelley		1		
Clive		2		
Tonia		3		
Ryan		5		
Shawn			1	
Wayne				3

She mentions Brown's research where the following questions were asked of students after they had solved the problems: "(1) What is the answer? (2) How did you get it? and (3) Why did you add, subtract, or multiply?"[29] Hollander, in her own research, used the "talking aloud as the subject worked the problem" technique but also "asked a series of questions, funnelling from nonstructured to structrued, relating to the problem solving strategies employed."[30] Gorle, after giving students eight possible problems to do, asked the following questions:

> 1. What is your answer? 2. How did you get it? 3. Why did you work it that way? 4. Do you think your answer is right? 5. Why do you think it is (or is not)?[31]

It should be noted that when these researchers generally ask questions, those questions are repeated to every subject. In this way there is some standardization in terms of the researcher's role from interview to interview. The absence of such regular patterns is viewed by many researchers as invalidating or potentially decreasing the validity of the results of the research. If statistical comparisons are being made between groups, this perspective may be accurate. However, this research is principally exploratory, an attempt to recognize processes and their related errors made by subjects while solving certain nonstandard calculator problems. To a degree these processes are demonstrated in the calculations performed without any activity on the part of the researcher. ~~But~~, to probe more deeply into the reasoning of the subjects, the researcher took an active role after the problems had been attempted.

It is interesting to compare the role of the investigator in this research with the position assumed by the error researchers employing Newman's scheme for error analysis. Newman described her approach as follows:

> The interviewer began by asking the pupil to rework the task [which had previously been answered incorrectly]. . . . After the pupil had attempted to solve the problem the interviewer then asked him to: (1) read the question; (2) say what the question meant or was asking him to do; (3) define the meaning of specific terms in the question; (4) explain how he obtained the answer, if the transformation stage was applicable; (5) show and verbalize, his workings for the question.[32]

Casey also used the Newman error categories, and Clements notes an important modification Casey made to Newman's method:

> An important difference between the interview procedures used by Newman and Casey is that in Casey's study the interviewers were required to help pupils over errors.[33]

There are those researchers who would criticize Casey for the interventionist role assumed by the investigator, but Clements explains the reasoning behind Casey's design, in noting that "in Casey's study, a pupil could make a number of errors on the one question."[34] The point raised is very important. It may be that unless the investigator does get involved in the process, the complete picture of the problem solver's understandings and attempts at solution may never be exposed.

To clarify the position of the interviewer in the retrospective interview, a classification scheme was developed to analyze the actions of the investigator. The different categories of interjection are now included with a brief description of each.

Table 9.

Interjection Classification.

Category	Code	Description
Explanation	01	a statement (comment) that explains a question or solution to the subject. Answering a question by the subject.
Repetition	04	statements that reword, without any substantial change, what the subject or the interviewer said.
Review	11	a statement or group of statements that repeat what was previously established in the discussion or printout.
Related question	05	a secondary question developed spontaneously during the interview that usually probes the subject's understanding of the question and is often related to the subject's knowledge and/or understanding of the calculator or mathematics.
Clarification	09	a statement that requests the subject to explain his reasoning on certain points or to answer a brief, simple question.
Correlation	10	a statement or question that relates the discussion to the problem solving process by making reference to specific aspects of the calculator printout.
Interruption	02	a statement that clearly interrupts what the subject is saying (usually prefixed by a dash in the preceding line of the protocol).
Affection	03	a statement made by the investigator that relates to the emotions of the subject, usually encouragement or a compliment.
Contradiction	06	a statement that establishes a line of reasoning opposite to that proposed by the subject or a correction of the subject's statement.

Certainty	07	a statement that tries to establish how sure the subject is that he has the solution or is reasoning correctly.
Hint	08	a clearly identified clue that helps the subject solve the problem.
Direction	12	statements that guide or push the subject to think or alter his explanations in a direction determined solely by the investigator.
Extension	13	using what the subject has stated or calculated as a basis for hypothesis (i.e., stating more than the subject implied) or questioning. Sometimes an extension into teaching.

Rather than focus on the obvious overlap that exists between categories, it is more useful to view certain interjections as related in terms of the overall effect on the interview. The interjections of explanation, repetition and review do nothing more than establish the state of the interview. The related question, clarification and correlation categories just probe the reasoning of the subject and develop aspects of the interview that were deemed unclear. The next four interjections listed are perceived as interrupting the flow of the interview; they all add to the interview, but not necessarily in a negative way. The direction and extension categories are seen as negative interjections in that they alter the interview. It is the last two categories that can be seen as potentially altering the course of the interview and so casting doubt on interpretations that can be made about the protocol.

Lankford, in a paper discussing the role of oral interviews for teachers, lists some guidelines for interviewing and it is relevant to compare these suggestions with the analysis of interjections here. He suggests that initially the subject be given a problem that he can solve and that he be complimented on his response. Lankford suggests that

> It is better not to interrupt the pupil as he computes. When he has finished and written an answer it is better then to ask him questions that require him to clarify or elaborate his thinking.[35]

The following caution relates well to the extension category mentioned here:

> . . . a diagnostic interview is not a teaching exercise in which pupils are led to correct their errors through a series of Socratic-type questions.[36]

It is important to note that this researcher taught the grade 9 subjects working in this research. The direction category is also identified by Lankford when he suggests that the interviewer "Avoid giving clues or leading the pupil through a series of questions."[37] This is not linked to the hint interjection category which was done by the interviewer when he felt the subject would progress no further without some assistance.

Examples of the interjections will now be given to illustrate the rationale behind the categorization.

As an example of the explanation interjection consider the following excerpt from Mardi's problem 1 attempt where the investigator explains why the pattern generated uses very large numbers:

Line	Protocol Text
101 I:	You'll notice that in the patterns I picked (i.e., in the question given) I used very large numbers –
103 M:	Yeah.
104 I:	Because something funny happens with the calculator when you completely load up the display . . . do you know what I mean? So you've ah . . . you haven't completely loaded the display – you used 10 million alright . . . or did you – that's 1 million . . . did you know that? . . . OK, try another pattern – any pattern you want.

This explanation comes at the end of the protocol and is the researcher's attempt to explain the idea behind the question before the subject stops entirely.

The repetition interjection is extremely common. Consider the following part from Evan's work on problem 1:

Line	Protocol Text
05 I:	Could you describe those numbers – just the numbers with stars – what pattern is there, what kind of sequence is that – what's happening?
08 E:	. . . they're increasing by 1.
09 I:	So . . . what you're doing is adding 1 to each number. That's pretty obvious. Can you tell me what you're doing each time you hit the equal sign?

The repetition occurs in line 09. It is merely a rewording of what the subject has already stated, indicating that that part of the calculations have been discussed and showing what will be discussed next. The last line would be coded as a related question, a section that will be discussed shortly.

The review classification of interjection is demonstrated in Sherri's efforts with problem 1. Review statements were used to piece together calculations that had already been discussed.

	Printout	Line	Protocol Text
No.9.	888888. x	243 I:	Six zeros with a 1 . . . . so if you keep on hitting equals, watch what happens.
	1.001 =		
	889776.89 **		
	889776.89 =		CALCULATOR OPERATING (No.9)
	890666.66 **		
	890666.66 =		Are you getting a sequence?
	891557.33 **		
	891557.33 =	246 S:	Not really - it's just decreasing by a certain amount each time.
	892448.89 **		
	892448.89 =		
	893341.34 **	248 I:	Is it the same amount each time?
	893341.34 =		
	894234.68 **	249 S:	. . . no.
	894234.68 =		
	895128.91 **	250 I:	There's many ways of looking at the sequence. Like . . . . . . sort of have to be careful. See like if you've got 8's becoming 9's . . . you've got 9's becoming 0's becoming 1's becoming 2's becoming 3's becoming 4's (note the thousands place). There's sort of a pattern in there.

In lines 250 to 255 the interviewer is reviewing the printout the subject generated. Again, this occurs at the end of the protocol and is really just a summation of the pattern generated in No.9.

Interruption interjections are not that common but are disruptive in that they tend to interrupt the subject's train of thought. Usually they are attempts by the interviewer to make a point in relation to something the subject has said. Most interruptions occurred with problem 4 where subjects seemed to have difficulty understanding the implications of the wording of the question (i.e., that the malfunctioning 6 key was <u>not</u> operating). Consider an interruption from Shelley's protocol of problem 4:

<u>Line</u>	<u>Protocol Text</u>
59	I: OK. Does that tell you anything? Can you conclude anything from that?
61	S: Well, well there was no 6 in that one.
62	I: Well you know that now though.
63	S: Yeah, but I was just —
64	I: When you first . . . . did you . . . the fact that the two numbers agreed, did that tell you anything?
66	S: It told me that all these numbers had to be . . . working on the . .

Here the interruption occurs in line 63 and is indicated by a dash. The interviewer feels the subject is postrationalizing, and the interruption is to clarify exactly what conclusions the subject made at the time of the calculation. However, it would have been interesting to see what the subject's explanation would have been and it is difficult to perceive the interruption as adding to the interview constructively.

Generally most interruptions are not of a serious enough nature to classify and are indicated by dashes in the text of the protocol rather than being coded each time. These "normal" features of the interview are not to be confused with statements that clearly block the explanation a subject is offering. The latter are classified as interruptions.

Affective statements are meant to encourage the subject. Affective comments by the subjects were identified for the reasons previously specified, and it would seem reasonable that similar positive comments by the interviewer should be recorded too. It also seems logical that creating as supporting an atmosphere as possible during the interview will encourage the subject to be more open in describing his attempts at solutions. As an illustration of the type of statement that is classified as affective, consider the following segment from Ruth's protocol on problem 4:

Line	Protocol Text
143 I:	OK. So you still haven't found out what they did that was wrong but you know it's the 6.
144 R:	It's the 6.
146 I:	OK. I think your reasoning so far is good. I want to leave you just to try to figure out now what you could do to get those numbers there (i.e., in problems 2, 3 and 4).

Here the affective comment occurs in line 146, where the subject is being encouraged before asking her to try a second part of the question.

A contradiction occurs when the interviewer takes exception with something the subject has said, usually because there is an error involved and the researcher is trying to see if the subject has good reasons for his statement. A typical contradiction is found in the following excerpt from Wayne's protocol of problem 8:

Line	Protocol Text
54	W: I put 264 plus 10, that's 274. Then I put 600 plus 282 and that's what . . .
56	I: Ah, but what's . . . you used an 8 didn't you?
57	W: For what? It's supposed to be 882.
58	I: Yeah . . .
59	W: Oh!
60	I: So what did you do that was wrong?
61	W: I forgot I wasn't supposed to use an 8.

In line 56 the interviewer contradicts the subject in that he points out an error that the subject didn't realize he had made. Such contradictions make the subject aware of some fact or concept that he has missed.

There are occasions where it is important to understand if the subject has confidence in his answer or calculation. There are points in the protocols where subjects do not understand what they have done and the investigator needs to determine how certain the subject is of his work. The certainty comment performs that function.

An example of a certainty comment is found in Bruce's work on problem 7:

Line	Protocol Text
68 I:	In other words you're trying to spot really the constant operations.
	CALCULATOR OPERATED (Nos. 1 to 9)
	OK. So you think you've got the problem then Bruce?
71 B:	Yep.

From the record of the time we see Bruce has been left alone from 3:32 to 3:37, a 5-minute period during which he performed the calculations numbered 1 to 9 by himself. The investigator then approaches Bruce. At the beginning of the interview the certainty question determines here how Bruce feels about his work; basically, does he feel he was successful?

If the subject is deemed to be at a point where he can progress no further a hint is given. Identifying the hint permits evaluations beyond that point to be considered accordingly. As an example, the following segment from Tonia's work on problem 4 will be reviewed:

Line	Protocol Text
217 T:	I added problem 2 up and instead of 267 I used 207 -
218 I:	So what have you done then?
219 T:	I said that the 6 wasn't working at all and that there was no number punching out from it.
221 I:	You said there was a 0 coming out,

The hint here is given in line 221 where Tonia is being indirectly told that her assumption in line 220 is correct--that the 6 key was not working and that replacing the 6 with a 0 is to assume something else. Grasping this idea is central to changing 267 to 27 and producing the error of 2358 in part 2 of the problem. As previously mentioned, these categories are not mutually exclusive, and there could be a good argument in saying the hint is really a contradiction of what Tonia has stated. True enough, but the value of the statement as a hint was subjectively determined to be greater.

In the course of the interview the investigator often found opportunities to question the subject on some related aspect of the calculations or statements. This added an open-ended quality to the interview and is seen by this researcher as fundamental to probing the reasoning and understanding of the subject. As mentioned previously in the discussion of the research on the clinical method, Opper sees such related questioning as invaluable in "provid[ing] useful information on the child's view of reality and his thought processes."[38] Opper gives a very good description of the role of what is here classified as related questions:

> If further clarifications are required, he [i.e., the interviewer] asks additional questions or introduces extra items. Each successive response of the child thus guides the interviewer in his formation of new hypotheses and consequently in his choice of the subsequent direction of the experiment.[39]

By adding such questions, each interview assumes an idiosyncratic character and it becomes difficult to compare interviews. As previously detailed, such comparisons are not a prime objective of this research.

The following segment from Ruth's efforts on problem 8 is typical of the presentation of a related question:

Printout		Line	Protocol Text
No.3.	441. x	52	I: So this . . . so you represented the 882 by multiplication, the 1028.
	2. +		
	514. x		
	2. +	54	R: By multiplication.
	264. +		
	10. =	55	I: And this one (i.e., 274) by addition and how come you only get 274 (i.e., as the starred number)?
	274. **		
No.4.	441. x		
	2. +	57	R: Well because it only added those two.
	514. x		
	2. +		
	264. +		
	10. =		
	2184. **		

This question is probing Ruth's understanding of the calculator's operation. It is interesting that No.3 cannot be reproduced here as there is no explanation this researcher can find for the result produced in No.3. It is important that Ruth was able to answer the question and more significant that she repeated the same operations a second time to produce the result in No.4. Related questions generally try to determine the depth of understanding of the subject.

Clarification comments attempt to get the subject to develop some statement or calculation further. Rather than digress into other areas like related questions do, these queries merely ask for elucidation of something which is not clear to the investigator or potentially ambiguous. An example would be the following section from Earl's work on problem 8:

Line	Protocol Text
31 I:	Alright. So in the first one what did you do in No.1?
32 E:	Well I thought that was 274 + 882 = 1028 so I was just checking to see if that was right or not. And ah . . .
34 I:	You thought what? Say it again.
35 E:	I thought it was supposed to be the first two added together to equal ah . . .
36 I:	Oh I see, you thought it was 1028. So you had this plus sign as an equation. And it didn't equal . . .

By asking for clarification in line 34 a very important reading error Earl had made was identified. Clarification comments are quite useful in that they lead subjects into expositions of their reasoning.

Correlation comments are probably the most commonly coded interjection. These statements by the interviewer relate the discussion to the printout of calculations made by the subject. A typical correlation statement would be the following from Bruce's efforts on problem 7:

Printout	Line	Protocol Text
No.5. 140. – 3. / 3. = 139. **	211 I:	No way. If it's got Algebraic Operating System - that's what you mean by AOS right it should do divide, it will do the order -
	214 B:	No, no.
	215 I:	Oh yes.

216 B: OK, so it'll divide by 3.

217 I: Which - what number did it divide by 3 in No.5?

218 B: (Pause) ah . . . I don't know, it should have divided by 3.

220 I: Divided by 3, but what number did it divide by 3?

221 B: 3.

The role of correlation statements is here demonstrated as constantly referring the discussion to the calculations the subject has made. A related question is then asked, based on the printout as a reference, to see if Bruce understands the order of the sequence of operations.

The last two interjections of direction and extension are viewed as having potentially negative effects on the analysis of subjects' calculations and responses. In effect, these interjections offer suggested paths of explanation for the subject (i.e., direction) or add implications to what the subject has stated that are not necessarily implied (i.e., extensions). Where previous interjections may have affected standardization and comparison of interviews, not necessarily in damaging ways, these last two interjection categories introduce elements into the interview that may obscure and possibly invalidate conclusions that the research on that interview might generate. However, though a factor that has to be considered, these categories are not perceived by the researcher to have general influences that affect the main body of the research. It is the objective of the interjection-classification scheme to identify such negative effects so that their local influence can be appreciated.

A typical direction comment is given in Lisa's attempts at problem 1:

Printout	Line	Protocol Text
No.5. 9999999. / 1.0000001 = 9999998. ** 9999998. =	118 I:	You've got it decreasing by 1. What was your idea? This is now No.5. What was your idea?
9999997. ** 9999997. = 9999996. **	120 L:	To find something that . . . would get it to decrease by 1.
	122 L:	Cause that's the pattern heh? So why did you think divide right away?
		PAUSE (5s)
	124 L:	I don't know . . . easier to work with than subtract.
	125 I:	You could have subtracted too heh . . . Do you think of dividing as making things smaller?
	127 L:	Yeah.

In line 118 the investigator has commented on what the subject has done in the calculation rather than asking the subject to explain what she did. It may be that the decrease is not the main feature of the calculation that the subject will describe. The interviewer has directed the reasoning of the subject. This is not seen to invalidate what the subject has to say so much as determining what aspect of the calculation the subject will discuss. Directions tend to structure the interview in paths the interviewer sees as important while the subject, if not manipulated by the direction, might actually see other aspects of the calculation as significant.

The interjection of extension represents comments where the interviewer adds more to the discussion than can be attributed to the subject, in effect, inflating the protocol. For example, the following excerpt from Shelley's solution to problem 4 is a case in point:

Printout		Line	Protocol Text
No.2.	131. +	125 I:	. . . in both No.2 and No.3, 2598 does not agree with what's given in the problem. . .
	267. +		
	538. +		
	934. +	127 S:	Yeah.
	728. +		
	2598. =	128 I:	2358. Did you notice anything or conclude anything from that?
	5196. **		
	5196. -		
	2598. =	130 S:	Well I noticed that there was just, like it was a difference of 200 and . . .
No.3.	131. +		
	267. +	132 I:	40.
	538. +		
	934. =	133 S:	Yeah. I just . . . I really didn't understand it then so I went on to see what the difference was (see No.6).
	1870. **		
	1870. +		
	728. =		
	2598. **	135 I:	So it's not as if you're looking at 2598 and 2358 and coming up with any ideas.
		137 S:	No.
		138 I:	You just see right away that they don't agree.

The interviewer is gradually taking over the commenting from the subject and rather than just repeating what the subject has to say, the interviewer has added more to the discussion than is actually directly stated by the subject. The interviewer suspects that Shelley is not analyzing the results of her calculations and in an attempt to establish this has not exercised the patience needed to have Shelley develop this fact in her own words and at her own pace. From the protocol as a whole and a review of the calculations, there is ample evidence that Shelley does not sufficiently analyze her printout, but extensions like that illustrated do little towards understanding the reasoning of the subject.

Comments Made by Subjects

Days' original checklist also had a section where the comments subjects made about the solution were recorded. The processes in this comment section included

> . . . questions existence of solution; questions uniqueness of solution; questions necessity/relevance of information; expresses uncertainty about final solution; says he or she doesn't know how to solve problem; and says the problem is difficult.[40]

As the subjects responded to retrospective questions posed by the interviewer in this study whereas Wheatley and Days' work involved the recording of subjects' comments "while" they solved the problem, most of the comment categories of the checklist were not used in this research.

It was however considered important to note certain categories of comments made by subjects. The table below gives a list and description of these comments along with the number used to code them.

Table 10

Subjects' Comments.

Code	Comment	Description
60	Questions solution existence	the subject doubts whether a solution actually exists.
63	States failure	the subject openly states that he cannot solve the problem or some part of it.
64	States success	the subject makes a statement that he has solved the problem.
65	Describes reasoning	comments made by the subject help in clarifying the thinking he used in solving the problem.
66	Makes affective statement	the subject makes a comment that displays his feelings about the problem.
50	Answers related question	the subject responds to a direct, related question posed by the interviewer.

It was originally believed that subjects who had difficulty with the problem might eventually try to resolve their perplexed state by questioning whether the problem, as phrased, actually existed. This comment was not very common and probably should have been omitted from the study. A rare example of its existence is in Earl's work on problem 4:

Line	Protocol Text
12 I:	So you think you got it solved Earl?
13 E:	Yeah ah . . . am I the first one to work on this or has someone else done this?
15 I:	Some one else has worked on it. Why?
16 E:	Because well, I worked it out that the 6 is not working.

18 I: OK.

19 E: And if you do that with problem 2 you get an answer 2538, and what you have printed down is 2358.

21 I: Yes.

22 E: That's more than just a coincidence, that's more than a type o (i.e., typing error).

Earl is questioning the form of the question, for though he has identified the malfunctioning key, he cannot find a method to reproduce the erroneous 2358, so he looks for an error in the question itself.

Because the interviewer left the subjects to work on the problems alone and then discussed their work afterwards, it was important at the beginning of the interview to determine how they felt they had done on the problem. Comments about their success or failure could then be compared to the record of their printout as the interview progressed. More importantly, much of the isolation of processes and their concomitant errors is developed within the context of what the subject says about his work. Comments about the success or failure on a problem are often corroborating evidence used in deciding what process or error was involved.

A typical success comment is in the following extract from Earl's work on problem 1, part 2, where he is trying to find the factors involved in creating a sequence that increases by 3:

Line	Protocol Text
37 I:	Give it a try and see what happens.
	CALCULATOR OPERATING (No.2) (20s)
38 E:	Bingo.
39 I:	So you get it heh? Very good, (advancing and removing the printout). So the numbers that you picked to multiply are good . . .

From line 38 it is fair to assume that Earl recognizes he has the answer.

Comments about failure are often helpful in determining the tone of the interview in that if the subject has not been successful there is a tendency not to want to discuss failures, and it is important that the interviewer overcome this initial problem. A good example of this is in Wayne's attempts at problem 8:

Line	Protocol Text
16 W:	Mr. Leesinsky, I can't get this problem.

The subject is having difficulty. The interviewer calmed the student and went on to discuss the calculations that the subject had done and was able to categorize the work.

It is often from what the subject has to say about his ideas that support is gathered for inferring that deductive or intuitive reasoning has been used. It is from the response the subject gives to a related question that the interviewer determines how well the subject understands the problem.

For example, in problem 7, where the interviewer is trying to determine if Shawn understands what the constant operations are, the following segment indicates the use of related questions and the significance of answers made to them:

Line	Protocol Text
06 I:	Now what I've done here is I've made a sequence. Do you know what numbers form the sequence?
09 S:	4 and 1.
10 I:	Nope. The numbers you started with - 33, 133 - what else?
11 S:	533 and 2133.
12 I:	In other words, the number you started with and all the numbers with stars. Now to make that sequence I used two constant operations. What are they?
15 S:	The 4 and the 1. Multiply by 4 and add 1.

It is vital that the subject understand what is meant by the terms of the sequence and the constant operations in problem 7. In this example, the interviewer uses related questions and the responses given to them to determine if the subject has the prerequisite knowledge needed to begin the problem.

Another comment category that will be recognized in this section of subjects' comments is the affective comment. These comments are significant because they reflect the attitudes and motivations of the subject. Corle[41] recognized the importance of a subject's confidence in his work by questioning subjects on how certain they felt they were right. In the protocols there are instances where the subject demonstrates a lack of confidence, usually by making disparaging comments about himself. Although not a major objective of the research, it was decided that such comments should be identified. Newman recognized the role of motivation generally by acknowledging a motivation error when "The pupil did not attempt to get the correct answer even though he could have done so if he tried."[42] Casey, in modifying Newman's error classification scheme, described motivation errors as "known blocks" which

> . . . contains all those trackable causes of errors which are outside the sequence (1) - (7) [i.e., the sequence formed by progressing through Newman's steps as outlined previously].[43]

Casey notes also that "the teacher has some chance of doing something about remedying errors in this category."[44] This was the perspective assumed by the researcher, and it is interesting to note how such comments made by subjects are dealt with by the researcher. Consider Sherri's comment at the beginning of problem 7:

Line	Protocol Text
108 I:	Yes. OK, so you're going to explain to me how you got it. Which one did you do first (there were two separate printouts)? This stuff here or that stuff?
111 S:	This stuff.
112 I:	So what I'll do is I'll number them, you're going to try your best to explain to me what you're doing.
114 S:	Knowing me I won't be able to.
115 I:	Oh, you'll do a good job. So in this first part here what are you trying (i.e., No.1)?

It is significant that Sherri does lack a certain amount of confidence in herself as demonstrated in line 114. The interviewer tries to be supportive at this point.

Concluding Statements

The major objective of the research, initially, was to analyze the printouts of calculations produced by subjects working on nonroutine calculator problems and to isolate certain processes within the calculations. The Wheatley modification of Days' checklist was modified again for that purpose. Generally, certain processes were isolated and shown to be common in many of the protocols. As well, unsuccessful applications of the processes were linked to errors, and common errors were identified in many of the protocols.

A comparison of the calculated printouts and the retrospective discussion confirms both as adequate methods for describing the problem solving techniques of the subjects. When both methods are used together a very clear picture of what each subject tries during the problem is demonstrated. It is suggested that this technique of having the subject work alone initially with a device that records his work followed by a discussion could be profitably used by other researchers.

Very significant in the research is not only the commonality of processes used but also the common sequence in which these processes are applied. Though the nature of the problems used in the research must be a considered factor, the research generally supports the claim that many of the subjects used the calculator in the same way. In particular, the similarity of protocols for problem 4 is especially striking. From an error perspective, the common misinterpretation of the term "malfunction" by most subjects could be viewed as an error in the structure of the question itself. But comparisons across protocols indicates the similarity was produced by reasoning errors, and that subjects generally did not analyze previous checking calculations, which would have helped them to identify their reasoning error.

It is suggested that the most significant aspect of this research is its demonstration and analysis of the strategy of successive approximations. It was observed that most subjects made use of this strategy, at least initially, but that more capable pupils couple a successive approximations technique with an application of deductive reasoning. For many subjects, successive approximations is little more than a blind substituting with little analysis of the processes being repeated. It is this researcher's belief that the strategy of successive approximations has limited application.

The number of careless errors was found to be significant and it is the researcher's hypothesis that a study is merited that investigates the size and organization of the calculator keyboard in relation to misentries. The design of the calculator, in terms of the size of the display and the size of the keys, may have been responsible for much of the carelessness.

It was initially believed that the affective component of the problem solving attempts would be identified in this research. Though comments by the interviewer and the subjects were classified with such an objective, it is clear that such an analysis was not successful. From the interviewer's experience with each protocol, it is hypothesized that much of the affective component of the interview is not isolated by this research. Indeed, it is now believed that it will be very difficult to design a method that will achieve a clear identification of affective components.

The research was considered singular in that the retrospective interview was not common generally in most research. An analysis was made of the interviewer's comments to help determine the effects of this technique on the research. Except for certain instances, which the classification of comments identified, where direction or extension was performed by the interviewer, it is believed that the interviewer did not have any detrimental effect on the material generated by the research. It is also demonstrated that the retrospective technique in not only viable but successful as an information-gathering method.

In closing, this research was repeatedly labelled as exploratory and descriptive. Based on the processes isolated here, there is a need for in-depth research on the actual structure of certain selected processes. In particular, the very general process of "modifying an existing algorithm" would benefit from the definition produced by a specialized study.

This research indicates that most subjects, even before they are clear on the solution, like to perform certain exploratory manipulations. This is usually continued with modifications to those algorithms first produced. The basis for such modifications would seem to be a very fertile area for research.

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APPENDIX

Earl Problem 1.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:51				1	I: So what I want you to do ... if you look at part a it says use the calculator to multiply those numbers, so would you do that? And then it says ... just read what it says.
No.1.	10000000. x				
	1.0000001 =				
	10000001. **				PAUSE (5s).
	10000001. =				
	10000002. **	4: rephrases part a.	4		After you do it - you tell me what the sequence is like.
	10000002. =				
	10000003. **				CALCULATOR OPERATING (No.1), (30s).
	10000003. =				
	10000004. **			5	E: And then seven more times (talking to himself).
	10000004. =				
	10000005. **			6	I: Yeah, just press the equals seven more times.
	10000005. =				
	10000006. **				CALCULATOR OPERATING (No.1), (30s).
	10000006. =				
	10000007. **			7	E: I think that's seven.
	10000007. =				
	10000008. **				
		11-12: repeats question part a and clarifies.	4 9		I: So let's just get that out (advancing and removing the printout) and then I want you to tell me what the pattern is. ... now it's the ... you will form a sequence of numbers with stars beside them. OK. Can you describe that sequence to me?
		12-14: sees an additive pattern.	50	12	E: Well ... ... they're all 10 million plus 1 each time, like the first one goes 10 million 1, then the next one is 10 million 2 and so forth down to 10 million and 8.
		15: related question to test calculator understanding.	5	15	I: So what's it doing each time you hit the equals?
		16: sees pattern as addition.	50	16	E: It's adding 1.
		21-23: repeating question part b.	4	17	I: So that's basically the sequence - you just described it. So that's part a. We'll get you to do part b now. Suppose you wanted to get the pattern of numbers just below - this pattern right here (pointing to the pattern in question b) ... now which pattern are we talking about - the starred numbers, heh? Tell me what two numbers you think you'd multiply?

				PAUSE (30s).
	24: a correct prediction.	50	24	E: I think it would be 1 and 3 ten millionths.
			25	I: 1 – you mean like –
	26: makes correct change in the second factor. Reasons deductively and relates problem to No.1.	40 33	26	E: 1 – like this (i.e., 1.0000001) – except with 3 at the end. –
	27: clarification.	9	27	I: So 1 point 0000003 and what?
			28	E: And 10 million.
No.2. 10000000. x 1.0000003 = 10000003. ** 10000003. = 10000006. ** 10000006. = 10000009. ** 10000009. = 10000012. **				CALCULATOR OPERATING (No.2), (20s).
	29: knows he has the answer.	64		Bingo.
			30	I: So you got it, heh? Very good (advancing and removing the printout). So the numbers that your picked to multiply are good ... now we come to one that's a little bit different ... so you need to read it.
				PAUSE (15s).
No.3. 10000000. x 0.9999999 = 9999999. ** 9999999. = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. ** 9999996. = 9999995. **				CALCULATOR OPERATING (No.3), (15s).
				By a pattern I mean the numbers with stars.
				CALCULATOR OPERATING (No.3), (45s).
	35: repeating the question.	9		So what happened?
			36	E: I got it.
			37	I: You got it right off the bat. What was your idea?

No.3. 10000000. x 0.9999999 = 9999999. ## 9999999. = 9999998. ## 9999998. = 9999997. ## 9999997. = 9999996. ## 9999996. = 9999995. ##	38-41; No.3: strategy: compares first number of sequence to product when multiplied by 10000000 and uses decimal notation to equate the two. A bright idea.	65 41	38	E: I saw that ... first number in part c (i.e., 9999999) was a series of seven 9's ... so I was thinking well ... if you put a decimal in front of that (i.e., 9999999) and times it by 10 million ... well then you'll get ... ah ... 9999999.
	42: repeats subject's ideas.	4	42	I: In other words if you put a decimal in front of the first 9, heh?
			43	E: Right.
	44-46: related question to test calculator understanding.	5	44	I: So that's what you did, heh, and you multiplied 10 million by that ... each time you hit the equals sign ... do you have any idea what you're doing?
	47;49: correctly sees the second factor is constantly multiplied.	50	47	E: ... multiplying ... that number by ... the second number again.
	48: clarification.	9	48	I: Which is the second number?
			49	E: Point 9999999.
	50-55: extended explanation of what actually happens.	13	50	I: So you see what the equal sign will do ... is whatever you ... when you first hit multiply by point 9999999 ... each time you hit the equal sign it'll do that. So it's taking the answer you got before and multiplying that and making it smaller by 1. ... so the pattern in this one is different. How is this pattern different ... than the patterns we started with?
	56-58: obvious answer to a simple question.	50	56	E: Well this one goes lower in value while the other two went up - they became greater in value with each equals ... these ones are going down.
			59	I: What I'd like you to do in part d - you do anything you want with that calculator - is I'd like you to make up your own sequence of numbers, I'd like you to make up a sequence of your own.
				CALCULATOR OPERATING (No.4), (35s).

No.4. 10000000. x 0.8888888 = 8888888. ** 8888888. = 7901233. ** 7901233. =	62: algorithm did not produce what he expected.	92	62	E: It doesn't come out very good, that one.
	No.4: note the clear similarity between No.4 and No.3. Only change is 9 to seven decimal places has been changed to 8.	33	63	I: What did you want your sequence to be (advancing and removing the printout)? What did you want it to do?
			65	E: I was just seeing what - if the series - like if the series of decimal and seven of those (i.e., 10000000) in the number would produce the same things as 9 would when multiplied by 10 million.
No.5. 10000000. x 0.8888888 = 8888888. ** 8888888. = 7901233. ** 7901233. = 7023317.5 **	63-64: questioning reasoning. 65-67: exploratory manipulation. 68: clarification.	9 22 9	68	I: And did it happen?
			69	E: No!
	70: reference to printout using related question.	5	70	I: Do you have any idea why not?
				PAUSE (10s).
				It didn't go down by 1 each time ... but maybe your pattern does go down by the same amount each time ... do you know what I mean? Maybe there is a pattern ... how would you check that out?
	73: changed question to find out what he knows about patterns.	5		
	74: answers related question.	50	74	E: You'd continue on a bit.
				CALCULATOR OPERATING (No.5), (25s).
	77-78: checking pattern ideas.	9	75	I: It's getting miserable (laughing). We'll leave that one for now (advancing and removing the printout). You went a little bit further and then you could see that - what really convinced you that the pattern is no good?
	79: wants whole number in pattern. Identifies error. 80-82: repeating part d.	55 54 4	79	E: That decimal point in there (i.e., 7023317.5).
			80	I: The decimal 5 doesn't make you feel too good. Alright - do you want to try another one just to see if you can make up another pattern?
			83	E: Yeah, sure.

No.6.	10000000. x 1.0000004 = 10000004. ** 10000004. = 10000008. ** 10000008. = 10000012. ** 10000012. = 10000016. ** 10000016. = 10000020. ** 10000020. = 10000024. **	No. 6: pattern now resembles part a with variable (last digit) changed. He uses what he has already shown he knows. Uses method of related problems and repeats algorithm.	33 48	CALCULATOR OPERATING (No.6), (53s).
				84 I: So in No.6, have you got a pattern going now?
				85 E: Yeah.
		86: asked too late for valid prediction.	9	86 I: What pattern did you want before you started it?
		87-90: explains reasoning behind No.6: combined results of parts a and b.	65	87 E: Well I was thinking if you multiply 10 million ... by 1 point ... seven zeros and then 1 and got a pattern ranging by 1 ... if you did that instead of the second one you get a 3 ... maybe ... try that with a 4 ... see if that would work.
		91: repeating part a for this generated sequence.	4	91 I: How would you describe your starred pattern - what does it do?
No.7.	10000000. x 0.999999 = 9999990. ** 9999990. = 9999980. **	92: sees pattern as additive.	50	92 E: It's adding 4 each time.
				93 I: This is a lot then like the very first one I gave you.
				94 E: Yeah.
		96-99: asking for a change of approach.	5	95 I: One other thing I wanted to ask you - just as a little bit of an aside question ... let's go back to part c again ... could you think of another way ... take a look at what you've got there (i.e., No.3) ... could you think of another way in which you could get that pattern?
		100: questions conditions.	20	100 E: Where it lowers by 1 each time.
				PAUSE (30s).
		101-2: repeats algorithm of No.3.	48 33	Well if there were six 9's instead of seven 9's after the decimal point it would probably do that ...
				103 I: Give it a try.
				CALCULATOR OPERATING (No.7), (17s).

No.7. 10000000. x
0.999999 =
9999990. **
9999990. =
9999980. **

No.8. 10000000. x
0.1111111 =
1111111. **
1111111. =
123456.77 **

No.9. 10000000. -
1. =
9999999. **
9999999. =
9999998. **
9999998. =
9999997. **

No.10. 0.9999999 /
10000000. =
9.999999-08 **
CLEAR
CLEAR
0.9999999 /
10000000. =
9.999999-08 **
9.999999-08 =
9.999999-15 **

No.7; a pattern where the common difference is 10. Analyses sequence for pattern. 55

104: identifies error. 54
105-7: repeating question of lines 98-99. 4
133: repeats algorithm with data change. 48

109: clarification. 9

110-3;No.8: describes his general strategy. A variation of No.7. Seems stuck on this algorithm. 65 48 97

116: hint given. 8

118;No.9: bright idea. Changes approach and produces the obvious pattern that Evan produced. 41 53 47
119-21: demanding third method of solution. 1

No.10: subject picks division but switches the order in which this is done. Uses another method. 12

104 E: No, that's lowering by 10 though.

105 I: So this is No.7 (advancing and removing the printout). Can't think of another - another way that there might be of getting that pattern of numbers?

108 E: Lowering by 1.

CALCULATOR OPERATING (No.8), (45s).

109 I: What were you going to try Earl?

110 E: I was thinking if there was a pattern of seven 9's after a decimal point ... then possibly ... there'd be another number like that which would just go by 1's. And then I started thinking well maybe it (i.e., another number) would be 1's, but ...

114 I: You got an interesting idea. Now you know what I was thinking of ... I'm kind of pushing it maybe ... you know ... right away you're sticking to multiplying.

117 E: I got one! I ...

CALCULATOR OPERATING (No.9), (10s).

10 million minus 1.

119 I: Yeah, that'll do it, subtracting 1. But there's other ways too, you know ... ... and I don't mean adding negative 1 instead of subtracting 1. Call that one No.9.

CALCULATOR OPERATING (No.10), (32s).

OK, it's not working out so well dividing, heh?

122 E: No, with that particular number (i.e., 10000000) it doesn't ... it could work out some how with division.

Sherri Problem 1.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:19				1	I: OK, what I want you to do is read - we'll do this in parts - so don't worry about b, c and d just yet. Do what it says in the question for part a.
No.1.	10000000. x				
	1.0000001 =				
	10000001. **				CALCULATOR OPERATING (No.1), (20s).
	10000001. =				
	10000002. **	04-07: repeating the question.	4		You did basically what it said. Now what I'd like you to do is look at the numbers that have stars beside them - two stars - and how would you describe that whole sequence of numbers as you go down the stars?
	10000002. =				
	10000003. **				
	10000003. =				
	10000004. **				
	10000004. =				PAUSE (10s).
	10000005. **				
	10000005. =	08: answers the question.	50	8	S: Well, add 1. Adds 1 all the way down.
	10000006. **				
	10000006. =			9	I: Yeah, basically that's what happens, heh? You're just increasing by 1. Now we'll try part b - that wasn't very hard. Now, before you use the calculator, I'd like you to ah ... tell me what two numbers you think you'd multiply to get this ... pattern here, the ones with the stars (i.e., 10000003, 10000006, 10000009 and 10000012)?
	10000007. **				
		11-14: rewording and repeating part b of the question.	4		
No.2.	10000000. x				
	1.0000003 =				
	10000003. **				PAUSE (18s).
	10000003. =				
	10000006. **				
	10000006. =			15	S: That.
	10000009. **				
	10000009. =	16: clarification.	9	16	I: What's that?
	10000012. **				
	10000012. =	17-18: predicts what the answer is before using the calculator.	50	17	S: 10 million, right. Times ... 1 point ... ...ah ... 1 point 0000003.
	10000015. **				
	10000015. =				
	10000018. **			19	I: OK, try it out.
	10000018. =				
	10000021. **	20: affective statement.	66	20	S: Watch this - probably won't work.
					CALCULATOR OPERATING (No.2), (30s).
	10000021. -			21	I: OK. So you got it. How did you figure it out?

No.2. 10000000. x 1.0000003 = 10000003. ** 10000003. = 10000006. **	24-27: recalls related problem in past calculations. 26-27: reading error.	33 55 81	24	S: Well it said 10 million by, well 1 point 6 zeros and 1, and then when you look at the answers there, it adds 1 everytime you press equals ... so when you asked me to get ah ... 10 thousand 3, 10 thousand 6, 10 thousand 9 -
10000006. = 10000009. **	28: correction.	6	28	I: Actually 10 million. It's OK, anyways.
10000009. = 10000012. ** 10000012. = 10000015. ** 10000015. = 10000018. **	29-31: explains the process used. Discovers the algorithm in part a, a link between the last digit of the second factor and the common difference of the sequence.	55 47 65	29	S: 10 million, whatever. It added by 1 down here (indicating No.1) ... so the second one, to add by 3, instead of the 1 on the end the second number, put the 3.
10000018. = 10000021. ** 10000021. -	32-39: reviewing part c of the question.	11	32	I: You saw that all you'd have to change would be the second digit. Let's try part c. Now in part c you don't have to tell me about what you want to do: you can try whatever you want (removing the printout from the calculator). You can see whatever you've already done (giving her the printout). And what I want you to try to do is get this pattern of numbers. Which pattern? - the ones that have stars beside them. And you're only allowed to use two numbers to form the pattern. PAUSE (3s).
	40: questions conditions given.	20	40	S: Multiplied, right?
			41	I: That's up to you. PAUSE (45s). CALCULATOR OPERATING (No.3), (30s). CALCULATOR OPERATING (No.4), (30s).
	42: contradiction.	6	42	S: Well, I haven't got it. I just wanted to ask you something here.
			43	I: Well, you got it.
	44: recognizes error.	54	44	S: Yeah, OK, but the thing is it starts at 9 ... it doesn't start at 9.

Printout	Comment		Transcript
			45 I: But let me just see, have you got – oh! I see what you mean. Go ahead.
	47: illustrates deductive reasoning.	40	47 S: Change 10 million to ... ah ... 9 million 9 hundred ... 999.
	48-49: some direction given.	12	48 I: Yeah, OK, you took 1 off 10 million. Was there any reason for that?
			50 S: Well, it went from – I wanted it to go from ... a hundred million to 99 ... down here. No 10 million to –
No.3. 10000000. x 0.0000001 = 1. ## 1. = 0.0000001 ##	54-55: reference to printout.	10	52 I: What we'll do is this – let's just advance it (i.e., the printout) and we'll see like different ideas you have there. You'll tell me about some of them. So we're talking about No.3. What was your idea in here – what were you going to try to do?
	56-57: reading error.	81	56 S: I was gonna see if the 1 ... in the second number ... 1 point ... 1 millionth I guess it is.
No.4a. 9000000. x 1.0000001 = 9000000.9 ## CLEAR	58: correction.	6	58 I: 1 ten millionth.
No.4b. 99999999. / 1.0000001 = 99999989. ## CLEAR	59-61: strategy: first parts used the last digit to create pattern so here she uses the first digit. Makes the second factor smaller to create a decreasing sequence. Alters previous algorithm but adaptation is not correct.	49 94	59 S: 1 ten millionth. I was wondering if the 1 would make a difference so I took it out and I multiplied 10 million by ... you know (i.e., 0.0000001).
			62 I: And what'd you get?
			63 S: It didn't work – it came out to 1.
No.4c. 9999999. / 1.0000001 = 9999998. ## 9999998. = 9999997. ## 9999997. = 9999996. ##	62: clarification. 64-65: extending subject's ideas.	9 13	64 I: That's right (giggling) – you're just basically multiplying the number by its inverse.
			66 S: Yeah.
			67 I: So as soon as you got 1 – you didn't stop though, did you?
			68 S: I pressed equals again.

No.4a. 9000000. x 1.0000001 = 9000000.9 ** CLEAR	69: clarification.	9	69	I: Just to see if there was a pattern – that's a good idea. And what pattern did you get?
	71: problem naming decimals.	81	71	S: Well, it went back to ... ten –
No.4b. 99999999. / 1.0000001 = 99999989. ** CLEAR	72-74: related question to see if she understands constant factor.	5	72	I: One ten millionth. Can you actually see why you got this one a second time? ... when you hit the equals a second time, what are you doing?
	75: error: she does not see how the calculator generates the sequence.	85	75	S: You multiply it again by 10 million.
No.4c. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **	76: contradicts while explaining	6	76	I: No. It multiplies again by the second number.
			77	S: Oh ... ah yeah that's right.
	78: clarification.	9	78	I: So that was your idea in No. 3. Did that teach you anything?
			79	S: Yeah.
			80	I: What?
	81: affective statement.	66	81	S: I'm pretty stupid.
			82	I: (laughing) no, no. Did it give you any ideas?
			83	S: That, that the 1 had to be there (i.e., in the second factor).
	84: reference to printout.	10	84	I: And I see you stuck with the 1 after that (i.e., in No.4a). Let's take a look at No.4. What was your idea there?
	No.4: all parts show a process of successive approximations where changes are made to the first factor only. This is an exploratory manipulation and modifying of previous work.	55 49 22	86	S: I took the ... I took a million off ... the first number, like 10 million, I took a million off, to make 9 million, and then timesed it by 1 and .... you know (i.e., 1 and 1 ten millionths).
	89: clarification.	9	89	I: What was your idea there – what did you hope was going to happen?

No.4a. 9000000. x 1.0000001 = 9000000.9 ** CLEAR			90	S: Well, I thought maybe that it might work out - it might come to ... the answer but it didn't.
			92	I: Now, from working with all these zeros and stuff, I noticed the next step in No.4b, you're using a lot more 9's. Why?
No.4b. 99999999. / 1.0000001 = 99999989. ** CLEAR	94-97: recalls previous work.	33	94	S: Well, I noticed in No. 4a, when I timesed 1 and tenth of million that the 9 million - the answer came out to 9 million point 9, so the 9 - had to be there. And so I figured that if I put a whole bunch of 9's would come out in the answer.
	96-97: strategy: compares numbers in first factor with numbers in first term of sequence.	55		
No.4c. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **			98	I: OK. But you've got eight 9's in here.
	99: reading error.	95	99	S: Because it's eight digits in the first number -
	100-1: contradiction of subject.	6	100	I: But if you look at the pattern you want you've only got seven (i.e., 9's).
			102	S: I didn't realize that.
			103	I: Anyways, didn't you get any ideas from that though?
	104: explains the change made from No.4b to No.4c.		104	S: Yeah, that I had one too many because that came out ...
	105: clarification.	9	105	I: One too many what?
	106: identifies error and repeats algorithm.	54 48	106	S: One too many 9's.
	107: carelessness in reporting.	100	107	S: I put seven 9's and ~~timesed it~~ by I -
	108: contradiction.	6	108	I: Divided it.
			109	S: Divided it. Yeah, I divided it before that too.
	110: related question.	5	110	I: Why did you decide to divide instead of multiply?

No.4a. 9000000. x 1.0000001 = 9000000.9 ** CLEAR	111-4: there is a contradiction here. From lines 94-7 she says she gets the 9's she needs by multiplication, so multiplication was getting her somewhere.	53 54	111	S: Well, multiplying - I don't know - would just - seemed to be getting me nowhere so I tried dividing, it was a different step - you know ... to experiment sort of. And ... I ... timesed them, and then it worked.
No.4b. 99999999. / 1.0000001 = 99999989. ** CLEAR	She changes her approach. In the line, 114, she knows she has it.		115	I: Divided you mean.
	116-7: compares her answer to the printout given.	55	116	S: Divided it. And it worked. But I came out with an 8 on the end of it (i.e., in 9999998).
No.4c. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **			118	I: Instead of?
			119	S: The 9 that was supposed to start (i.e., 9999999).
	122-3: direction given.	12	120	I: So really what's happened is you've got the sequence of numbers except for the first term. So ... ... I wonder if ... what you would need to do ... well, can you get any ideas in here? Look ... if you start with seven 9's and you divided by that you get one number less ... so what do you think you might do? PAUSE (5s). You're really on the right track, here. You've just got I think to make a slight change. PAUSE (15s).
	127: comment about solution.	83	127	S: I don't know.
			128	I: Do you want me to leave it with you for a little while?
			129	S: I'm just thinking here.
			130	I: I think that's what I'll do. I'll leave you with it a while. I think sometimes it's better like that. I think you get more chance to think.

				CALCULATOR OPERATED (No.5 and No.6), (6m).
No.5a. 99999990. / 1.0000001 = 99999980. **	133: returned to subject after it was obvious she was having problems and made reference to printout.	10		Sort of stumped, heh? We'll take a look at what you've got here (advancing and removing the printout). And we'll see some of your ideas. We'll call this one No.5. OK, what did you try in No. 5?
No.5b. 9999999. / 1.000001 = 9999989. ** 9999989. = 9999979. **	No.5a: strategy: still altering dividend. Repeats algorithm with data change.	48	137	S: I tried changing the last 9 to a 0, but that didn't ...
	No.5b: corrects careless error.	55		
	Strategy: change divisor, instead of dividend. Make divisor larger.	48	138	I: That would give you seven 9's, is that why you did it?
No.5c. 99999999. / 1.000001 = 99999899. ** 99999899. = 99999799. **			139	S: Yeah, but ...
			140	I: What happened?
			141	S: It didn't work.
No.6. 999999. / 1.0000001 = 999998.9 **			142	I: An 80, heh (i.e., 9999980)?
			143	S: An 80 on the end.
No.7a. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. **	144-5: reference to printout.	10	144	I: This one though, this next one ... isn't this No.4 all over again?
			146	S: No, I took one of the zeros off the last number (i.e., 1.000001 instead of 1.0000001).
No.7b. 9999991. / 1.0000001 = 9999990. **	148-9: probing reasoning.	10	148	I: Oh, you divided it by a bigger number. Why? Did you have any reason for trying that? OK, if you look at what you first did in No. 5, heh?
	151: interjection interrupts.	2	151	S: I had to get ah -
	152-3: comparing No.5a to No.5b to determine strategy, but directing.	12	152	I: The answer you're getting is very big (i.e., 99999980). So were you trying to make it small or something?
	154: uncertainty expressed.	95	154	S: Well yeah, and also ... ah, I don't know.

No.4b. 99999999. / 1.0000001 = 99999989. ** CLEAR	No.5c: original strategies have been paired - returns to 8 digits and maintains divisor of No.5b. Compare No.5c to No.4b and the combination strategy is clear.	55 49	155	I: Well you can make the answer smaller either by multiplying - by making the dividend smaller or the divisor larger. Now in most of these, in No.4, you kept the divisor constant (i.e., 1.0000001). Now in No. 5 ... you started altering the divisor and sticking with the dividend (i.e., 9999999) and it decreased by 10 each time.
No.5c. 99999999. / 1.000001 = 99999899. ** 99999899. = 99999799. **			161	S: And then I did it again. Only I took another zero out.
	162: directing question to find if her changes in numbers are linked to getting larger or smaller quotients (i.e., to find if she is using successive approximations).	12	162	I: To make it even larger, heh?
			163	S: And make a hundred - it decreased by a hundred.
No.6. 999999. / 1.0000001 = 999998.9 **			164	I: So that made sense, didn't it? Like if you make the divisor larger by 10 then you'll make the sequence of answers smaller by 10 times what it was.
No.7a. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. **	163;No.5b: she analyses the results of her calculations.	55	167	S: Yeah.
	164-6: investigator is linking size to changes, not subject.	12	168	I: Now in No.6?
No.7b. 9999991. / 1.0000001 = 9999990. **	No.6: changes both dividend and divisor. Double change, like in No.5b.	48	169	S: I tried taking the 9 off and returning the second number to the full (i.e., 1.0000001 instead of 1.000001).
			171	I: But you only went one step in No.6.
No.4c. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **	No.7a: is exactly like No.4c. Error in repeating previous work.	86	172	S: Well it came to a decimal ... and ....
	172: analyses results.	55	173	I: You didn't like that.
			174	S: You know, it's not the answer so ...
			175	I: So now in No. 7. Looks like that's identical to No.4.
	176-8: there does not appear to be a plan behind the changes made.		176	S: I just wanted to get it on here so I could look at it and see how it sort of worked out. Then I tried putting a 1 in the place of a 9.

No.8. 10000000./ 1.0000001 = 9999999. ** 9999999. = 9999998. ** 9999998. = 9999997 ** 9999997. = 9999996. ** 9999996. = 9999995. ** 9999995. = 9999994. ** 9999994. = 9999993. ** 9999993. = 9999992. ** 9999992. = 9999991. ** 9999991. = 9999990. ** 9999990. = 9999989. ** 9999989. = 9999988. ** 9999988. = 9999987. ** 9999987. = 9999986. ** 9999986. = 9999985. **			179	I: Why?
	180: error: random response.	94	180	S: I don't know ... just seemed like something to do.
	181–91: an explanation given of the solution and some instruction.	1 13	181	I: What I was trying to show you – what I was trying to get you to see is look – you're getting the right sequence in No. 4 and No. 7. And you're getting the right sequence because the number you're working with ... the number, the dividend ... is 1 less than you want to start with. So I think if you made that (i.e., 9999999) 1 larger ... then you'd get this sequence starting 1 number larger. Do you follow what I mean? Like make instead of 9999999 – make it 10000000 and divide by the number you're saying to divide by (i.e., 1.0000001). (Investigator demonstrates what he means on the calculator).
				CALCULATOR OPERATING (No.8), (Demonstrated by interviewer).
				Then it should give you –
			192	S: Oh boy! (when she sees the number generated).
	193–200: rewording the question.	4	193	I: If you had made the divisor 1 larger ... you were very close to it, but couldn't see it. Now what I'd like you to do is this – from all of the things you've done, what I'd like you to do is make up – use two numbers to make up your own sequence, whatever sequence you want to make.
				CALCULATOR OPERATING (No.9).
				Can I interrupt you for a minute? Like you'll press the equal button once, then after that I'd like you to tell me what you think your sequence is going to be. And then you'll press it a couple of times.
				CALCULATOR OPERATING (30s).

```
No.9,   888888. x
          1.001 =
        889776.89 **
        889776.89 =
        890666.66 **
        890666.66 =
        891557.33 **
        891557.33 =
        892448.89
        892448.89 =
        893341.34 **
        893341.34 =
        894234.68 **
        894234.68 =
        895128.91 **
```

			So there's your first sequence with the two steps. What type of sequence are you trying to make, what's your idea?
204: note the random nature of what subject does is reflected in the problem she generates.	94	204	S: I don't have any yet (Laughing).
		205	I: Well you multiplied some numbers here; why'd you pick those numbers? ... ... what parts of the problems that you did is it like?
205-7: clear direction given.	12		
No.9: note what is constant and what changed: all numbers the same in the first factor; uses 1's to start and end second factor; uses three decimal places.		208	S: Like ... starting to get into No. 4. I changed the 9's to 8's.
		209	I: Well you kept them all the same though. How did you decide – you used six, right ... six 8's. And you used 1 with [illegible] so what's the second one (i.e., 1.001) like though?
		212	S: It's like the first number ... like 1 and ...
213-4: subject generated first term as asked to stop and predict by the interviewer.	1	213	I: Six zeros with a 1 ... so if you keep on hitting equals, watch what happens.
			CALCULATOR OPERATING (No.9).
			Are you getting a sequence?
		216	S: Not really – it's just decreasing by a certain amount each time.
217: related question.	5	217	I: Is it the same amount each time?
		218	S: ... no.
219-23: review of printout.	11	219	I: There's many ways of looking at the sequence. Like ... ... sort of have to be careful. See like you've got 8's becoming 9's ... you've got 9's becoming 0's becoming 1's becoming 2's becoming 3's becoming 4's (note the thousands place). There's sort of a pattern in there.

No.9.
```
 888888. x
   1.001 =
889776.89 **
889776.89 =
890666.66 **
890666.66 =
891557.33 **
891557.33 =
892448.89
892448.89 =
893341.34 **
893341.34 =
894234.68 **
894234.68 =
895128.91 **
```

No.10.
```
895122.23 *
895122.23 =
896017.35 **
```

224 S: And then the next number decreases ... 7, 6, 5, 4, 3, 2, 1 (in the tens and hundreds place). And the next number the same thing.

226 I: Now the way you can sort of see if you've got a pattern - a good way of checking if you've got a pattern - is whether you can predict what's going to come next ... like if you just advance that (advancing the printout). Do you think you could tell me what number would come next?

229-30: prediction question.

231 S: 8960122. ... ah, no I'd have to start this side (from the right of the printout) because you're going this way, you have to add on everything.

231: This answer makes sense in terms of what the sequence has done in No.9: the thousands increased by 1 and the hundreds decreased by 1. She is reasoning deductively.

PAUSE (10s).

3,2 point ...

PAUSE (5s).

235 I: Well you can do some work on the calculator if you want to find it.

PAUSE (30s).

So what we'll do is I'll punch in that number you've just given me, 895122.23, we'll just print it.

CALCULATOR OPERATING (No.10).

That's what you say it's going to be. So we'll just hit equals and it'll give you the next number. This is actually the next number in the sequence, the one that's just below that (i.e., 896017.35). It's very hard to predict it.

237-8: this is not correct. No. 11 indicates what should have resulted and comparing this with the prediction in line 231 (i.e., 8960122) we see the subject correctly predicted the changes in the thousands (5 to 6) and the hundreds (1 to 0) places.

```
No.11.   888888. x
           1.001 =
       889776.89 **
       889776.89 =
       890666.66 **
       890666.66 =
       891557.33 **
       891557.33 =
       892448.89 **
       892448.89 =
       893341.34 **
       893341.34 =
       894234.68 **
       894234.68 =
       895128.91 **
       895128.91 =
       896024.04 **
       896024.04 =
```

12:50

Mardi Problem 1

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:11		01-03: clear directions on what to note.	12	1	I: So what I want you to do Mardi - here's a problem ... I want you to read part a - don't worry about about b, c or d yet ... so you do what it says.
	No.1.10000000. x 1.0000001 = 10000001. **				CALCULATOR OPERATING (60s), (No.1)
	10000001. = 10000002. ** 10000002. = 10000003. **	04-06: repeating question part a.	4		OK, so you multiplied the numbers that it said in part a ... and what I'd like you to do now is describe the sequence of starred numbers - can you see the numbers with the stars?
	10000003. = 10000004. **			7	M: Yeah.
	10000004. = 10000005. ** 10000005. =	08-09: repeating the written question in part a.	4	8	I: (Advancing and removing the printout) how would you describe the sequence? What's happening as you go down the starred numbers?
	10000006. ** 10000006. = 10000007. **	10: clear, direct answer to the question.	50	10	M: Every number ... is increasing by 1.
				11	I: That's the sequence. That's basically part a. Now in part b - before you do it on the calculator ... I'd like you to tell me what two numbers you think you'd multiply. So you read it over, and there's the sequence you want to get ... and you tell me what two numbers you think you ... would multiply to get that ... and you can look at what you did in No.1 if that will help you.
		13-14: asking for a prediction.	5		
	No.2.10000000. x 1.0000003 = 10000003. ** 10000003. =	16-17: repeating the question. 17: trying to establish the printout as a reference.	4 10		
	10000006. ** 10000006. =				PAUSE (24s)
	10000009. ** 10000009. =				What two numbers do you think you'd multiply?
	10000012. **	18: note: place value not used to name the decimal.	50	18	M: Ten million and 1 point ... 0000003.
				19	I: So try it out.
					CALCULATOR OPERATING (28s), (No.2)
					So what happened?
		21: has confidence in the answer		21	M: It works good (advancing and removing the printout).

No.2.10000000. x	23: hesitation.		23	M: Well ...
1,0000003 =				
10000003. **	24: rewording of process question.	4	24	I: How did you figure out that was the thing to do?
10000003. =				
10000006. **	25: ostensive language.	33?	25	M: Well in this one –
10000006. =				
10000009. **			26	I: In No. 1, yeah –
10000009. =				
10000012. **	27–29: focuses on the second factor, linking increase to last digit of second factor. Works from sequence backwards to factors.	55	27	M: In No. 1 it increased by 1 when you multiplied by the one with the point 1 ... and then in this one it increased by 3 so it had to be added on 3 each time. So ... 3 ...
	30-31: related question probing calculator knowledge.	5	30	I: Can you tell me ... what the calculator's actually doing ... each time you hit the equal sign? You know?
			32	M: Multiplying by the same number.
			33	I: Which number?
			34	M: 1 point 0000003.
			35	I: So you see this second starred number that you get (i.e.: 10000006) ...
			37	M: Yeah.
No.3.10000000.	38: trying to determine if she sees factor [illegible]		38	I: Like 1 and six zeros and a 6? How did you get that number – what [illegible] you multiply by? ...
1.0000001 =				
9999999. **				
9999999. =	40: she knows what the second changing factor should be.	5?	40	M: [illegible] and [illegible] 1 point 0000003.
9999998. **				
9999998. =	41: this is adding to what the subject actually says.	13?	41	I: So the second digit, i.e., [illegible] number, that you're multiplying [illegible] is the one you multiply all these by.
9999997. **				
9999997. =			42	M: Yeah.
9999996. **				
9999996. =				
9999995. **				

No.3. 10000000. /
1.0000001 =
9999999. **
9999999. =
9999998. **
9999998. =
9999997. **
9999997. =
9999996. **
9999996. =
9999995. **

Comment		Line	Transcript
		44	I: Let's look at part c - I'll take these out so you can look at them (removing printouts No.1 and No.2) ... ... these you might want to look at these to help you see ... you can use - you don't have to tell me what - you can play around with the calculator a bit ... and see if you can produce this pattern of numbers (i.e., those given in part c of the question).
			CALCULATOR OPERATING (41s). (No.3).
			OK, let's just advance it (advancing the printout). You got the pattern heh? What was your idea?
51: should have tried to find if the subject knew she had the right answer before assuming it.	9	52	M: Well ... this No. - when you wanted to -
No.3: reverses previous operation in No.2.	47	53	I: Which No.
54-57: sees a relation between questions 2 and 3. Still seeing the second factor as determining the difference between terms.	31	54	M: In No.2 ... when you wanted to increase it you multiplied it by the number it was increasing it - like in the decimal part there (i.e., the second factor ended in 3 and the increase was 3) so in this part it's increasing by 1 so I divided it by ...
58: probably means decreasing. Demonstrates deductive reasoning.	47	58	I: So what did you really change?
59: rephrasing of process question.		59	M: The sign ... the ... function ...
60: clarification.	9	60	I: Instead of multiplying -
		61	M: I divided.
62: directing question.	12	62	I: Did you know that dividing would make it smaller?
		63	M: Yeah.
		64	I: How do you know that dividing will make it smaller?
65: reference to previous knowledge.	31 5	65	M: ... I don't know - I just learned that...

```
No.4.1000000. x
   1.00054 =
   1000540. ##
   1000540. =
   1001080.3 ##
   1001080.3 =
   1001620.9 ##
   1001620.9 =
   1002161.7 ##
   1002161.7 =
   1002702.9 ##
   1002702.9 =
   1003244.4 ##
   1003244.4 =
   1003786.1 ##
```

66: counterquestion to probe understanding. Also a regression into teaching.	6	68	I: But if I divided by a number less than 1 ... it would become bigger ... you divided by a number bigger than 1, heh, (i.e. 1.0000001) ...
		69	M: Uh hum.
74-76: very general question to create openendedness.	1	70	I: That's part c - you didn't have any trouble with that at all ... that just took you maybe a couple of seconds (41 to be exact). What I'd like you to do now - and this is the last part of the question (advancing and removing the printout) ... I'd like you to try - you make up ... any question - any pattern you want of numbers ... whatever you want to make up - in other words I'm asking you to make up a pattern ... any one you want to make.
			CALCULATOR OPERATING (53s), (No.4).
77: attempt at obtaining prediction but not asked soon enough as she was already operating.	7		Are you getting what you expected?
		78	M: Uh hum.
79: rewording question in 77 as doubtful about her answer in 78.	9	79	I: You saw all these decimals ... 3, 9, 7, 9, 4 and 1 coming?
80: funny refers to lack of whole numbers.		80	M: Not all of it - I knew it was gonna be funny but ...
		81	I: You knew it was gonna be funny?
		82	M: Different, like ... a whole bunch of numbers ...
83: a specific test of her ability to predict.	5	83	I: In the pattern you can figure out what the next number will be?
No.4: note the changes similarity between the sequence the subject generates and those parts already done.	33	84	M: Yeah.
		85	I: Can you tell me what the next number will be in your pattern?
			PAUSE (18s
86: error indicates she does not have control of this sequence?	93	86	M: 19 (laughing. -

No.5.10000000. /
1.0000003 =
9999997. **

No.6.9999997. =
9999994.**
9999994. =
9999991. **

87: contradiction. 6

89: questioning processes (idea) 10
behind the pattern generate. 100
90: random response.

-92: explaining the principle the 1
sequence. 13

99: prediction demanded before 9
any operating.

102: correct prediction. 50
No.5: note the resemblance to No.3 31
Subject has less change in this printout than in No.4. Has acted on the suggestions given.

87 I: I don't think so .... I don't see it - maybe you can see it ... you look at the starred ones ...

PAUSE (13s)

Why did you pick this 1.00054 ? ...

90 M: Just out of the ... nowhere.

91 I: You'll notice that in the patterns I picked (i.e., in the question given) I used very large numbers -

93 M: Yeah.

94 I: Because something funny happens with the calculator when you completely load up the display ... do you know what I mean? ... completely load up the display ... you haven't completely loaded the display - you So you've ah ... you haven't completely loaded the display - you used 10 million alright ... or did you? ... that's 1 million ... did you know that? ... OK, try another pattern - any pattern you want.

CALCULATOR OPERATING (185s), (No.5).

Just before you go (i.e., press the equal sign repeatedly) ... what do you expect to happen?

102 M: ... It'll probably decrease by 3 every time.

103 I: See, you got the first one -

104 M: Yeah -

105 I: And you're dividing by ... 1.0000003. Let's see what happens.

CALCULATOR OPERATING (7s), (No.6).

Is it doing it?

No.5.10000000. / 1.0000003 = 9999997. **			107	M: Yeah.
	108: related question.	5	108	I: So which pattern is this one like?
No.6.9999997. = 9999994.** 9999994. = 9999991. **	109: note the reference made to a previous printout perhaps indicating it was a model.	50	109	M: The pattern in No. 3.
12:22	No.5: modifies her procedure in No.4. Note the similarity to previous problems.	49 33		

Evan Problem 1.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:30					PAUSE (30 s while Evan reads the problem).
					CALCULATOR OPERATED (No.1.).
		01: explaining the question.	1	1	I: OK, so what I'd like you to do now is look at the numbers you've got there (advancing and removing the printout). You see the starred numbers?
No:1.	10000000. x				
	1.0000001 =				
	10000001. **			4	E: Uh uh.
	10000001. =				
	10000002. **	05-07: repeating and rewording part a.	4	5	I: Could you describe those numbers - just the numbers with stars - what pattern is there, what kind of sequence is that - what's happening?
	10000002. =				
	10000003. **	08: sees pattern as additive.	50	8	E: ... they're all increasing by 1.
	10000003. =				
	10000004. **	09: rewording what subject said.	4	9	I: No ... what you're doing is adding 1 to each number. That's pretty obvious. Can you tell me what you're doing each time you hit the equal sign?
	10000004. =	10-11: related question on constant feature of the calculator.	5		
	10000005. **				
	10000005. =				PAUSE (5s).
	10000006. **				
	10000006. =				
	10000007. **	12: sees what the constant operation but does not state multiplier Tone indicates uncertainty.	50 95	12	E: Multiplying again?
				13	I: Yes, by what?
				14	E: By the same number.
		15: developing question in line 11 more precisely.	9	15	I: Which same number?
		16: sees what multiplier is.	50	16	E: The second one.
		17-20: 10 million should be 10 million and 2. A related question that extends that in lines 11,15.	5	17	I: That's right. In the product heh? It's not the first number you multiplying by, but the second one each time. So like, how do you get this 10 million? The second starred number (i.e., in No.1) - what numbers did you multiply to get that?

No.1.
10000000. x
1.0000001 =
10000001. **
10000001. =
10000002. **
10000002. =
10000003. **
10000003. =
10000004. **
10000004. =
10000005. **
10000005. =
10000006. **
10000006. =
10000007. **

Comment	Code	Line	Transcript
			PAUSE (15s).
21: does not see the changing nature of multiplication. Misinterprets calculator operation.	83	21	E: The ten million at the top and the ten million and one.
22: contradiction and then an answer of the related question.	6	22	I: No I think you multiplied the 10 million and 1 by the second number (i.e., 1.0000001), like you told me. ... ... Do you see what I mean?
		25	E: Yeah.
25-26: asking same question as in line 19 but with different term of the sequence to test understanding.	5	26	I: To get 10 million and 2, multiply the answer you got by this one (i.e., 1.0000001). How do you get 10 million and 4?
			PAUSE (5s).
27: indicates understanding.	50	27	E: Multiply the 10 million and 3 by the second number.
28-35: explanation and teaching.	1	28	I: Yeah, by 1 and 1 ten millionth. So you don't actually see the multiplyings act there anywhere do you? All you see is the equal sign, but you have to realize what the equal sign is actually doing - it's doing constant function, the same thing each time. OK, that's No. 1. Now let's look at part b. I want you to look at these numbers - there's a sequence heh? - you look at the starred numbers - and ah .. I want you to tell me what two numbers you would multiply to get that pattern of starred numbers.
			PAUSE (12s).
36: subject seeks clarification.	83	36	E: I don't quite understand what you mean.
37-40: rewording the question.	1	37	I: You see the pattern of numbers, 10 million and 3, 10 million and 6, 10 million and 9, 10 million and 12? What two numbers would you multiply to get that pattern ... to start the pattern heh? ... what two numbers would you multiply?
41: Discovers pattern.	[illegible]	41	E: [illegible] going up by 3 each time.
		[illegible]	[illegible] and 1 and they're going up each time.

43: uses the first number as a constant factor and increases the second factor by 1 each time	43 E: Then 3 and 2, 3 and 3.
44: clarification	44 I: OK, what do you mean by 3 and 1?
	45 E: To get the 3 ...
45-47: related question	46 I: What would you multiply to get this first number, 10 million and 3?
	PAUSE (5s).
48: questions conditions.	48 E: So you go by this (i.e., the question)?
49: small hint	49 I: Yes, and you can use part a to help you.
	PAUSE (5s).
50: changes both factors from part a. An error as he sees multiplication as commutative here.	50 E: 1 and 1 millionth and 10 million and 2.
51-52: error by investigator in saying that second is unchanged. Related question.	I: So you would leave the second the same and what would you make this one (i.e., the first factor: 10000000)?
53: states wrong answer.	53 E: 10 million and 2.
	54 I: 10 million and 2. Alright, try it.
	CALCULATOR OPERATING (No. 2.).
55-56: explanation.	If you're going to see a pattern you'll have to hit it more than one time.
	CALCULATOR OPERATING (No. 2.).
57-58: error on the part of the interviewer. Subject entered what he said.	Go again. First of all, you missed off a zero here (i.e., entered 1.000001 instead of 1.0000001) heh?
	59 E: Right.

No.2. 1.000001 x
10000002. =
10000012. **
10000012. =
1.0000014 14**
1.0000014 14 =
1.0000016 21**

60-66: explanation and instruction. 1 13

No.3. 10000002. x
1.0000001 =
10000003. **
10000003. =
10000004. **
10000004. =
10000005. **

69: related question to see if he understands the commutative aspect of the problem. 5
70: answers question correctly. 50

No.4. 10000000. x
1.0000003 =
10000003. **
10000003. =
10000006. **
10000006. =
10000009. **
10000009. =
10000012. **

73: relating discussion to printout. 9
74: same in the sense that it increases by 1.
75-77: explaining and extending from subject's statements. 13
No.3: order of factors changed from No.2 but error is still made as change is not made to the correct factor (i.e., 1.0000001 to 1.0000003). 49 01

No.4: makes further modification and answers part b.
80-82: clarification.

66 I: But also you didn't get the pattern heh? - but also - I don't know if you know it or not - but there is a difference between multiplying them in this order (i.e., 1.000001 x 10000002) and then hitting the equals and switch - in part a - and switching them around and multiplying them .... do you understand? ... You see the equals each time you hit the equals, you're multiplying by the second number, right?

67 E: Right.

68 I: So if you switch these numbers around and you keep hitting the equals - you're multiplying by which one?

70 E: This one (pointing to 10 million).

71 I: Try this one again, and see if you can think of - with what I just told you - see if you could think of a way of getting that ... of getting the pattern in part b.

CALCULATOR OPERATING (No.3.), (30s).

So what did it do?

74 E: Same as the first one. Gives the same pattern.

75 I: And yet you've different numbers than in part a. So it doesn't matter if you increase the 10 million by small numbers. We'll get you to try something else (advancing and removing the printout).

CALCULATOR OPERATING (No.4.), (20s).

What happens?

E: Same sequence as on the page (i.e., in the question).

I: Yeah, you got it. Let's just take a look at this. And I'd like ... to tell me ... and in No. 2 ... idea was ... did you want ... the six zeros?

			83	E: Yes I did.
	84: calculator misentry error in No.2.	100	84	I: So that's an error. Even if it was six zeros - your idea was to change - alright, first of all you switched around the order in which you multiplied ... and I explained to you that that makes a difference. So you needed to switch that around (i.e., reverse 1.000001 x 10000002), heh? If it goes up by 3 each time, why did you want to multiply by 2 more?
	89: in other words, why was the factor 10000002 chosen?	5		PAUSE (5s).
				Is there a reason? ... why did you pick 2?
				PAUSE (5s).
				You just picked it?
	91: reference to previous problem.	30 55	91	E: Well, it was because of this.
	92: questioning reasoning.	9	92	I: Because of No.1. Why did you pick 2 again? What happened in No.1 that made you want to pick 2?
	No.4: strategy: uses third step in part a to generate 10000003 which is successful but other terms are not created.		93	E: Because when you multiply the 10000002 by 1 millionth it gives 3 (i.e., the second step in No.1).
No.3. 10000002. x 1.0000001 = 10000003. ** 10000003. = 10000004. ** 10000004. = 10000005. **	95-98: repeating subject's ideas.	4	95	I: Oh the next step here (i.e., in No.1). So like ah ... you thought if you had that (i.e., 10000002 and 1.0000001) you'd get this (i.e., the sequence wanted). I see what you're getting at. But it didn't work, heh? Alright, in No. 3, what was your idea?
	98: reference to printout.	10		
	99: done after this was explained.	33 49	99	E: Well, I fixed up the errors in the other one (i.e., No.2).
	100-1: questions certainty.	7	100	I: So you switched the order back and you got the six zeros in. Now did it do what you wanted it to do?
			102	E: No -

	103: contradiction.	6	103	I: It did, you got the first one didn't you -
	104: realizes what his error is.	54	104	E: Yeah, but then it only steps 1 (i.e., from 10000003 to 10000004 to 10000005).
	106: related question to see if he understands constant factor, changing factor and increases that will result.		106	I: Do you have any idea why it went up by steps of 1?
			107	E: Cause it's only multiplying by 1.
	107: probably means adding 1 instead of multiplying by 1. 109: related question.	5	108	I: And although you changed the 10 million to ten million 2 you get the same pattern you were getting in No.1. Do you have any idea why that is?
	111: explaining his strategy.	65	111	E: Well I just left out the first part of the sequence. To get the 10 million 1, 10 million 2, went right to 10 million 3 and 4.
No.3. 10000002. 1.0000001 = 10000003. ** 10000003. = 10000004. ** 10000004. = 10000005. **	113-9: reviewing calculations.	11	113	I: That's right too, it's not identical (i.e., to No. 1), is it? Like you missed out this part here (i.e., 10000002 is not printed twice). Basically what you're doing ... ah ... if you look at the multiplication in there ... in No. 3 ... you're multiplying 10 million and 1 10 millionths ... and that's adding 1, OK? Now when you multiply 10 million 2 by that (i.e., 1.0000001) it still adds 1 -
			120	E: Adds 1.
No.4. 10000000. x 1.0000003 = 10000003. ** 10000003. = 10000006. ** 10000006. = 10000009. ** 10000009. = 10000012. **	121-5: reference to printout.	10	121	I: So you started with a bigger number and that's why you missed out one turn (i.e., in the sequence). Let's take a look at No. 4 and this is where you got it, heh? ... what happened there ... well You decided - can you tell me what your idea was there - why you picked those two (i.e., 10000000 and 1.0000003)?
	126-7: No.3 and No.4 related to the increases they produce. Explains reasoning.	[illegible]	12[illegible]	E: Saw that it'd be 2 if I kept it as 1 it would just go up in steps of 1 so I had to use -
			12[illegible]	I: If you kept the second factor, heh?

Printout	Commentary		Line	Transcript
No.4. 10000000. x 1,0000003 = 10000003. ** 10000003. = 10000006. ** 10000006. = 10000009. ** 10000009. = 10000012. **	129-31: strategy: change the last number in the second factor to the difference you want in the sequence		129	E: 3. Cause 1 - it would go up in steps of 1 so I raised it (i.e., the second factor) to 3 ... it (i.e., the sequence) would go up to threes.
	132-3: repeating and extending.	4 13	132	I: So you saw it was the second number you were multiplying by that controlled how much it went up.
			134	E: Yeah.
	135: more extending beyond what the subject actually said.	13	135	I: And you kept it 10 million, like in the very first one?
			136	E: Right.
No.5. 9999999. - 1. = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. ** 9999996. = 9999995. **	No.5: strategy: use the simplest solution possible, just subtract 1 repeatedly. Indicates intuitive reasoning. Not identical to given printout as is noted by subject in line 144.	41 54 47	137	I: And it worked. Now we're going to come to a problem a little bit different. And I'll leave this (i.e., the printout) for you to look at. And what I want you to do is you use the calculator and see if you can come up with this sequence of numbers - the starred ones, heh. Can you make this sequence using numbers and whatever you want to do? PAUSE (30s). CALCULATOR OPERATING (No.5). You got it heh (looking at the display)?
			144	E: Not quite.
	145: questions certainty.	7	145	I: Oh, you're just a little bit. Or are you? You've got it, don't you?
	147: notes he wants a sequence where 9999999 has a pair of stars beside it.		147	E: Well I should have started with 10 thousand (actually means 1 million).

```
No.6. 10000000. -
             1. =
       9999999. **
       9999999. =
       9999998. **
       9999998. =
       9999997. **
       9999997. =
       9999996. **
       9999996. =
       9999995. **

No.5.  9999999. -
             1. =
       9999998. **
       9999998. =
       9999997. **
       9999997. =
       9999996. **
       9999996. =
       9999995. **

No.7.        2. x
           200. =
           400. **
           400. =
         80000. **
         80000. =
      16000000. **
```

154: affective statement. 3

155-9: repeating of part d of the question 4

160: seeking problem definition. 20

161-2: explains the problem. 1

163: questioning certainty. 7

No.7: note how different this sequence is from the work done in parts a, b and c.

165: clarification. 9

166: giving direction. 12

148 I: OK. Ah so ... that's your first attempt. Try again and see if you can get it. You've got everything except the first number.

PAUSE (20s).

CALCULATOR OPERATING (No.6).

Did you get it (advancing and removing the printout)? Just explain to me what you did in No. 5. What you were doing - oh you were subtracting. Of course it's going to work. You're just taking away 1. That's excellent. In No. 6 you did the same thing, started with 10 million, of course you'll get it. That's what I was looking for - it's a very smart way to do it. One last thing I want to get you to do - right now you use any two numbers you want - start off any sequence you want. You can look at what you did, look at the problem, make it out of your head, whatever you want. Just make any sequence you want right here.

160 E: Any number at all?

161 I: Yeah, you can start off with any two and you just make any sequence you want.

PAUSE (40s).

CALCULATOR OPERATING (No.7).

Is that the sequence you wanted? Did you get what you wanted to?

164 E: I didn't think it would go up that rapidly though.

165 I: What did you want it to do - your sequence?

PAUSE (5s).

Did you want it to keep multiplying by [illegible] each time? Or did it just turn out like that

168-9: exploratory manipulation.	22	168	E: Well it was ... actually it was ... experimenting with the exponent key ... I was just gonna ... ...
170-1: related question.	5	170	I: You wanted to do something with the exponent key? ... Which key is that? ... Do you know how it works?
		172	E: That's just why I was going to experiment.
		173	I: (Laughing) well how'd you know it was going to give you a sequence if you didn't -
		175	E: Well ...
176-8: direction given.	12	176	I: So how'd you end up with this then - 2 times 200 equals, equals, equals - you didn't use the exponent key for that ... ... did you?
179: using multiplication by 200 as his constant operation. Related to previous work.	33	179	E: Well I put that into the second number.
180: clarification.	9	180	I: ... you put what?
		181	E: That one into the second number.
182: clarification.	9	182	I: Which into the second number?
		183	E: This.
184: clarification wanted on ostensive language.	9	184	I: Show me how you pressed the keys - what'd you do again? .... You pressed 2 times 2 exponent 2.
			CALCULATOR OPERATING (No.8).
			You pressed 2 times 2 exponent ... you pressed next ...
		197	E: Uh huh.

No. 8. 2. x 200. = 400. ** 400. = 80000. **	190: related question.	5	188	I: Exponent 2. And then what? Equals? And then equals again. Oh! And the button doesn't even show EXP, doesn't write EXP beside the number on the printout. Did you know what the exponent would do?
	192: random response.	94	192	E: Well, took a guess.
			193	I: What do you think it does?
	194: error: he still does not understand the functioning of the exponent key even after working with it in No. 8.		194	E: Doubles the number each time.
	195: contradiction. 196-7: related question to show knowledge subject has of exponents.	6 5	195	I: By 200. When you say exponent 2, you're telling me the power of ten you want to multiply by. So it's 2 times 10 squared. You know what ten squared is?
			198	E: Yeah.
	199: clarification.	9	199	I: What?
	200: has basic knowledge of exponents. 201-2: explanation.	50	200	E: A hundred.
3:44			201	I: So it's really 200. So you multiplied by 200 each time. Alright, that's very good.

Bruce Problem 1.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:41					CALCULATOR OPERATED (No.1).
No.1.	10000000. x 1.0000001 = 10000001. ** 10000001. = 10000002. **	01-04: repeating question part a.	4	1	I: OK. So let's just advance it. Now what I'd like you to do is describe - you see the numbers that have the stars by them? - describe that pattern of numbers, what's happening as you go down the starred numbers.
	10000002. = 10000003. **	05: discovers pattern.	50 55	5	B: They all increase by 1.
	10000003. = 10000004. **	06: rewording question part a.	9	6	I: So in effect, what are you doing?
	10000004. = 10000005. **	07: answers question.	50	7	B: Adding by 1, I suppose.
	10000005. = 10000006. **	08: direction given.	12	8	I: Each time you hit the ...
	10000006. = 10000007. **			9	B: Equals.
		10-13: related question to test calculator knowledge.	5	10	I: I've got a question for you. You see the second starred number? 10000002? How did you get that number? Like you just keep hitting the equal signs of the calculator but can you tell me what the calculator did to get that second number?
				14	B: It has something to do with - we were working with this last time (i.e., the last calculator problem Bruce did). Cause you remember we were playing with that and I hit equals once and it gave some funny number. OK, where I typed in 1,... whatever that is ...
		17: error: reading ability (number recognition).	81		
		18: explanation (contradiction).	6	18	I: 1 ten millionth.
		19-20: comprehension error: he does not see that one factor remains constant and the other factor changes.	35	19	B: OK, 1 ten millionth, then it kept multiplying ten million times 1 ten millionth -
		21: clarification.	0	21	I: Each time?
				22	B: Each time I hit the equals button.
		23-24: repeat of question in lines 10-13.	4	23	I: So how did you get that second starred number then (i.e., 10000002)

No.1.

```
10000000. x
1.0000001 =
10000001. **
10000001. =
10000002. **
10000002. =
10000003. **
10000003. =
10000004. **
10000004. =
10000005. **
10000005. =
10000006. **
10000006. =
10000007. **
```

25-26: says he can't solve it. No response to question. 63 92

27-28: related calculator question. 5

29: partial understanding: deals with the obvious pattern.

31: hint given. 8

32: responds to hint. 50

33-35: explanation of calculator operation and extension into teaching. 1 13

43-45: this is a slightly different form of the question given to other subjects in that the same type of question as in part a is repeated in part b.

46-48: discovers pattern and predicts correctly. 55

49: questions reasoning. 9

25 B: The one more? I don't know ... maybe it has something to do with the constant function –

27 I: Do you know what the constant feature's doing – what it is doing each time you hit the equals?

29 B: It's adding 1 more –

30 I: OK, that's what it ends up doing. But it's not really adding 1 more – it's multiplying by something each time.

32 B: ... it's multiplying by 1 ten millionth each time?

33 I: That's right. And what it does it multiplies like the first starred number by 1 ten millionth. So the way you get 10 million and 2 is by multiplying 10 million and 1 by 1 –

36 B: Ten millionth –

37 I: Ten millionth – catch on?

38 B: Yep.

39 I: So I just wanted to see if you understood what it was doing each time. OK, so let's look at question b, and I think we'll just take this out so you can look at it (advancing and removing the printout) ... and we'll call this No.1. ..., now you have to tell me, if you wanted to get ... ah ... if you multiplied those two numbers (i.e., 10000000 and 1.0000002), what pattern you would get.

46 B: (Immediately) I think I'd get the same type of pattern but it would be increased by ... I think it might be increased by instead of just 1, I think would be increased by 2.

49 I: Why do you think that?

	50: affective statement.	66	50	B: Well I think it has something . – well that's pretty dumb – but I think it has something to do with the last number entered into the thing that you multiplied out ... but if it was just – unless it was based on that –
	51–54: shows that he understands what variable is responsible for determining the difference in the sequence. Explains processes employed and shows reasoning.	40		
		55	54	I: On what?
	54: clarification.	9		
	55–58: explains reasoning.	65	55	B: Or ..., er the first – the number left of the decimal point. Then it would just increase by 1 but that's ... OK, if it was 10 million times 1 – 1 ten millionth then it should be ten million – I can see how that gets there ...
	No.2: calculator misentry.	100		
			59	I: OK. Let's try it – see what happens.
				CALCULATOR OPERATING (No.2).
No.2. 1000000. x 1.0000002 = 1000000.2 ** 1000000.2 = 1000000.4 ** 1000000.4 = 1000000.6 **	60: identifies error.	54	60	B: Ah, I think I mucked up. It shouldn't have a ten million ... er –
	61: related question to printout.	9	61	I: What's it increasing each time?
				PAUSE (5s).
				The one you've just done.
	63: number recognition error.	81	63	B: It's increasing by 2.
	64: contradiction.	6	64	I: Not by 2.
			65	B: No?
	66: related question but with direction.	12	66	I: 2 what?
	67: questions conditions.	20	67	B: What do you mean by 2 what?
			68	I: Well, just advance it (i.e., the printout) a minute so you can see it, what place is the 2 in?

No.2. 1000000. x 1.0000002 = 1000000.2 ** 1000000.2 = 1000000.4 ** 1000000.4 = 1000000.6 **	70: mathematics error in place value.	85	70	B: It's in the tens.
	71-74: explanation and correction of errors.	1 6	71	I: The tenths. So it's increasing by 2 tenths each time. You're right in saying it's increasing by 2 but when you usually say increasing 2 you mean 2 ones. OK, you've made an error in there (i.e., No.2).
	75: questions conditions.	20	75	B: An error like in the printout?
	76: explanation.	1	76	I: An error in what you entered ... can you see it?
	77: identifies error.	54	77	B: ... I put in 1 million instead of 10 million.
			78	I: That's right.
			79	B: So it should be –
	80-84: explanation.	1	80	I: We'll start again, we'll clear it.
No.3. 10000000. x 1.0000002 =				CALCULATOR OPERATING (No.3).
10000002. **	81-2:No.3: note encoding error in there is a gap in the printout.	95		You only pressed it one time. You won't see a pattern if you only press it one time.
10000002. = 10000004. ** 10000004. = 10000006. ** 10000006. = 10000008. ** 10000008. = 10000010. ** 10000010. = 10000012. **				CALCULATOR OPERATING (No.3). OK, and it did what you said it would do. Let me ask you a question, right. ... you see this last number, 10 million and 12?
			85	B: Uh hum.
	86: related question to test calculator knowledge.	5	86	I: How did the calculator get that? What did it do and to what?
	87-88: error in comprehending how the calculator generates the sequence. The same two factors are not constant.	85	87	B: It multiplied 10 million times 1.000 - 1 and 2 ten millionths. It did that seven times ...

No.3. 10000000. x 1.0000002 = 10000002. **	89-91: explanation and contradiction based on what the subject said.	1 6	89 I: This is what I was trying to get across to you last time. To get the sixth starred number (i.e., 10000012) you multiplied the fifth starred number by 1.0000002 ... do you follow?
10000002. = 10000004. ** 10000004. = 10000006. ** 10000006. = 10000008. ** 10000008. = 10000010. ** 10000010. = 10000012. **			92 B: I see. 93 I: You didn't multiply the 10 million – it multiplies the starred number preceding – 95 B: So it multiplies 10 million – 96 I: And ten – 97 B: Times 1 point – er 1 and 2 ten millionths.
	100-2: asks for prediction.	4	98 I: Right. It's a hard number (i.e., very long to read out). So that is part b and you worked that out. Now, in part c of the question when (investigator reads part c outloud). So you tell me the two you think first (i.e., the two numbers that the question asks the student to find).
	103-4: predicts correct answer.	50	103 B: It'd have to be 10 million ... and 1 point ... I'd say 1 point – 1 and 3 ten millionths.
	105: contradiction.	6	105 I: 10 million and 1 and 3 ten millionths. OK, try it.
No.4. 10000000. x 1.0000003 =			CALCULATOR OPERATING (No.4).
10000003. ** 10000003. =	106: states success.	64	106 B: So it does work out.
10000006. ** 10000006. = 10000009. **	107-8: questioning reasoning.	9	107 I: You got the pattern, heh? Can you tell me very quickly how you figured out it was that (advancing and removing the printout)?
10000009. = 10000012. **	109-10: recall of previous problems and repeats algorithm of part 1b.	33 55	109 B: Well, cause the ones previous to it showed that the point number that I entered in er the –
	111: rewording of subject's idea.	1	111 I: Decimal number.

No.4. 10000000. x 1.0000003 = 10000003. ** 10000003. = 10000006. ** 10000006. = 10000009. ** 10000009. = 10000012. **	112-5: explains reasoning used. No.5: note that the order of the operation in No.4 is reversed. He is testing commutativity.	65 47	112	B: The decimal number - whatever millionths - was the number that ended up with the ones position in the sequence of numbers. So I figured that if I entered in 1 and 3 millionths, it would come out being 10 million 3.
	116-9: related calculator question.	5	116	I: Right. OK ... do you have any idea why the calculator - even though you're multiplying ... the way you are - why the calculator is ending up adding by 3 each time? Do you have any idea why it's doing that? This is still in part c.
No.5. 1.0000003 x 10000000. = 10000003. ** 10000003. = 1.0000003 14 ** 1.0000003 14 = 1.0000003 21 ** 1.0000003 21 = 1.0000003 28 ** 1.0000003 28 = 1.0000003 35 **	120-1: not answering the question. Exploratory manipulation.	22	120	B: I think it has to do with the sequence that you've entered - like if I entered one ... is it alright if I did an example?
			122	I: Alright, do an example. We'll call this No.5.
				CALCULATOR OPERATING (No.5).
			123	B: I point (hitting the calculator as he speaks) six 3, times ... OK, if I did it that way ...
	125-6: prediction demanded.	5	125	I: Yes, what's gonna happen (asked before the calculator actually prints out a result)?
	127-8: expresses uncertainty.	92	127	B: I think it's gonna increase by ... ah ... (looking at the printout).
	129-30: encouragement.	3	129	I: Go ahead, cause that's interesting (encouraging the subject to verbalize his consternation).
			131	B: I thought it would increase by -
	132: directing the subject.	12	132	I: Hit the equal sign a few more times - watch what happens.
				CALCULATOR OPERATING (No.5).
	133-4: related calculator question.	5		Bruce, do you have any idea why it's doing that? ... this is still No.5.

No.5.	1.0000003 x 10000000. = 10000003. ** 10000003. = 1.0000003 14 ** 1.0000003 14 = 1.0000003 21 ** 1.0000003 21 = 1.0000003 28 ** 1.0000003 28 = 1.0000003 35 **	135: no response indicating he doesn't understand.	63 83	135	B: ... no I don't.
		136-7: repeating question in lines 133-4.	4	136	I: OK. Which one (i.e., of the two factors) is the calculator multiplying by, repeatedly?
		138: error in understanding how sequence is generated. Already classified.		138	B: 1 point - or 1 and ...
				139	I: No.
		139: hint given by contradiction.	6		
				140	B: No? So it's multiplying by 10 million.
		141-4: explanation of the related question posed.	1	141	I: That's right. So, what it's really showing you is you cannot switch the order of these (i.e., the two factors: 1.0000003 and 10000000) around ... and get like repeated multiplication. It's not commutative. Do you understand what I mean?
				145	B: Yeah.
				148	I: Because like in this case the equal sign is geared to multiplying by 10 million. It's whichever factor is second that it's geared to.
				149	B: Uh hum. I see.
				150	I: Do you follow?
				151	B: So in these instances it has ... a raised number of whichever term (i.e., this subject understands the exponential meaning given to the spaced notation in the printed number 1.0000003-14) ... so it's ... that's meaning ...
		155-6: related calculator question and hint.	5 12	155	I: What's happened here: you've actually got a pattern ... if you take a look at it ...
				157	B: It means it's increased the number of zeros too ...

No.5. 1.0000003 x 10000000. = 10000003. ** 10000003. = 1.0000003 14 ** 1.0000003 14 = 1.0000003 21 ** 1.0000003 21 = 1.0000003 28 ** 1.0000003 28 = 1.0000003 35 **	158-9: repetition of the question in lines 133-6.	4	158	I: Do you have any idea what it means when you've got a space there like that (i.e., referring to the space in 1.0000003-14)?
	160: answers related question.	30	160	B: It means an exponent or whatever –
			161	I: Power of 10.
			162	B: Power of 10, yeah.
	163-4: clear direction given.	12	163	I: So really what you're increasing by is – can you see a pattern – you've got 14, 21, 28, 35 ...
	165: responds to direction and sees a pattern.	55	165	B: You're increasing by seven zeros or powers each time.
			166	I: Because you're multiplying by 10 million. You are multiplying by 10 million each time. You can see the pattern in here (i.e., referring to the printout).
			169	B: So it all is dependent on the last number that you enter into –
			170	I: Multiply by. OK. Here comes a really good question. All of these lead up to this question (advancing and removing the printout). And I'll leave these (i.e., the printouts) out so you can look at them if you think they'll help you at all ... read that question – question d.
			175	B: Out loud?
			176	I: No, to yourself. PAUSE (20s). Just hold on a sec – I want you to tell me about it. So I'll just press the print button – that's what you were going to do – and we'll hold it that way (i.e., stop the calculator from advancing).

No.6.	9999999. x -1.999991 = -19999908. ** -19999908. = 39999636. **	181-4: gives an answer but his algorithm is incorrect.	50 92	181	B: I think I'd multiply 9 million 9 hundred and 99 times ... uh ... I think it might have to be a negative number, I'm not sure. To get it to go down ... it might have to be ... say negative ... point - or negative 1 and that'd be 1 ...
		185-7: subject appears to be rambling and left to collect ideas.		185	I: OK, let me let you try that by yourself for awhile. I'll stay here (the investigator remained with the subject while he did the problem). You try that, see if you can get that problem, OK?
No.7.	10000000. x -1.0000001 = -10000001. ** -10000001. = 10000002. **	No.6: performs exploration. Uses negative factor to get decreasing pattern.	20 49 92		CALCULATOR OPERATING (No.6).
		calculator operation indicating lack of prerequisite knowledge.		188	B: Do I have to hit the negative before or after I hit the number?
				189	I: Ah ... like ... you want to change its sign?
		188: indicates lack of calculator knowledge by asking for help.	24 85	190	B: Uh huh.
		191: explanation.	1	191	I: Hit the number and then hit the plus or minus button.
					CALCULATOR OPERATING (No.6).
		192: knows his answer is wrong.	55 54	192	B: There goes that idea. So that doesn't work.
				193	I: So let's clear that (hitting the clear key on the calculator). And you can look back at that (advancing and removing the printout) or ...
		196-7: questions conditions.	20	196	B: Or maybe it's 1 million ... 10 million times ... something - this has to be times, heh? Or multiplication?
		198: after Bruce, this question was altered so that the operation was not specified. Here, however, multiplication was stated.		198	I: Not necessarily (actually the question does specify multiply). ... it does say multiply, though, doesn't it.
					PAUSE (10s).
					CALCULATOR OPERATING (No.7).

No.7. 10000000. x -1.0000001 = -10000001. ** -10000001. = 10000002. **	200-5;No.7: strategy: changes factors, making one larger and second smaller, but still negative.	47 92	200	B: I think it might still have to be with this ... 10 million ...
				CALCULATOR OPERATING (No.7).
				And to get the numbers to go down.
				PAUSE (15s).
				How to get the numbers to go down ...
	203: changes the condition of the problem. Reasons intuitively.	82 41		So I think it has to be division ...
No.8. 10000000. / 1.0000001 = 9999999. ** 9999999. = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. ** 9999996. + 9999995. ** 9999995. = 9999994. **				CALCULATOR OPERATING (No.8).
				Divided by ...
				CALCULATOR OPERATING (No.8).
	205: knows he has the answer.	54		Voila.
				CALCULATOR OPERATING (No.8).
	206-8: questions reasoning.	9	206	I: So you got it. Let's just go over (removing the printout) ... with basically what you did. OK. What was your idea in here (i.e., in No.6)?
	209-11: explains reasoning.	65	209	B: OK, I was thinking that - take 9999999 and times it by - this is pretty abstract - but 1 and 99999 ... whatever, negative, to see if I could get something there but it didn't really work.
	212: reference to printout.	9	212	I: Why did you want to make it negative?
	213: explains reasoning. See comment for No.6.	31	213	B: Well, cause I thought it had to do with going down and getting numbers ...
			215	I: So you really wanted to subtract, heh?

```
No.8.  10000000. /
       1.0000001 =
        9999999. **
        9999999. =
        9999998. **
        9999998. =
        9999997. **
        9999997. =
        9999996. **
        9999996. +
        9999995. **
        9999995. =
        9999994. **
```

215: this is an error on the investigator's part. Bruce clearly wanted to change the sign of a factor. Directing.	12	216	B: Yeah.
		217	I: Cause the pattern you have here – what does the pattern actually do?
		219	B: It goes down instead of up –
220-1: explanation of a principle that he should know.	1 6	220	I: By 1. So you were hoping by negative – but Bruce look, any time you multiply a positive number by a negative number –
		222	B: I was ...
223-4: affective statements.	3	223	I: It's OK, anyway. It's your idea, so you try it. Don't worry about it. Then you pressed the button a couple of times and it didn't work out. So we'll try this here in No.7. In No.7 what was your idea – you're still with a negative number –
225-6: reference to printout.	10		
227-8; No.7: repeats previous algorithm.	48 92	227	B: Yeah, I still was with a negative number. I multiply 10 million times negative 1 and ten millionths.
		229	I: OK. Why did you try that? What was your idea –
		230	B: Cause I figured with the pattern – I was hoping the negative would get it to go like the other way ...
		232	I: Down.
233-4: this is the same idea as in No.6. Why did he change factors?		233	B: Too much on the computer brain but – I was hoping to get it go down instead of up so ... I tried the negative.
235: questioning reasoning.	9	235	I: OK. Why'd you start with 10 million?
No.8: related to No.7 where first factor is the same. Operation has been changed to create a decrease showing deductive reasoning and an analysis of results calculated.	47 40 55 33	236	B: Well, cause I figured it had some relevance because ... the number we were using was just one off ten million.
		238	I: In other words 9999999. And why did you pick 1.0000001?

```
No.8.  10000000. /
       1.0000001 =
        9999999. **
        9999999. =
        9999998. **
        9999998. =
        9999997. **
        9999997. =
        9999996. **
        9999996. +
        9999995. **
        9999995. =
        9999994. **
```

		239	B: Cause I figured that also had to have some relevance with it because ... it was what we used in the other sequences to go up: I ... so I figured if I used something similar I could get it to go down 1.
243: reference to printout.	10	243	I: OK, now in No.8 is where you got your brain wave -
		244	B: And divided it.
		245	I: OK, why'd you divide it?
246-8: identifies error and uses deductive reasoning.	54 40	246	B: Cause then it clicked - if a negative number didn't work then I'd have to divide to do the opposite of multiplication to get it to go down.
249-50: extending subject's words.	13	249	I: Because we had been multiplying up here (i.e., in the previous part). ... and it did what you wanted it to do.
		251	B: Uh huh.
252-6: problem generation demanded.	4	252	I: Ah ... so that's pretty good. So what I'd like you to do - it'll just take a minute or two - I'd like you to make up any sequence you want to. You pick two numbers - here - let's see what it says in here (investigator starts reading the question outloud to the subject). How would you get that?
257: questions conditions.	20	257	B: How would I do that?
		258	I: You can make up anything you want.
259-60: conditions still questioned and recalls related problem.	20 33	259	B: Like something similar to this (referring to part a of the question)?
		261	I: Anything you want.
		262	B: And I could have it increasing by anything I want?

No.9. 1000000. x
1.000009 =
1000009. **
1000009. =
1000018. **
1000018. =
1000027. **

			CALCULATOR OPERATING (No.9).
			And I can make this any number I wanted there (referring to the second factor he is using).
			CALCULATOR OPERATING (No.9).
			And then it would keep doing it (pressing equal key repeatedly).
266-7: post-predicting demanded.	9	266	I: OK. What did you want it to do each time (i.e., that the equal key was pressed)?
268;No.9: note the relation between this and No.2 and No.4 with the following change: both factors have been reduced by 1 decimal place. Changes existing algorithm successfully.	33 49	268	B: Wanted it to increase by 9.
		269	I: So when you wanted it to increase by 9 - what I'm interested in - let's just take a look at the pattern you've got (advancing and removing the printout). I don't just think you picked any numbers at all ... I think you were very careful - or you could have picked any numbers you wanted - but once you picked this, (pointing to the first factor) -
269-74: review of previous work.	11		
		275	B: 1 million -
276-77: questioning reasoning.	9	276	I: Once you picked this 1 million ... did you - does that tell you what the second number has to be?
278;No.9: illustrates deductive reasoning and an analysis of results to discover a pattern.	40 55	278	B: It has to be ... some number - millionths.
279-80: repetition of subject with some inference.	13	279	I: Millionths. In other words, as many zeros as there are in the first factor there has to be decimal places in the other one.
		281	B: Uh huh.
		282	I: And so you saw a relationship in that. That's excellent. And that's the whole thing.

		284	B: OK.... at first I was thinking of doing a number line instead of 9 like doing 12 or 14.
286: clarification.	9	286	I: Why did you decide not to do that?
4:00		287	B: Actually I decided after I must have checked out 5 zeros (i.e., 1.000009).

Lisa Problem 1.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:36					PAUSE (20s) (Question given to Lisa to read).
No.1.	10000000. x 1.0000001 = 10000001. ** 10000001. = 10000002. ** 10000002. = 10000003. ** 10000003. = 10000004. ** 10000004. = 10000005. ** 10000005. = 10000006. ** 10000006. = 10000007. **			1	I: Alright, so there's all those parts and I want you try part a.
					CALCULATOR OPERATING (No.1). (54s).
		02: questions subject's confidence in her answer.	7		You got it?
				3	L: I think so.
		04-05: rephrasing question part a.	4	4	I: Just look at the starred numbers (advancing and removing the printout) and describe to me any pattern that you see.
		06: pattern seen as additive.	55 50	5	L: Like each one of them ... goes up 1 more. Like 1, 2, 3 and 4.
		07-10: repeating subject.	4	7	I: Increases by 1, heh? Alright, Lisa, do you know what numbers you're really - what you're doing - like when you got this second starred number (i.e., 10000002), right - you keep on hitting the equals don't you?
				11	L: Uh huh.
		12-13: related calculator question to see if she understands how pattern is generated.	5	12	I: When you get this second starred number like, 10 thousand 2, what two numbers did you multiply to get that?
					PAUSE (3s)
				14	L: That one -
		15: clarification.	9	15	I: Which is that one?
		16: error: does not comprehend how the calculator generates the sequence. Answers related question.	50 83	16	L: 10 million and 1 point zero, zero ... 1.
			9	17	I: That's what you got to get the first starred number -
				18	L: Yeah.
		19: contradiction.	6	19	I: But to get the second starred one you didn't multiply those two.

No.1. 10000000. x 1.0000001 = 10000001. ** 10000001. = 10000002. ** 10000002. = 10000003. ** 10000003. = 10000004. ** 10000004. = 10000005. ** 10000005. = 10000006. ** 10000006. = 10000007. **	20: error: does not understand that though the sequence is additive, operation is constant multiplication.	85	20 L: You add ... looks like it's added that one. 21 I: It's added 1 like you said - 22 L: Oh, yeah.
	23-26: explaining how the sequence created and extending.	1 13	23 I: But what you're really doing is you're really multiplying the first number with the stars by 1.0000001. So each time you hit equals it really multiplies by the second number (i.e., the original second factor). Do you follow that? 27 L: Yeah. 28 I: So what you've done here is you've multiplied every successive starred number by 1.0000001. You see why it would - by multiplying by that - would make it 1 larger each time? PAUSE (5s).
	31-32: related question on operations within the sequence.	5	Do you see anything about those two numbers (i.e., 10000000 and 1.0000001)?
	33-34: answers related question showing misunderstanding.	50 83	33 L: Well ... ... they're kind of the same but ... looks like they just add both of them together and put 1 at the end.
	35-40;42-3: further explanation but within sequence.	1 13	35 I: They look very much the same except the second number, the second factor, is really 1 so when you multiply a number by 1 and something, the part that's multiplied by 1 doesn't change, does it? So what you're doing is you're multiplying it by itself (actually by 1) so it stays the same, and then you're multiplying by 1 ten millionth, of course, 1 ten millionth of ten million is 1. 41 L: Oh, OK. 42 I: Then when you go to multiply ten million and 1 it treats it as if was ten million and adds another 1.

		44	L: OK.
45-48: asks for prediction and repeating part b.	4	45	I: So let's look at part b now. And in here – suppose you wanted to get the pattern that I have – the starred numbers ... what two numbers do you think you would multiply (reading the question to Lisa)?
			PAUSE (7s).
49: predicts correct answer.	50	49	L: 10 million and 1.0000003?
		50	I: Try it.
			CALCULATOR OPERATING (No.2), (20s).
51: questions certainty.	7		You get it?
		52	L: Yeah.
53-54: questions reasoning.	9	53	I: You can tell me how you knew it was those two numbers – how you figured out it was those two numbers?
		55	L: Well ... they're just adding 3 to each number.
56-58: still questioning reasoning but some direction.	9 12	56	I: And ah ... OK, you know that you have to add 3 so did that tell you which numbers to pick or which one of the numbers you had to pick ... the fact that you had to add by 3?
59: explains reasoning.	65	59	L: Well, since the first part was 1 in each ...
		60	I: At the decimal –
		61	L: The decimal was 1 –
		62	I: At the end.

No.2.
10000000. x
1.0000003 =
10000003. **
10000003. =
10000006. **
10000006. =
10000009. **
10000009. =
10000012. **

	63-64: strategy: connects the increase in the sequence with the last digit of the second factor. Explains logic. Indicates a related problem.	55 40 33	63	L: And it increased by 1 each time you used 1 so I thought maybe if you put 3 it would increase by 3.
	65-66: related calculator question.	5	65	I: Ah ... do you think it would have made a difference if you had put this number (i.e., 1.0000003) first and the ten million second?
	67: error: does not understand that second factor will be a constant factor.	85	67	L: Shouldn't.
No.3. 1.0000003 x 10000000. = 10000003. ** 10000003. = 1.0000003-14 **	68: new question using the calculator.	5	68	I: OK. Try it out. Switch them around and see what would happen. CALCULATOR OPERATING (No.3), (20s). Does it make a difference?
	69: repeating question in 65-66.	4		
	70;No.3: analysis of printout and identifies error.	55 54	70	L: Yeah.
	71-76: repeats question in 65-66 now using the results produced by the calculator.	9 10	71	I: (Laughing) alright, let's give those numbers (advancing and removing the printout) so I can keep track of them. Can you tell me why in No.3, it doesn't work out, when you switch the factors around? Can you tell me why it doesn't work out, if you look at them (showing No.2 and No.3 for comparison purposes): Cause you don't get the same pattern, heh? PAUSE (14s).
	77-78: shows she understands the operation of the calculator in the sequence.	50	77	L: Cause you're multiplying by ... this one over again (indicating 10000000)?
	79-84: repeating subject's ideas and instructing.	1 13	79	I: Ten million each time, that's right. So it does matter which one you write first, heh? In a problem like this usually like you'd say three times two is six two times three is six, really doesn't matter. When you keep hitting the equal signs ... then the one that came second is the one you keep on multiplying by each time. Follow?
			85	L: Yeah.

	86-88: repeating part c.	4	86	I: So let's do part c. Now, I'd like you to get this pattern of numbers and you decide how you're going to do it: use two numbers any way you want ... ... to get that pattern.
				PAUSE (5s).
	89: questions conditions and gets a bright idea.	20 41	89	L: Can you divide?
			90	I: Yes.
				PAUSE(45s)
				CALCULATOR OPERATING (No.4).
No.4. 9999999. / 1. = 9999999. ** 9999999. = 9999999. **	91: expresses uncertainty.	95	91	L: Might not work.
	92: questions reasoning.	9	92	I: What was your idea?
	93: questions reasonableness of solution.	54	93	L: Divide by 1 but you just keep getting the same answer.
	94-5;No.4: error: should be clear that 1 is the identity element of division. Not analyzing results.	96	94	I: Cause 1 is the identity element of division. OK, but maybe that's not such a bad idea.
No.5. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **	No.5: maintains dividend and operation but changes divisor. Strategy: increases the size of the divisor.	48 92		CALCULATOR OPERATING (No.5), (50s).
				So what's happened? ... you're getting it aren't you? Except you didn't get the -
			98	L: Yeah.
			100	I: First starred number did you? Did you notice that?
			101	L: Yeah.
	102-3: questioning reasoning.	9	102	I: You've got it decreasing by 1. What was your idea? This is now No.5. What was your idea?

No.5. 9999999. / 1.0000001 = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. **	104: explains reasoning.	65	104	L: To find something that ... would get it to decrease by 1.
	105-6: questioning reasoning.	9	105	I: Cause that's the pattern, heh? So why did you think divide right away?
				PAUSE (5s).
	107: shows that she sees another method, thus protocol expands hidden element of printout.	65	107	L: I don't know ... easier to work with than subtract.
	108-9: too much direction.	12	108	I: You could have subtracted too, heh ... do you think of dividing as making things smaller?
			110	L: Yeah.
			111	I: OK, I was just wondering ... so what's happened is you've got the pattern except you've started one too small. How could you fix that up?
				PAUSE (17s).
	114: questions conditions of problem.	20	114	L: You mean get it so you can find the first number?
			115	I: Yeah.
	116;119: reasons deductively.	40	116	L: You could try dividing 10 million by ...
	117-8: related question.	5	117	I: The number you've got there - the second. Why would you pick 10 million?
			119	L: Cause it's 1 more than 9999999.
			120	I: Try that. I like that. I just wanted to see ... if you understood it.
				CALCULATOR OPERATING (No.6), (14s).
	122: questions certainty.	7		You get it?

No.6. 10000000. /	123: identifies answer.	54	123	L: Yeah.
1.0000001 = 9999999. ** 9999999. = 9999998. ** 9999998. = 9999997. ** 9999997. = 9999996. ** 9999996. = 9999995. **	No.6: strategy: last sequence was sufficient in that it decreased by 1 but was deficient as first term (i.e., 9999999) was not generated. Increase dividend by I.	49 55	124	I: Let's just hit it a few more times to see if you got it (the investigator hits and advances the calculator). We'll call this part d. I want you to make up any pattern you'd like to - from all the stuff we've just done. You make up your own pattern. CALCULATOR OPERATING (No.7), (55s).
	No.7: note the similarity between this sequence and part a. The changes: both factors have been reduced a decimal place, indicating this may have been seen as a needed condition in the original pattern.	33 49	128	L: (Laughing) just the same.
			129	I: Let's advance it. You tell me what kind of pattern you wanted to make - did it turn out to be what you wanted it to?
No.7. 1000000. x	128: compares solution with general known result.	55	131	L: Uh huh.
1.000001 = 1000001. **	132: questioning reasoning.	9	132	I: What did you want it to do?
1000001. = 1000002. **	133: repeats algorithm.	48	133	L: Increase by 1.
1000002. = 1000003. ** 1000003. = 1000004. **	134-5: questioning reasoning.	9	134	I: And it did that. Ah ... like how did you decide which number to pick? PAUSE (3s).
	136-7: support for strategy claim in No.7 above.		136	L: I just took the numbers from part a (i.e., No.1) and took off a zero.
	138-42: some direction as reasoning of subject investigated.	9 12	138	I: So just wanted to see if they were slightly smaller - if it would increase. Ah ... were you careful in the numbers like - when you picked the first number, did you - when you went to pick the second number then, did you look at anything then in the first number to decide what you would pick in the second number?
			143	L: How many zeros there were there.

No.7. 1000000. x 1.000001 = 1000001. ** 1000001. = 1000002. ** 1000002. = 1000003. ** 1000003. = 1000004. **	144: clarification.	9	144	I: So what did you try to do?
	145: explains processes used. Validates assumptions stated about No.7 earlier.	65	145	L: Keep the same ... amount of numbers were the same. Like if you would put one more zero in the second number it wouldn't have worked.
	148-9;151;153: extending subject's reasoning.	4 13	148	I: So you want, like if there are six zeros in the first, you want five in the second?
			150	L: Yeah.
			151	I: So what you want is as many decimal places –
			152	L: Yeah.
3:50			153	I: Past the decimal. As there are zeros in the first.

Earl Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:22				1	I: What I'm going to do ... I want you to read the problem over and if you think you understand it I'm going to leave you to do it by yourself and I'll be right up there (at the front of the room). If you want to ask me any questions or if you have the answer.
				5	E: OK (reading the problem). I understand this.
				6	I: You understand it? OK. So I'm going to leave you awhile with it. Alright. And when you think you have the answer or if you really can't figure it out, then you'll let me know.
					CALCULATOR OPERATED (Nos. 1 to 26), (7 minutes).
		09: questioning certainty.	7	9	I: So you think you got it solved Earl?
		10-11;13;15-16: questions existence of solution.	60	10	E: Yeah ah ... am I the first one to work on this or has someone else done this?
				12	I: Someone else has worked on it. Why?
		13: has the solution.	50	13	E: Because well, I worked it out that the 6 is not working.
				14	I: OK.
		15-16: error in understanding the question. Analyzes results.	83 55	15	E: And if you do that with problem 2 you get an answer 2538, and what you have printed down is 2358.
				17	I: Yes.
				18	E: That's more than just a coincidence, that's more than a type o (i.e., typing error).
		20: repetition.	4	20	I: Oh! You thought it was a typing error?
				21	E: Yeah, because well ...
		22: contradiction.	6	22	I: Oh, no - no - no.

			24	I: OK. You've got a lot of printout here. OK. So what I want to do with you is ... you're going to ask me that question again, OK.
			26	E: Number 2?
No.1. 41.7 + 5079. = 5120.7 **	27-31: questioning related to printout.	10	27	I: Yeah. What you were telling me there about the typing error. We'll go over that here (i.e., in No.2 of the printout). OK? Just ... I'll number these (the printouts) and you'll just tell me what you did, what your ideas were in the different steps, heh? So what were you doing in this first part?
No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	32-33; Nos. 1 -2: exploratory manipulation. Develops checking algorithm and analyzes results. 34: questioning certainty.	22 49 55	32	E: In this part I was just checking if problem 1 was right because it seemed to be right.
			34	I: Was it right?
			35	E: Yeah.
	36: reference to printout.	10	36	I: So you just did exactly what problem 1 said. What did you do in No.2?
	38: evidence for stated analysis of calculations.		38	E: Same thing except the answer wasn't correct.
	39: clarification.	9	39	I: Which answer wasn't correct?
			40	E: Ah ..., in that particular case, No.2 there, it's that one (pointing).
	42-43: clarification.	9	42	I: 2598 is not right (i.e., the answer found on the printing calculator)?
	44-46: analyzes results; deductive reasoning demonstrated.	55 40	44	E: Yeah, because it didn't match up with this (i.e., 2358) so I knew that ... OK ... well it's the first one is correct so the key that's wrong is not one that's in the first one.

No.2.	131. + 267. + 538. + 934. + 728. = 2598. **	47-49: explaining which sum is incorrect. Extending from what the subject said.	1 13	47	I: OK. Just hold on. I see what you're getting at. But let's get back to No.2. What I want you to see is this 2598 that you got on the printing calculator is OK.
				50	E: Yeah, I know.
				51	I: The error is in the problem.
				52	E: Involved in that (i.e., the given problem). That's what I meant.
		53-54: demanding clarification.	9	53	I: Now, what you're telling me ... say again what you saw when you looked at No.1.
No.1.	41.7 + 5079. = 5120,7 **	55-58: deductive reasoning demonstrated.	40	55	E: With No.1 I saw that the answer with both (i.e., the given problem and produced printout) matched right. So when I figured it all ... if that matches then the key that is not working is not one that is No.1.
		59: clarification.	9	59	I: OK, when you say matched you mean the answer you got in No.1.
				60	E: Yeah.
				61	I: And the answer that's there are the same.
				62	E: Yeah, they are the same.
		63: questioning reasoning used in No.1.	9	63	I: And so ... what?
		64-66: deductive reasoning cont'd.		64	E: So then the ... keys used in problem 1 – they were all working fine so ... the key that is not working is not one in No.1. That took me about 15 minutes to figure out there (laughing).
No.3.	82.6 x 85.91 = 7096.166 **	67: reference to printout.	10	67	I: OK. So let's go back to the printout. In No.3, what are you doing?
		68: repeating checking algorithm.	48	68	E: Checking it again.
		69: clarification.	9	69	I: Checking which one?

No.3.	82.6 x			70	E: Problem 3 again.
	85.91 =				
	7096.166 **	71: questioning reasoning.	9	71	I: And what happened?
		72-73: good illustration of deductive reasoning.	40	72	E: The answer on the sheet and the answer on the printout didn't match up so ... I knew that one of those keys had to be wrong too.
		74: repetition of subject.	4	74	I: One of them had to be wrong.
				75	E: Yeah.
No.4.	307. x	76: reference to printout.	10	76	I: And then in No.4 here?
	64. =				
	19648. **	77;No.4: repeats checking algorithm.	48	77	E: Same thing.
				78	I: Checking?
				79	E: Problem 4.
		80: review of previous work.	11	80	I: So the first four are just checks of the problems.
				81	E: Right.
No.5.	2598. -	82: reference to printout.	10	82	I: Now, what are you trying in No.5?
	2358. =				
	240. **	83-85;87: error: this algorithm will not yield a solution.	47 92	83	E: Well, OK .... ah ... I figured well No.1 is OK. So ah ... in No.5 I took the answer ... I had done on the printing calculator for adding up the sums -
				86	I: For problem 2, yes.
				87	E: And try to find the difference from the answer they gave here.
				88	I: OK.
				89	E: And got 240.

No.5. 2598. − 2358. = 240. **	90: question that directly probes the subject's ~~reasoning.~~	5	90	I: Was there any reason why you were going after that difference?
	91-92;No.5: strategy: find the place value of the error by finding the difference.	9 32 47	91	E: I thought ... you know ... it might help me to figure roughly which place ... the value was wrong in.
	93: repetition of subject.	4	93	I: Ah, so you're doing subtracting to ... ...
			94	E: Right.
	95: repetition of question in line 90.	4	95	I: Did you get anything - when you saw this difference of 240?
	96-98;100: good illustration of deductive reasoning.	40	96	E: Well ... well, OK, since it's 240, no number is in the ones place so that means all the numbers in the ones row must be correct then.
			99	I: Oh!
			100	E: So it must be something in the tens and hundreds.
	101-2: repetition.	4	101	I: Oh, I see what you're saying. In other words they both ... ah ... 2598 and 2358 both have an 8.
			103	E: Right.
	104;106-7: extending from the ideas of the subject.	13	104	I: So the 8 comes from work done in the units place.
			105	E: Right.
			106	I: So you're saying because those are the same, then your error must be where?
			108	E: In the tens and hundreds place.
	109: reference to printout.	10	109	I: OK. So what did you do in No.6 now?

	Comment	Code	Line	Transcript
No.6. 131. + 167. + 538. + 934. + 728. = 2498. **	110-5: explains basic ideas demonstrating deductive reasoning.	65 40	115	E: What did I do ... OK ... I ah ... well I was working with the idea that since ... ahm ... the values that would work to get the difference there ... would be in the tens and hundreds spot, I was trying to spot ahm ... some numbers which were the same ... ahm that were in both the hundreds and the tens spot and in this case (i.e., No.6) that was 2, I believe.
			116	I: OK, You're looking ... let me see if I understand you, OK.
	No.6: note the 267 has been made into 167. Indicates an exploration and the developing of an method that is not successful yet.	22 47	117	E: Wait a sec! I never added that right.
		92	118	I: What did you do wrong in it?
	117;119: identifies error.	54	119	E: That ... like I subtracted ... like that is supposed to be 267.
	Calculator misentry.	95	120	I: Yes.
	119;121;123;125-6: explains reasoning	65	121	E: And since 2 is in the hundreds there and the tens there ...
	122: clarification.	9	122	I: Ah which one? So 2 is in the hundreds in 267.
			123	E: And the tens place in 728.
			124	I: Yes.
	125-6: error explanation cont'd.		125	E: So I said, well, substitute one but I forgot to put it in one the printout for 728.
			127	I: OK.
	128: good indication that results are analyzed.	55	128	E: That didn't work anyways.
	129: clarification.	9	129	I: But you only realized that error now, though.
	130-1: indicates part of the strategy of No.6.	65	130	E: Yeah. But that didn't work anyways back then because it was still too large.

No.6.	131. +	No.6: strategy: change digits	47	132 I: OK. So what does that tell you?
	167. +	that are in the hundreds and tens.	92	
	538. +	2 changed to 1 because 2598 is		133 E: The 2's are not the one that's wrong.
	934. +	larger than the 2358 answer given.		
	728. =	Successive approximations shown.		134 I: Does the fact that the value's too large tell you anything?
	2498. **			
		132: probing reasoning.	9	135 E: Yeah ... well ... I don't know.
		133: good illustration of deductive reasoning.	40	136 I: OK.
		134: clarifying ideas but with definite direction.	12	137 E: Well you know ... I was just starting to guess around ... trying ... to ...
		137-8: randomness indicated.	94	
No.7.	131. +	139: reference to printout.	10	139 I: OK. Go on. This was No.6. Now in No.7, what have you done?
	265. +			140 E: OK. Ah ... No.7 ...
	538. +			
	934. +			
	528. =	141: extending on the basis of calculations.	13	141 I: Looks a lot like No.6.
	2396. **	142-3: No.7: strategy: substitute 5 for 7 in the hundreds and units place. Wants a digit that is in two places to make 2598 into 2358. Using the same algorithm as No.6 but with different digits.	48 55 92	142 E: It was except I forgot all about the ones place. And started working with the idea ... well maybe the value of 7 is wrong.
				144 I: So you're not working ...
		145: good indication of analysis.	55	145 E: Cause there are two 7's.
		146: clarification.	9	146 I: What do you mean by two 7's?
		147: answers question.	50	147 E: Well OK. Here is a 7 in 728 and in 267, right.
		148-9: use of counter-example. Omission indicates lack of understanding.	6 92	148 I: Yeah, but if you [illegible] back to No.1 (i.e., the check of problem 1), isn't there a 7 in there too?
				150 E: Yeah, I just noticed that about 15 minutes after.
		151: clarification.	9	151 I: OK. So you hadn't realized that yet.

No.7. 131. +
26[illegible]. +
538. +
934. +
528. =
2396. ##

		152	E: Right.
		153	I: OK. Go on.
154-5: explains reasoning.	65	154	E: And so I tried substituting the 5 in instead of the 7, but that didn't work out.
156: related question to probe reasoning.	5	156	I: Any reason why you picked the 5 instead of the 7?
157: more indication of randomness.	94	157	E: Well, I just picked a rough number to substitute for the 7.
158: clear example of direction.	12	158	I: Well, did you think the 5 key was working? You didn't know, heh?
		159	E: Well, I thought the 5 key was probably working, yes.
160: probing reasoning.	9	160	I: Is there any reason for that?
		161	E: No.
162-3: reference to calculation.	10	162	I: OK. So you just tried something in. OK. Alright. And what happened then? You got 2396.
164-5: indicates successive approximations strategy.	55	164	E: Which was closer to 2358 so I knew I might be on the right track there.
166: clarification.	9	166	I: The right track in what sense?
167-8: continues reasoning of lines 164-5.	65	167	E: Well that ... maybe it was the 7's that's wrong, maybe it's just that it needs a lower value.
169: clarification.	9	169	I: Why do you think a lower value?
		170	E: OK. The number that came out in that (i.e., No.7) is 2396.
		171	I: Yes.

No.8. 131. +
267. +
538. +
934. +
728. =
2598. **
2598. SM

No.2. 131. +
267. +
538. +
934. +
728. =
2598. **

No.9. 131. +
260. +
538. +
934. +
28. =
1891. **

170;172-3: Analyzes results and indicates successive approximation as method that is repeated.	55 48	
174: reference to printout.	10	
175-7: exploratory manipulation.	22	
178: review of what subject did.	11	
182-4: counterexample of simpler method. Subject repeats previous work.	6 86	
186-7: reference to printout.	10	
188-9; No.9: repeats algorithm and recalls No.7. Still using successive approximations. Very good deductive reasoning. Error in that 260 should be 26 if key is not working. Making 728 into 28 shows she partly understood this.	48 33 92 40	
192: indicates analysis of results.	32 55	
193-4: reference to calculations.	10	

172 E: And in problem 2 it's 2358. And those are ... how far apart? ... 38 apart. So I thought I was getting closer.

174 I: So what'd you do in No.8 now?

175 E: No.8 ... what did I do in No.8? I don't know what I did in No.8. ... I got off the 7's for a second there. Oh yeah. I was just going to store it in memory.

178 I: So you redid the whole problem (i.e., No.2).

179 E: Right.

180 I: And you stored what the answer should be:

181 E: In memory.

182 I: So in No.8 we're just redoing problem 2, so you can store it in memory. Why didn't you go back and look at the answer you got in No.2 and store that?

185 E: I never thought of that.

186 I: OK. So I thought maybe you were checking again. That's alright. What did you do in No.9 now?

188 E: In No.9 I ... well I went back to the idea that maybe the 7 is wrong. And I substituted in ... well 0.

190 I: OK.

192 E: Just to check: But then the value went far below so I knew ...

193 I: OK. Let's get straight where you put the 7 in. You put a 0 in for the 7 in 260. And anywhere else?

			195 E: And 728.
No.9. 131. + 260. + 538. + 934. + 28. = 1891. **	196-8: extending from subject's statements.	13	196 I: You really figured ... ah so in other words there's a 0 there but it wouldn't show on the calculator. On the calculator 028 and 28 are the same.
	199: if he did know, 260 would be 26.		199 E: Yeah, I know.
	200: clarifying printout to see if he understands how malfunction originates.	9	200 I: Can you remember, did you punch in 028?
			201 E: Ah ... no, I knew it wouldn't printout 028 so I just typed in 28.
	202: reference to calculation.	10	202 I: OK. And you got 1891.
	203-4: further continuation of strategy of successive approximations.	55	203 E: Which was way too low so I knew that if the 7's were wrong it had to be a higher value than that (i.e., 0).
No.10. 2598. RM	205: reference to printout.	10	205 I: Just below that ... in step No.10, what did you do?
	206: recalls previous problem.	33	206 E: I just recalled the sum from memory for a second.
	207-8: probing reasoning with some direction.	12	207 I: And when you saw ... going back to No.9 ... when you saw 1891 was way lower than what you wanted, what was your conclusion again?
	209-11: good illustration of deductive reasoning.	40	209 E: Well OK, if the 7's were wrong then the value couldn't be 0. It would have to be higher than that. And since I had 5 in before I knew that it had to be higher than 0 but lower than 5.
No.11. 1. + 2. + 5. + 9. + 7. = 24. **	212: reference to printout.	10	212 I: OK. I got you. Now what are you doing in No.11 here?
	213-4: exploratory check.	22	213 E: Well, I added up the hundreds place to see what I would get in the thing.
	215: clarification.	9	215 I: OK, which hundreds place? In problem 2 still?
			216 E: Yeah.

No.11. 1. + 2. + 5. + 9. + 7. = 24. **	217: repetition of subject.	4	217 I: So you're adding like the 1 in 131, the 2 in 267.
			218 E: Etc.
	219: reference to printout.	10	219 I: And you got an answer of 24. OK.
	220-1: modifies the algorithm in Nos. 7 and 9 and analyzes the result.	49 55 92	220 E: And I was thinking ... ah ... well you know ... lower the 7 by one after that and you'd have 23, be half way there.
	223: extending from subject's statements and calculations.	13	222 I: Ah, in other words you'd get the 23 that was in 2358.
			223 E: Right.
No.12. 131. + 266. + 538. + 934. + 628. = 2497. **	224: reference to printout.	10	224 I: Ah, I got you. So in No.12 what are you doing?
	225-6: identifies error by analyzing calculation.	54 55	225 E: I substituted 6 in for 7 but ah ... I never noticed that I'd get carry from the tens spot.
	227-8: reference to printout.	10	227 I: Ah so, it made it back up to 24, heh? So when you saw that, so in in No.13, ah ...
No.13. 131. + 265. + 538. + 934. + 528. = 2396. **	229-30: error: repetition.	86	229 E: I lowered it (i.e., the 7) to 5 again, I forgot ... (i.e., this is identical to No.7).
	No.12: strategy: change 7 to 6 so that 24 hundreds becomes the 23 wanted. This is an error in that only part of the problem is dealt with.	48 92	231 I: So you're just replacing the 5 wherever the 7 is, so there's only two numbers where you're replacing the 5 with the 7, heh?
			233 E: Ah in -
	231-2: review of calculations.	11	
	234: probing reasoning.	9	234 I: And you got 2396. So did you conclude anything from that?
	235: identifies error.	54	235 E: Ah ... I just realized that I already had done 5 before.
			236 I: Where?
			237 E: Up on No.7.

No.13. 131. + 265. + 538. + 934. + 528. = 2396. **			239	E: Just now.
			240	I: So you didn't realize it then?
			241	E: Right.
			242	I: That's alright.
No.7. 131. + 265. + 538. + 934. + 528. = 2396. **			243	E: Ah ...
			244	I: So No.13 is really a duplication of No.7.
	245: changes approach when he switches problem though not necessarily method.	53 33	245	E: In No.14 I decided well; let's switch over to problem 3.
			246	I: OK.
No.14. 7096.166 SM 7096.166 RM	247: did this in No.8 too.	48	247	E: And I just stored the answer I had gotten before on the printout.
	No.14: does not repeat his error of No.8. Removes 2598 from the memory. Checks by recalling memory.	48	248	I: In No.3.
			249	E: Stored in memory. And ... that's a different step there.
No.15. 7096.166 RM 7096.166 - 7044.62 = 51.546 **			250	I: So you're storring and recalling No.14. And now in No.15, what are you doing?
	No.15: returns to the difference strategy so that he can use successive approximations with a different problem.	48 92	252	E: In No.15 I recalled it again and subtracted what the answer should have been. And ah ... got what sum was left.
	254: probing reasoning.	9	254	I: OK. Why are you doing that? Is there any reason?
	252-3;255-6: explains calculator operations.	65	255	E: Well, to get a rough approximation of what ... ah ... the difference is that I needed.
			257	I: In other words how much more you had to remove or ...
			258	E: How much more I had to remove.

No.15. 7096.166 RM 7096.166 - 7044.62 = 51.546 **	259-60: reference to printout.	10	259	I: So 51.546. Looking at these numbers, did you get anything from that?
	261;263-4: notes decimal places in result.	55	261	E: I saw well, three digit in there. So ...
	262: clarification.	9	262	I: Which three?
			263	E: In the answer. And ... on ... in the answer that I got when I subtracted it and from the one before-hand.
	265-6: clarification.	9	265	I: Just a minute. When you say three digits this is ... you mean these last three?
	267;270: is looking for a pattern in comparing 7044.62 and 7096.166 and the fact that they have different numbers of decimal places. A new algorithm developing	47	267	E: Yeah. The last three ... into decimal places.
			268	I: That's what you should have gotten, heh?
			269	E: Right.
			270	I: But you only ... the answer there.
			271	E: But the answer has two.
	272: probing reasoning.	9	272	I: So what did you conclude from that?
			273	E: Ah ... what I concluded from that was ... OK maybe it's the 1 in 85.91.
No.16. 82.6 x 85.9 = 7095.34 **			275	I: Sorry, say it again.
			276	E: Well I concluded maybe it's the 1 in 85.91 which I did in No.16.
			277	I: Say that again Earl.
			278	E: OK. Since there were three decimal places, right, ... I thought well ... OK ... two in here, one in there.

No.16.	82.6 x 85.9 = 7095.34 **	278-9;281-3;No.16: explains strategy behind No.16. Deductive reasoning illustrated. I appears to be selected at random.	20 94	281	E: OK. There's two in 85.91 and one in 82.6. So if I got rid of the 1 that would leave me with two decimal places and maybe that was the problem.
		284: repeating subject's ideas.	4	284	I: Oh. I see. So you want to get rid of the 1 in 85.91.
				285	E: Which I did in No.16.
		286-8: extending from subject's calculations.	13	286	I: Now ... in the next step, that's what you're doing. You're trying to make an answer that's smaller by getting rid of one decimal place.
		289: indication of successive approximations.	55	289	E: And still wasn't small enough.
		290: related question to probe reasoning.	5	290	I: So in other words you got 7095.34 but you wanted what?
				291	E: I ... ah ... 7044.62. so I was off there.
		292: probing reasoning.	5	292	I: So did you conclude anything from this?
		293: excellent demonstration of deductive reasoning.	40	293	E: Well ... you can't change the value of the 1. So that means –
				294	I: Why can't you change? Oh, in other words you can't make it small – you've already made it 0.
				296	E: You made it 0.
		297: probing reasoning.	9	297	I: So then what are you going to do?
No.17.	82.6 x 85.91 = 7079.646 **	298-9;No.17: uses same algorithm he has all along and reduces 0.9 to 0.7 to get smaller answer.	48 92	298	E: I went to No.17. In No.17 I tried changing the value of the 9. In 85.91.
		300: extending from calculations	13	300	I: You made it 85.71.
		301: indicates analysis of work.	55	301	E: Tried that and I came out with three decimal places.
		302: definite direction.	12	302	I: So you were happier? No you didn't want –

No.17. 82.6 x 85.91 = 7079.646 **			303	E: No I didn't want that.
	304-5: using printout to probe reasoning.	10	304	I: And also you're still larger, heh? So what did you conclude when you saw this 7079.646?
	306-7: illustrated deductive reasoning. Gets the answer. Has worked with bottom two decimal places so now works with top decimal place. Modifies algorithm.	40 49 54	306	E: Well I thought then ...OK if you want only two decimal places for the answer that must mean that the 6 is wrong.
	308: directing subject: he did not necessarily mean this.	12	308	I: In other words get rid of a decimal place now in the top one.
	309-10: explanation of reasoning.	65	309	E: In the top one. Then you'd have two decimal places in the lower one.
	311: reference to printout.	10	311	I: So you took away the 6 in No.18.
No.18 82. x 85.91 = 7044.62 **			312	E: In No.18 ah ... the answer turned out to be correct.
	No.18: uses his modified algorithm to find the solution.	55 64	313	I: Ah ... so you hit one where you got the answer the same as the person who got it wrong.
	313-4: review.	11	315	E: Right.
	316: reasoning probed.	9	316	I: And so what did you conclude then?
	317-8;320-1: identifies error.	54	317	E: OK. I misinterpreted the instruction a bit. I thought it was a different key for each one.
	319: repeating subject's ideas.	4	319	I: Oh I understand ... each problem had ~~a~~ different key wrong.
			320	E: Later on ... down a bit, when I was doing problem 4 ... I finally thought heh maybe it's the same key for all of them.
	322-3: clarification.	9	322	I: So you feel then in No.18 you found the key that was wrong just for which problem?
	324: answers question.	50	324	E: For problem 3.

No.19. 307. x 64. = 19648. ** 19648. SM	325: reference to printout.	10	325	I: So then you're going in No.19 and what are you doing there?
			326	E: Ah ...
			327	I: You're working with ...
107. x 64. = 6848. **	No.19: redid calculation No.4, makes a change in 307 to 107 showing he did not understand original question. Uses the strategy of reducing again.	86 83 48 92	328	E: I worked it out again so I could store it in memory.
			329	I: OK. So really it's a duplication of what's in No.4, heh?
No.4. 307. x 64. = 19648. **			330	E: Yeah.
	328: retraces steps and makes routine check: Former procedure (No.14).	33	331	I: OK. And so you stored that in memory. Now in the second part (i.e., of No.19) what are you doing?
	331-2: reference to printout.	10		
	333-4: repeats algorithm and uses successive approximations.	48 55	333	E: In the second part of that I ... ah ... I thought well OK maybe the 3 is too large in that.
			335	I: Yeah.
			336	E: Because it's ... because if you got ... the answer that comes out in problem 4 ... you get 19648 and the answer you want is 1228.
			338	I: So that's way bigger.
	339: deductive reasoning cont'd.	40	339	E: So that's what I figured ... so I thought get rid of a hundred.
	340: repetition of subject's idea.	4	340	I: Ah. You're working in the hundreds place?
			341	E: Right.
	342: reference to printout but with direction.	12	342	I: So you change 307.
			343	E: To 107.
			344	I: And you're multiplying.

No.19.	307. x 64. = 19648. ** 19648. SM	345: still concerned with comparing sizes. 346: reference to printout.	55 10	345 346	E: By 64. And I got 6848. Which is still much too large. I: Now that's No.19. Now in No.20?
	107. x 64. = 6848. **	347;No.20: inductive reasoning illustrated. Uses algorithm on second factor, on same key as previously found (No.18).	41 54	347 349	E: In No.20 I got the brainstorm maybe it's the same key for all of them. I: For all the problems.
No.20.	307. x 4. = 1228. **	350: says he got it. 351: questioning reasoning to probe misunderstanding of malfunction (i.e., 0 or nothing at all replacing the 6.	64 9	350 351 352	E: So I took away the 6 ... and I got the correct answer. I: OK. So you changed the 6 key. You left the 6 off? E: I changed it to a 0 because in problem 3 it would work out if the 6 wasn't there.
No.2.	131. + 267. + 538. + 934. + 728. = 2598. **	352-3: error in understanding that would have blocked solution if he tried on problem 2. 354-5: reference to printout.	10	354 356	I: So you know you got the answer. Now, you were coming back to problem 2 in No.21. And what did you do there? E: I found the only 6 in that and traded that for a 0, In there, and just added up what it would be.
No.8.	131. + 267. + 538. + 934. + 728. = 2598. ** 2598. SM	358: questions certainty. 360: reference to printout.	7 10	358 359 360 361	I: Did you get it? E: No. I: You got 2538 still. E: Yes.
No.21.	131. + 207. + 538. + 934. + 728. = 2538. **	362-3: probing reasoning. 364-6: questions existence of solution by looking for an error in the problem. Analyzes results and misinterprets problem.	9 60 55 83	362 364	I: Oh I see. You had 2598 (i.e., No.2 and No.8). So what did you think when that happened? E: OK. Cause they have the same digits ... just because ... this goes 2538 and the one in the problem is 2358. I thought maybe it was a typographical error.

No.21	131. + 207. + 538. + 934. + 728. = 2538. **	367: clarification.	9	367	I: Which was the typographical error?
		368: answers question.	50	368	E: This problem sheet itself.
				369	I: I see.
				370	E: Cause maybe this (i.e., the problem sheet) was like a rough copy.
No.22.	131. + 207. + 538. + 934. + 728. = 2538. **	371-2: repeats what subject says and reference to printout.	4 10	371	I: I got you. So you thought I made a mistake when I wrote 2358; that I meant to write 2538. OK. So what did you do with No.22?
		373: repetition error.	86 48	373	E: I did it again just to ...
		374-5: reference to next calculation.	10	374	I: To make sure. So in No.21 and No.22 are just identical work. So then you're giving up on problem 2, aren't you?
				376	E: Yeah.
No.23	41.7 + 5079. = 5120.7 **	377: reference to printout.	10	377	I: And you're going in No.23 to what?
		378-9: continues checking of line 373.		378	E: Just checking them all back again. Ah ... with the value of 6 as a 0.
		380: clarification.	9	380	I: This is which problem (i.e., No.23)?
		381: answers question.	50	381	E: That's problem 1.
		382-3: reference to printout.	10	382	I: Oh. You had already done that. OK. And then in No.24, what are you doing?
		No.23: repeats all parts replacing 6 with 0. Error in understanding in that he does not see there will be no difference in first, third and fourth problem.	96	384	E: In No.24 I was substituting 0 in for the 6 there.
				385	I: You did it again. So you did exactly what you did in No.21 and No.22. And then in No.25?
				387	E: Checking that one (i.e., problem 3) again.

No.24. 131. + 207. + 538. + 934. + 728. = 2538. **	No.24: a repetition of No.21 and No.22.	86	388	I: You had already –
			389	E: Put 6 as a 0. Yeah. Then in No.26 I did the same thing (as in No.20) just to check ... for myself.
			391	I: So really at this point you haven't still ...
			392	E: Figured out problem 2.
No.25. 82. x 85.91 = 7044.62 **	393: repetition of original question.	4	393	I: Do you think you see now what it is?
			394	E: No.
No.26. 307. x 4. = 1228. **				
12:55				

Sherri Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:12		01-05: introducing the question and establishing the atmosphere.	1	1	I: What I'd like you to do is read the problem and see if you understand it. Then if you understand it I'm going to leave you by yourself to try and work on it. If you get the answer you're going to call me, if you have any trouble with it ... like you know, you reach a real block, then you'll call me to help you.
				6	S: Yeah.
				7	I: Alright. So see if you understand it.
					PAUSE (20s)
		08: has not comprehended the question as the 4 problems are	82	8	S: OK. Which problem do you want me to do?
		09-10: explanation.	1	9	I: All four problems were done with that one calculator. OK. So I'm going to leave you with it a while.
					CALCULATOR OPERATED (Nos. 1, 2, 3, 4 and 5), (9 minutes).
No.1.	41.7 + 5079. = 5120.7 **	11: clearly asserts that she has the answer.	64	11	S: I got it.
		12-16: explanation.	1	12	I: OK. So I want to do with you exactly what I did with the other problem (this student also did the fourth problem). I want to isolate certain parts of this (i.e., the printout), you've nicely separated it into parts for me. You're going to try to tell me what you did. Let's call this part No.1.
No.2.	131. + 267. + 538. + 934. + 728. = 2598. **	17-19;23-24: explains reasoning.	65	17	S: OK. I looked at them and then I added them up. Like I did what it said. The first said 41.7 plus 5079, I added it. And it came to the right ... answer.
		No.1 - No.4: routine checks repeated.	22 48		
		20: clarification.	9	20	I: So this is then just the first problem.
				21	S: So that was right. So then I did the second one.
				22	I: So this is this part here (referring to No.1).
				23	S: Then I added it all up and it came out to 2598.

No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	25: related question to see if she understands the error in the question.	5	25	I: OK. So which one is wrong Sherri?
	26: answers related question.	50	26	S: The one that I came ... the second one is wrong. Like the one that I did on the calculator.
	28-29: clarification.	9	28	I: You're telling me that this one (i.e., the one on the printout) is the wrong answer?
No.3. 82.6 x 85.91 = 7096.166 **			29	S: Should come out to this. But there's a number in here that should not be in here.
			31	I: What do you mean?
No.4. 307. x 64. = 19648. **	32-35: shows that she understands the problem by separating parts of the condition.	21	32	S: Well, it says here that there's a key that's not working. So ... when the person pressed the key ... like it didn't go through the computer, or the calculator. It just pressed it but it wasn't printed out.
	36: restates the problem.	9	36	I: Which number was that?
	37: she has the answer and knows it.	50	37	S: The 6.
	38-40;42-45: explaining related question.	4	38	I: Yeah, that's right. So you've got the answer. Now I want to try to figure out ... I think you're slightly mixed up. This, in No.2, is OK. That's right.
	41: she has not understood which is wrong, the printout answer or the given answer.	83	41	S: Is it?
			42	I: It's this one that's wrong (i.e., the 2358 in the problem). This is the one that has the calculator not working, problem 2 on the piece of paper I gave you. This one (i.e., the problem given) should be 2598. That is the answer you want.
			46	S: Oh, OK.
	47-48: reference to printout.	10	47	I: Well, anyways, go ahead. So in No.2 you're just adding up what's there.

No.3.	82.6 x 85.91 = 7096.166 **	49-52: continues checking and compares the given and calculated answers.	55	49	S: Yeah and seeing ... got that. And in No.3 multiplied them and got that answer (i.e., 7096.166) which was different from this answer (i.e., 7044.62). And I multiplied problem 4 and got a different answer from what it says on the sheet.
No.4.	307. x 64. = 19648. **	53-54: review of what the subject did and said.	11	53	I: OK. So in the first 4 you've redone the problems on a printing calculator.
		55-57: explains reasoning and in reveals deductive reasoning. Clarifies the thinking applied to Nos. 1 to 4 as a whole.	65 40	55	S: So ... what I did was I looked back on the first problem which was correct and then I figured out which numbers weren't in the problem.
		58-59: questioning reasoning.	9	58	I: OK. So in problem 1 where you had 41.7 plus 5079 equals 5120.7 you looked in that problem and what did you look for?
		60-61: deductive reasoning shown.	40	60	S: The numbers that weren't in 41.7 plus 5079. Like there's not a 2, 3, 6 or 8.
		61: related question established by contradiction.	5 6	61	I: But there's a 2 there in 5120.7.
		62;65-6: answers related question and distinguishes between error in input and output keys.	50 40 65	62	S: But I was just taking the numbers ...
				63	I: Oh, I see, the numbers that you put in not the numbers that are printed out.
				65	S: Yeah. And so I tried putting in like or taking out all those numbers. Like first I tried the 2 -
No.5.	131. + 67. + 538. + 934. + 78. = 1728. **	67-68: reviewing what the subject said previously.	11	67	I: OK. Let's take a look at the printout again. I see your reasoning, right? So in No.5 you're redoing No.2?
				69	S: No.2.
		70: related question.	5	70	I: Why didn't you redo No.1?
		71: answers related question illustrating reasoning.	50 65	71	S: Well because it worked out the same.

No.5.	131. + 67. + 538. + 934. + 78. = 1728. **	72-73: extending from what the subject did previously. 74;No.5: illustrates the bright idea to solve the problem. Wrong operation though the algorithm is good.	13 41 47 92	72	I: (Laughing) I just wanted to see what you're thinking. So in No.5 you're redoing and what change have you made?
				74	S: Ah, took out the 2.
				75	I: So instead of ...
				76	S: Of the second number which was 267, I just added 67.
		77: review of what the subject did and stated.	11	77	I: So you've eliminated the 2.
				78	S: Yeah. And on the last one it was 728 and I took out the 2 to make it 78.
				80	I: And what happened?
		81: printout clearly analyzed.	55	81	S: And it didn't come out with the same answer either.
		82: probing reasoning.	9	82	I: So what do you conclude from that?
		83: clearly indicates deductive reasoning.	40	83	S: It's not the 2.
		84: probing reasoning.	9	84	I: Did you try the 2 in any other ones?
No.6.	8.6 x 85.91 = 738.826 **	No.6: error in that she did not comprehend the conditions specified in the question where it is only 1 key and the 2 key needed to be tested only once.	82	85	S: Yeah. I tried the 2 in No.6.
				86	I: OK. Just hold on. So in No.6 you're still working with the 2.
No.7.	11. + 267. + 538. +	86: clarification.	9	87	S: Yeah.
				88	I: Even though you see in problem 2 that it isn't ...
		89: repetition error.	86	89	S: Oh, I just needed more evidence.
				90	I: OK. Fine. So then what have you done, you've left the 2 out, heh?

				91	S: Yeah, on 82.6 I made it 8.6. And then times it by 85.91. And that didn't work out either.
				93	I: OK.
		94: error: did not understand the significance of this which really meant that a 2 test was unnecessary.	96	94	S: So I just stopped there because there's no 2 in problem 4.
				95	I: Alright. Now in No.7 here, what have you done?
		95: reference to printout. No.7: applying same strategy developed for No.5. Changes existing algorithm unsuccessfully.	10 48 92 54	96	S: I tried the 3 next.
				97	I: So this then is which problem you're redoing?
				98	S: Problem 2.
				99	I: Problem 2.
		100-1: calculator misentry. Identifies error.	100 54	100	S: Well, see what happened was ... I put the 3 in there (i.e., 538) so I had to clear and start all over again (laughing).
No.8.	11. + 267. + 58. + 94. + 728. = 1158. **	102-5: review of work and reference to printout.	11 10	102	I; That's OK. Alright. I need to see that. In other words ... ah I see, where you were leaving the 3 out you put it in the 538. In No.8 you're just redoing the work you were doing in No.7. OK. Explain to me what you were doing in No.8.
		106;108;110-11: explains the reasoning of No.8.	65 48 92	106	S: In the first number, 131, I took out the 3 to make it 11.
				107	I: Yes.
No.9.	131. + 267. +			108	S: And then the third number 538 I took out the 3 to make 58.
				109	I: I follow you.
				110	S: The fourth number, 934, I took out the 3 to make it 94. And then I added that up and it didn't come out the right answer.

No.8.	11. + 267. + 58. + 94. + 728. = 1158. **	112: specific reference to printout and some direction.	10 12	112	I: OK. Now then ... so you got 1158 which did not agree with 2358.
				113	S: Right.
				114	I: So did you make any conclusions from that?
		115: illustrates deductive reasoning.	40	115	S: It couldn't be the 3.
No.9.	131. + 267. +			116	I: OK.
		117;No.9: calculator misentry.	100	117	S: OK. So then the next I tried ... in ... see I went back, I did that, I put the 6 in again (laughing).
		119-20: clear reference to printout	10	119	I: Hold on a minute. So in this part here, in No.9, this little part (i.e., 131 + 267 + ...) what were you going to try there?
		121-2: she does not repeat the error of No.6 and illustrates deductive reasoning.	54 40 49	121	S: I was ... OK ... problem 3 didn't have a 3 in it so I decided the 3 wasn't there so I wasn't going to go on with that.
		123-4: rewording of subject's ideas	11	123	I: Say that ... so what you're telling me then is you wouldn't continue on with the 3 because there was no 3 in problem 3?
				125	S: Yeah.
		126-7: review continued.		126	I: So then you knew it couldn't be the 3 key that's causing the error because there is no 3 in problem 3.
				128	S: Right.
		129-30: reference to printout.	10	129	I: OK. So what are you doing in this part of No.9 (i.e., the last part)?
				131	S: Ahem, the next number that isn't in problem 1 is 6. So I was going to try that one. And I put the 6 in the 267 and I had to clear (laughing).

No.10.	131 + 27. + 538. + 934. + 728. = 2358. **	134-5: reference to printout.	10	134	I: Alright, that's fair. Don't worry about those, because I just want to know what you're doing. In No.10, you're doing what?
		136-8;No.10: explains reasoning and identifies error. Repeats previous algorithm but with different key. Knows she has the answer.	65 54 48 54	136	S: OK. Then I ... I took problem 2 and I took out the 6 and it was only in the second number which was 267 and I made it 27. And added up and it came out with the right answer.
No.11.	82. x 85.91. = 7044.62 **	139-40: questioning reasoning.	9	139	I: Oh! OK. So now you get something interesting. What did you decide from that then?
		141;143: not certain of having the answer.	95 82	141	S: That it could be the 6. OK. But then I went on -
				142	I: What do you mean by it could be the 6?
No.12.	307. x 4. =			143	S: Well, ... I ... you know ... I just needed more evidence.
				144	I: OK, but like -
				145	S: It probably was.
		146-7: related question to see if subject understands not just key that's wrong, but nature of mal-function.	5	146	I: What do you mean by the 6 though? Explain to me like what do you think may be going wrong.
		148-50: shows understanding of nature of error.	50 40	148	S: That the 6 key wasn't working and that when all these numbers were punched in the 6 ... didn't ... you know, it wasn't working so it wasn't punching in or anything.
		151: repeating subject's calculations.	4	151	I: So instead of 267 you've put in 27.
				152	S: 27.
				153	I: OK.
		154: knows she has the answer.	54	154	S: And added it up. And it came out with the right answer.
		155: probing reasoning.	9	155	I: Is that the only change you had to make in there?

Problem	Entries	Comment	Ref.	Line	Transcript
No.11.	82. x 85.91. = 7044.62 **			156	S: Yeah, it was the only 6.
		157: reference to printout.	10	157	I: Alright. So in No.11 what are you doing?
		158-9;No.11: continued testing of 6 key. Makes routine check.	48 54 55	158	S: Problem 3. I took out the 6 in 82.6 and make it 82. And then times by 85.91 and it came out with the right answer.
No.12.	307. x 4. =			160	I: Ah! OK. And you know what I find very interesting in here too? You remember way back you were telling me you said you weren't worried about the 2 in problem 1 that was in the final answer?
				163	S: Yeah.
		164-5: related question to probe understanding.	5	164	I: This 6 in 7044.62. You notice there's a 6 in there? That didn't bother you, heh?
				166	S: No.
		167-8: directing reasoning of the subject.	12	167	I: In other words I think what's upsetting ... what you think may be wrong is it's impossible to enter a 6.
		169-70: answers related question. Already illustrated this in line 62.	50	169	S: But it ... it will work in the computer ... like the computer itself will put the 6 in.
				171	I: Yes, it'll generate a 6. Very good.
		172-4: continues algorithm.	48	172	S: Then in problem 4, 307 times 64 I changed to 307 times 4, and it came out to the right answer so I concluded that it was the 6 that was wrong.
		175-7: review of what subject has previously stated with some extending.	11 13	175	I: So in this No.12, so what you've done then, you weren't satisfied with just having it work out in one; you had to have it work out in all of them.
				178	S: Yeah.
		179: probing reasoning.	9	179	I: Is there any particular reason for that?

		180: shows No.11 and No.12 are routine checks continued from lines 158-9.	48	180 S: Just to make sure it works.
		181-2: review that attempts to clarify thinking involved.	11	181 I: OK. Going through this whole thing, right, where would you say that you realized you had the answer?
No.10.	131 + 27. + 538. + 934. + 728. = 2358. **	183-5: knows when she has the answer.	65	183 S: Ahm ... ... No.10. Where I added up problem 2 without the 6. Because I couldn't ... well ... I almost had it figured out. I had it narrowed down to four numbers.
				186 I: Yeah.
No.3.	82.6 x 85.91 = 7096.166 **	187-92: recalls related problems. Explains reasoning and demonstrates deductive reasoning.	33 65 40	187 S: In ah ... which one was it ... No.3 and No.4, can't remember ... when I added up 2 and ... like I did them all out ...and then I went back to problem 1 because it was the only one that was right and I figured out which numbers weren't in there that were in the ones. That's when I had it narrowed down to the four numbers it could be.
No.4.	307. x 64. = 19648. **	193: testing previously answered question.	4	193 I: Which four numbers were they?
				194 S: 2, 3, 6 and 8.
		195-6: related question to test understanding of conditions of problem.	5	195 I: I can see that. OK. Once you found 6, heh, you didn't bother trying 8, heh?
				197 S: No.
				198 I: Why?
		199-200: separates parts of conditions in question.	50	199 S: Well, it says which key (i.e., singular) isn't working not which keys aren't working.
12:40				201 I: Yeah. Good for you (both laughing).

Evan Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12.15				01	I: What I'd like you to do is read this problem and just tell me if you understand the problem.
				02	E: OK.
				03	I: So I'd like you to try and figure out which key isn't working. Now when you think you've figured it out, or let's say you're having really a lot of problems and you're stuck, I'm gonna be up there and you just give me a call. I think you'll work better if you work alone.
		08: questions conditions.	20	08	E: And you use all these ones (i.e., the problems) to figure it out.
					CALCULATOR OPERATED (Nos. 1 to 4), (4 minutes).
		09: says he has it.	64		OK. I think I've got it.
		10: questions certainty.	7	10	I: You think you've got it?
				11	E: Yeah.
				12	I: Alright, so what I want to do with you, I want to take a look at your printout, in other words what you did here (advancing and removing the printout). Then I want you to explain what you did as best as you can. Like what your idea was, what you were trying to do. Boy, you sure don't have a lot of work here, but, you think you've got it here, heh? Let's take a look and see what happens. What are you doing by ... up to the first stars (i.e., No.1)?
No.1.	41.7 + 5079. = 5120.7 **	18: reference to printout.	10		
		19; Nos. 1 to 4: routine checks of all four problems given.	47	19	E: Well just trying to find out what the real answers are for each one.
No.2.	131. + 267. + 538. + 934. + 728. = 2598. **	21: clarification.	9	21	I: So this is the real answer for which one?
				22	E: For this problem here.
		23: repetition.	4	23	I: Problem 1.

No.1. 41.7 + 5079. = 5120.7 **	25-27: probing reasoning with implied meaning (i.e., direction)	5 12	25	I; So the one that's in the problem and the one you got on the printout is the same. Did you get anything from that? Like did you get any ideas from that?
No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	28-29: demonstrates deductive reasoning.	40	28	E: Well probably ... ahm ... the numbers that ... weren't working properly was not any of the numbers from problem 1.
	30: related problem.	5	30	I: So which ones - I follow you - which ones are working properly?
	31: deductive reasoning.	40	31	E: 4, 1, 7, 5, 0 and 9.
No.3. 82.6 x 85.91 = 7096.166 **	32: related problem to probe his understanding.	5	32	I: Why not the 2 on this side (i.e., the other side of the equation)?
	33: demonstrates deductive reasoning and an analysis of results.	40 55	33	E: Because it's in the answer.
No.4. 307. x 64. = 19648. **	34-36: repetition of the original question.	4	34	I: OK. Excellent. What could be the key that's not working from problem 1? Do you have any idea which key it is or keys that could not be working?
			37	E: Could be 2, 3, 6 or 8.
	38: probing reasoning.	9	38	I: Excellent. You're just checking the first one. Did you get any of those ideas - when you saw the 5120.7 - did you figure out, did you get that idea you were just talking about?
			41	E: Well, at first, before I started doing anything.
	42: clarification.	9	42	I: What do you mean by that?
	43-44: explains reasoning and refers to a general plan.	65	43	E: At first, before I started, I decided to check out all the answers and I sort of got the idea that -
	45-46: interruption to maintain sequence of reasoning as shown in printout.	2	45	I: From the first answer though, when you got the first answer, did you know which keys should be working?
			47	E: Yeah.

No.2.	131. + 267. + 538. + 934. + 728. = 2598. **	48-50: reference to printout.	10	48	I: You did. Now in this next part, like everything between the first two stars and the second two stars, what were you doing in here? We'll call this one No.2.
		51;No.2: explains reasoning; continues checking procedure.	65 48	51	E: That's the addition for problem 2.
				52	I: And what happened?
		53-54: demonstrates analysis.	55	53	E: The answers weren't the same as on the page (i.e., the problem given).
		55: clarification.	9	55	I: Is it?
				56	E: No.
		57: related question based on results.	10	57	I: So 2598, what you got on the printout, that's the real answer?
				58	E: Yeah.
		59-61: some direction given.	12	59	I: And 2358 is the answer they gave you. From these two different answers, when you see the answers are different, did you get any ideas?
		62-63: illustrates deductive reasoning.	40	62	E: One of the numbers in that, in the numbers that are being added is the one that isn't working.
		64: related question.	5	64	I: Do you have any ideas which one it could be still?
				65	E: Still can't really decide. Same ones ...
		66-67: reference to printout and review of previous statement.	11	66	I: You mentioned in problem 1. But you haven't reduced it down any. Like you said in No.1 it could be 2 ...
		68: demonstrates deductive reasoning during the interview.	40	68	E: I'd probably leave out the 8.
		69: probing reasoning.	9	69	I: Why?

	70: place value strategy emerging as a test of integers.	55	70 E: Well ... cause the first line of addition is the same number (referring to the ones column).
	72: clarification.	9	72 I: OK. What about the units in the two answers?
			73 E: The same. Well I thought that would mean the 8 was probably correct.
	75: tries to determine if reasoning is after calculator work.	9	75 I: OK. Excellent. Did you figure that out when you did it?
			76 E: Yeah.
	77: reference to printout.	10	77 I: Alright, so you've eliminated the 8. It's probably working, heh? So, what did you do then? After you did this one. Did you go on to problem 3 then?
			80 E: Yeah.
No.3. 82.6 x 85.91 = 7096.166 ***			81 I: So we'll call this one No.3. So what are you doing in No.3?
	82;No.3: checking procedure continued.	48	82 E: It's multiplication, problem 3.
	83: reference to printout.	10	83 I: So you multiplied it. Did the two answers agree?
			84 E: No.
	85: probing reasoning but with direction.	12	85 I: OK. When they didn't agree, what could you figure out from that?
	86-87: excellent use of deductive reasoning. Note: he combines the results of No.2 and No.3.	40 55	86 E: That ... the one that was wrong must be in 2 (i.e., problem 2) and also in problem 3.
			88 I: And, do you know any more than that?
			89 E: Not yet, well ...
	90-91: repetition of what subject said.	4	90 I: Just whatever you noticed. You just noticed that it had to be the same one in each one, heh?

			92	E: Yeah.
	93;95-7: reference to printout. Probing reasoning but with direction.	10 12	93	I: So then what'd you do, you went on and checked problem 4?
			94	E: Yeah.
No.4. 307. x 64. = 19648. **			95	I: So let's take a look at problem 4. Now, you did problem 4, heh, and once again the answers didn't agree. Did you figure out anything from that?
	98-99: as defined here, Evan is not guessing.	40 54	98	E: Ah, well at this point I guessed that it was the 6 that wasn't working.
	100;102: clarification.	9	100	I: At which point you guessed it would be the 6?
			101	E: As soon as I finished.
			102	I: As soon as you finished this No.4?
			103	E: Yeah.
	104: probing reasoning.	9	104	I: OK. Can you tell me why you guessed it was the 6?
	105: excellent demonstration of how to solve the problem by deductive reasoning used in analyzing the calculated results.	40 55	105	E: Because the 6 shows up in all the problems that weren't right.
	106-7: related question.	5	106	I: Is there any other one (i.e., number) that shows up in all the problems that aren't right?
			108	E: No.
	109-10: repetition of related question.	4	109	I: Well you told me it could have been the 2. Is the 2 in all of them?
	111: illustrates past analysis of results.	55	111	E: The 2 isn't in problem 4, though.
	112: related problem.	5	112	I: OK, what about the 3?

	113: answers related question.	50	113	E: It isn't in problem 3.
	114: reviewing with subject.	11	114	I: OK, so then you've eliminated. And you eliminated, you told me –
			115	E: 8.
	116-21: a new question explained.	1	116	I: Was not in problem 2. So it's 6. So you found the key and that's what the question asked for. I'm going to ask you a second part to work on. Can you figure out, OK, the 6 key isn't working. You found that out, heh. Can you figure out what's going wrong in problems 2, 3 and 4? Like what they're really adding or multiplying or subtracting or whatever.
	122;124: questions conditions.	20	122	E: Instead of the 6?
			123	I: Heh?
			124	E: Instead of the 6.
			125	I: Yeah. What's happening. So try problem 2 and see if you can tell me what's happening there. Do you want me to leave you alone for awhile?
			128	E: Sure.
				CALCULATOR OPERATED (Nos. 5 to 9), (.8 minutes).
	129-32: review of calculations.	11	129	I: OK. So let's just go over it a little bit and see if that doesn't help you a bit. You're just going to tell me what you were trying to (advancing and removing the printout). You kept on trying the same problem again and again.
	133: indicates change of approach.	53	133	E: Yeah. Then I decided to work on the next one , but ...
No.5. 82.4 x 85.91 = 7078.984 **	134-6: reference to printout.	10	134	I: OK. Let's take a look at this first part here. We're going to call it No.5. What were you trying to do in there? You're trying which problem?

No.5. 82.4 x 85.91 = 7078.984 **	137–9;No.5: replaces the 6 with a 4. Misinterprets the malfunction as meaning printing 4 instead of 6, rather than as nonfunction.	47 92	137	E: First of all I looked at problem 2 and thought they might be printing a 4 instead of 6, so in problem 3 I substituted a 6 for a 4. And tried that.
			140	I: Oh, I see. Did it work?
	141: analyzed results.	55	141	E: Didn't work out.
	142–4: extending subject's ideas on the basis of No.5 and discussion.	13	142	I: Alright, when it says which key isn't working like, when the key isn't working you think it might mean that it's putting in another number, right? Is that what you thought?
			145	E: Yeah.
	146–7: hypothesizing on the basis of calculations and feedback from subject.	13	146	I: So then you want to figure out what other numbers could go in besides the 6, heh?
			148	E: Yes.
No.6. 82.8 x 85.91 = 7113.348 **	149: reference to printout.	10	149	I: And you tried 4 in there and now what did you do in No.6 here?
	150;No.6: employs same strategy as in No.5, but replaces 6 with 8.	48 92	150	E: I tried the 8.
	151–2: probing reasoning.	9	151	I: When you tried the 4 it didn't work, is there a reason why you tried 8?
	153;155: reference to printout and return to former problem.	55 33	153	E: Well, OK, I looked at this one and saw that they got –
	154;156: clarification.	9	154	I: You looked at which one?
			155	E: At this one.
			156	I: Which one?
			157	E: At 9 in there.
			158	I: Problem 2.

159;160-1: reasoning is as follows: replacing 267 with 287. This actually produces a total of 2618 so Evan has made a computation error.	40 85	159	E: I saw that they had a 9 so I -
		160	I: Instead of the 5 on the paper, in problem 2.
		161	E: Right. So I put an 8 in there ... I added these up and found how I could get a 9 as a second digit instead of a 5.
163-4: contradiction.	6	163	I: Oh, OK. I think it's the other way around you mean. How to get a 5 instead of a 9. Right?
		165	E: Yeah.
166: checking certainty with direction.	12	166	I: Would 8 have done that? You're not sure, heh?
		167	E: Not sure.
168: review of subject's previous statements.	11	168	I: So really in No.6 like you're using, you're checking problem 3, but you're using problem 2 to help you.
		170	E: Uh hum.
171-2: probing reasoning and extending from statements.	9 13	171	I: Like what happens in problem 2 to find the numbers to check in problem 3. Is that right? Do you know what I mean?
		173	E: Yeah.
174-5: reference to printout.	10	174	I: OK. What are you doing in No. ... OK, you didn't get the right answer.
		176	E: No.
177-9: probing reasoning with reference to printout but giving direction.	12	177	I: OK, from the answer that you got and the answer that they have, like they have 7044.62 and you have 7113.348. Did you get anything from that?
		180	E: I thought the number should be lower than the one I was trying.

No.6. 82.8 x 85.91 = 7113.348 **	180; Nos 6 and 7: explains reasoning and illustrates deductive reasoning. 82.8 changed to 82.2 as he uses method of successive approximations. Is he still using results of problem 2? Doubtful.	40 55	181	I: So what did you do to make it lower? You tried in No.7, 2.
			182	E: Uh hum.
No.7. 82.2 x 85.91 = 7061.802 **	183-5: clarification sought through extending from subject's calculations.	13	183	I: Instead of .. oh, I see, this No.5, it's also higher than what you want, isn't it, 7078. So is that why you made this one lower than even 4? Did you look at those kind of things?
			186	E: Yep. That's what I did.
	187: reference to printout.	10	187	I: So 82.2 instead of 82.4, heh? What happened?
	188: indication of analysis.	55	188	E: Didn't get the right answer yet.
	189: extending subject's ideas.	13	189	I: And you're still too big, heh?
			190	E: Yeah.
No.8. 307. x 44. = 13508. **	191: reference to printout.	10	191	I: So, did you want to - OK, then what'd you do? You checked another one, heh. In No.8 here you're not checking problem 3 anymore, are you?
	194: changes approach.	53	194	E: I decided to try it on ...
	195: probing reasoning.	9	195	I: Problem 4. What change did you make now?
No.5. 82.4 x 85.91 = 7078.984 **	196; No.8: changed problem tested but error in that 4 had already been tried in No.5 as a possible replacement for 6.	48 86	196	E: Tried the 4. Changed the 6 into 4.
	197-9: direction towards difference strategy.	12	197	I: You had already done that one time in No.5, heh? So did it ... what happened? You got 13508. Did that tell you anything? What did that tell you?
			200	E: (Laughing) not much besides I goofed.
No.9. 307. x 4. = 1228. **	201: reference to printout.	10	201	I: Kind of big, heh? So then you quit on problem 4, and in No.9 -
	202: bright idea.	41	202	E: I know what it is now!

Printout	Comment	Code	Line	Transcript
			203	I: What?
	204: gets the solution.	54	204	E: The 6 isn't working. Not doing anything.
	205d: probing reasoning.	9	205	I: How'd you figure that out?
	206: explains reasoning.	65	206	E: Cause it's just multiplying 4 there (i.e., in problem 4).
			207	I: OK. How'd you see that?
	208-9: uses mental computation and difference strategy to find the answer.	55 40	208	E: Cause that number is way too small (i.e., 1228) to do that so 4 times 307 is the answer.
			210	I: And you know that by just looking at it. Would you want to try it again in the calculator to make sure you're right. You are, I think right, but let's just see what happens.
				CALCULATOR OPERATED (No.10).
	213-5: subject directed to find other errors.	12		OK. So you found out the mistake in problem 4. See if you can find the mistake in problem 3.
				CALCULATOR OPERATED (No.11).
				Did you find it? You got one more to find the mistake in. That's problem 2.
				CALCULATOR OPERATED (No.12).
No.12. 131. + 27. + 538. + 934. + 728. = 2358. **	216-7: important point of clarification.	9		Did you find it? Yeah, OK. I noticed when you printed 27, you stopped, heh? What were you trying to decide?
	218-9: logical reasoning explained.	65 40	218	E: Well I didn't know for sure if the 6 was actually a 0 or just nothing.
	220: probing reasoning based on printout.	10	220	I: What made you decide to print 27 instead of 207?

			221	E: Well ...
No.12. 131. + 27. + 538. + 934. + 728. = 2358. **	222-4: reference to printout.	10	222	I: This one will bee No.10, this one No.11. So we're talking about No.12. Why did you decide not to print the Q? Why did you print 27? Is there a reason?
	225-6;228-31;No.12: still using problem 2 as a check for numbers. Reasons that the sum of the tens column (i.e., 3 + 2 + 3 + 3 + 2 = 13 in problem 2). This was also done in No.6. Deductive reasoning well illustrated.	55 33 40 48	225	E: Well I moved the 2 in here, I was looking back to ... this one over here -
	227: clarification.	9	227	I: Which No.?
			228	E: Well actually it refers to all of these 6 and 7 but, ... all this added up to 13. I found a number that gave it a 5 there, it was the 2 so I just moved the 2 over there, so it would become 7 (i.e., referring to problem 2).
No.9. 307. x 4. = 1228. **	232-4: reviewing with reference to printout and some direction.	11 12	232	I: (Laughing) so this was mental work you had done: 5, 6 and 7. That's excellent. Can you just tell me quickly what you were doing in No.9? You're checking problem 3 again, heh?
			235	E: Uh hum.
	236: probing reasoning.	9	236	I: What change are you making?
	237;No.9: returns to replacing strategy of Nos. 5, 6 and 7. Not using previous results in combination with difference strategy as 82.4 was too large, so why use 82.7?	96 48	237	E: Change the 6 to a 7.
			238	I: You remember you telling me you knew it wasn't the 7 key because it worked up here (i.e., in No.1) and all that.
	238-9: counterexample used to explore error.	6	240	E: Yeah (laughing).
No.10. 307. x 4. = 1228. **	241-3: reference to printout.	10	241	I: So really you're experimenting in there. Now let's look at what you've just done (advancing and removing the printout). And in No.10, what are you doing?
	244;No.10: developing elimination procedure.	47	244	E: Well, I took problem 4 and just left out the 6 entirely.

No.11. 82. x 85.91 = 7044.62 **	245-6: reference to printout.	10	245	I: That it wasn't working at all. And you got the right answer. In No.11 what'd you do?
	247; No.11: repeats elimination procedure.	48	247	E: Left out the 6 again.
	248: clarification.	9	248	I: In which problem?
			249	E: Problem 3.
			250	I: So instead of 82.6 ...
			251	E: Just 82.
	252-7: reviewing reasoning and calculations.	11	252	I: Just 82 and it worked out, heh? And then in No.12 we just talked about that. And you changed 267 to 27. And you say you moved the 2 over because of work you had done in problem 2 where you realized to get 15 you needed to make 6 a 2. And is that why in No.7 you used the 2? No. Is it? See you've changed 6 to a 2 in No. 7.
				PAUSE (10s).
	259-60: trying to determine reasoning behind inhibition.	97		Maybe, heh. 4, 8 and 2 you changed it to. And 7. Well you liked problem 3 to check it in. How come you liked problem 3 to check all your stuff in?
			261	E: I don't know.
12:44			262	I: You only tried problem 4 once. You did most of your work in problem 3. And you were working with problem 2 in your head.

Shelley Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
1220		01: probing reasoning of statements made before taping.	9	1	I: I gave you a chance to read the problem and what did you say?
		02: understanding error.	83	2	S: I don't understand the question.
		03: clarifying.	9	3	I: OK. What don't you understand about it?
				4	S: Uhm ... well I don't ... how ... how do they mean like it ... how can they tell?
		06-7;09-10;12-4: direction, hints and an explanation of the question	8 1 12	6	I: Alright. They're telling you that. They're telling you that this ... there is a calculator that makes mistakes.
				8	S: Oh. OK.
				9	I: But it's consistent. That is, the same mistake will be continued in different problems.
		12: restates problem.	21	11	S: Oh, it's the same one ...
				12	I: That's right. There are 4 problems. Some may have mistakes, some may not, but if there is a mistake, it's the same mistake in each problem.
				15	S: And I have to find it?
		16-7;19;21-2: cont'd explanation.	1	16	I: Yeah. Now the reason there is a mistake is because a key isn't working.
				18	S: Yeah. One of these.
				19	I: Now, on this calculator I'm giving you, it's working.
				20	S: Yeah.
				21	I: But they're telling you there's another calculator and this is what it did.
				23	S: Oh, and I have to find out which key, OK. I understand.

	24-26: explaining a calculator prerequisite.	1	24	I: Now I want ... did I show you how to use the print button (a key that will print the display without performing operations with it)?
	27: starting to explain her plan.	40	27	S: I just go along and add all these and ...
	28-29: clarifying.	9	28	I: Well, that's what you're going to decide, OK, how you are going to find the mistake.
				CALCULATOR OPERATED (Nos. 1 to 12), (14 minutes).
	30;32: expresses certainty and states the answer.	64 54	30	S: I got it.
			31	I: You got it, OK.
			32	S: The number 6.
	33: reference to printout.	10	33	I: It's what?
			34	S; 6.
			35	I: It's the 6, heh? So you're right. Now what I'd like to do with you Shelley is ..
			37	S: Go over what I did.
No.1. 41.7 + 5079. = 5120.7 **	38-40: reference to printout.	10	38	I: Yeah. Go over what you did, and if you can, try to remember as well as you can all the different things you did in these steps. OK. So what are you doing in this first one here (i.e., No.1)?
	41-2;No.1: check of problem 1; logic illustrated.	47 40	41	S: I just wanted to see if that one was ... if there was anything wrong with that one. Like if it added up.
	43: clarification.	9	43	I: Which one?
			44	S: Problem 1.

45: repetition of what subject said.	4	45	I: So in printout No.1 you're checking problem 1.
		46	S: Yeah.
47: clarification.	9	47	I: OK. Did you discover anything?
48-49: explains reasoning.	65	48	S: It was the same. Like ... but ... it added up. I was just going through and seeing if they would add up.
		50	I: OK.
		51	S: To this number (pointing to the question).
52-4: repeating what subject said.	4	52	I: This first ... OK. These first ones here you added 41.7, 5079 and you got that. And you're saying the answer in the problem and the answer on the printout agree.
		55	S: Yeah.
56-57: probing reasoning.	9	56	I: OK. Does that tell you anything? Can you conclude anything from that?
58;63: illustrates deductive reasoning.	40	58	S: Well, well there was no 6 in that one.
59;61-2: clarifying.	9	59	I: Well you know that now though.
		60	S: Yeah, but I was just -
		61	I: When you were first there. Did you ... the fact that the two numbers agree ... did that tell you anything?
		63	S: It told me that all these numbers had to be ... working on the ..
64: related question.	5	64	I: Which ones?
65: answers related question indicating understanding error.	50 82	65	S: 4, 1, 7, 5, 0, 9 and 2.

	66;68: contradiction and hint.	6 8	66	I: The 2 is in the second part of the equation, heh?
			67	S: Yeah.
			68	I: And you don't have to punch that in.
	69-70: identifies error.	54	69	S: Yeah. OK. Yeah, that's right. So 4, 1, 7, 5, 0 and 9 are all working.
	71: clarifying.	9	71	I: Did you realize that when you did it?
			72	S: No I didn't. I was just going through just to see if it –
No.2. 131. + 267. + 538. + 934. + 728. + 2598. = 5196. ** 5196. – 2598. = 2598. **	73: reference to printout.	10	73	I: So you didn't realize that in No.1 yet. Alright, that's OK. Now in No.2?
	75;79-80;No.2: checking answer to problem 2. Note calculator misentry. Identifies error and analyzes result of calculations.	48 54 55 100	75	S: OK. In No.2 I went through and just added them.
			76	I: The ones in problem 2?
			77	S: Yeah.
	78: clarification.	9	78	I: OK. And then what happened?
			79	S: It didn't equal ... or OK, see in this part here I pushed ... ah ..., plus and then equals.
			81	I: Ah.
	82-83: error identification previously coded.		82	S: And it printed out ... so then ... I realized what I did so I subtracted 2598 from that.
	84: repetition of subject.	4	84	I: Because you had added 2598 too much.
			85	S: Yeah.

	90-91: related calculator question	5	90	I: So then what is the answer to this sum in No.2? Where do you see the answer to this sum?
	92: answers related question.	50	92	S: There, at the bottom.
	93-94: clarification by reference to printout.	10	93	I: In No.2, when you subtracted 2598, then you knew that this (i.e., 2598) was the sum.
			95	S: Yeah.
No.3. 131. +	96: reference to printout.	10	96	I: OK. When you see that ... OK, so what are you doing in No.3 then?
267. + 538. + 934. = 1870. ** 1870. + 728. = 2598. **	97-101; No.3: routine check of problem 3. Thinks an omission may have been made.	47 92 82 55	97	S: OK. In No.3 I was going through to see ... like I was just adding them up and equalling them to see what numbers - I was just gonna - seeing if ... if maybe they left out a few numbers to get it. To get the answers. Like maybe they just deleted one number and then I realized that they didn't.
	102-3: repeating what subject said	4	102	I: Oh, you thought maybe they had left out one of the numbers they were adding.
			104	S: Yeah.
			105	I: So what you did then is you left out which number to check it?
	106: continues explanation of reasoning.	65	106	S: Well I went through, I went down to 934 and then I equalled it.
	107: clarification.	9	107	I: So you're leaving out which number?
			108	S: Ah ... 728.
	109-11: explanation/review of what the subject did.	1	109	I: And checking to see if that was what was left out. But you realize when they leave a whole number out they're three keys out, heh? So. OK. You're just trying that an as idea.
			112	S: Yeah, I was just going through -

112;114: explaining reasoning; modifies algorithm for correction.	49 54	113	I: And then when you saw 1870, what did you realize?
113: interruption; directed question.	12 2	114	S: It was too low to be 2538 so I added 728.
115-6: related question.	5	115	I: So when you do that, what does this one become like? No. 3, when you do this, when you add ...
117: answers related question.	50	117	S: Same as No.2.
118-20;122: probing reasoning.	5	118	I: Right. Now, what I was going to say to you? So far in the problem ... ah ... OK, when you see that in both No.2 and No.3, 2598 does not agree with what's given in the problem ...
		121	S: Yeah.
		122	I: 2358. Did you notice anything or conclude anything from that ?
123-4;126-7: explains reasoning; difference strategy emerging.	40 96	123	S: Well I noticed that there was just, like it was a difference of 200 and ...
		125	I: 40.
		126	S: Yeah. I just ... I really didn't understand it then so I went on to see what the difference was (see No.6).
128-9: clarifying.	9	128	I: So it's not as if you're looking at 2598 and 2358 and coming up with any ideas.
		130	S: No.
131: putting words into the subject's mouth.	13	131	I: You just see right away that they don't agree?
		132	S: Yeah. And then I , I kind of thought about that for a while and then I decided I'd go on and see what happened with the other ones.
135: previously stated demand.	4	135	I: But it didn't mean anything to you?

				136	S: Not, not then it didn't.
No.4.	82.6 x 85.91 = 7096.166 **	137: reference to printout.	10	137	I: So then in No.4 what are you doing?
		138-9; No.4: check of problem 3.	48	138	S: OK. In No.4 I went and tried this to see if it would equal the same.
		140: clarification.	9	140	I: You tried problem?
		141;143: analyzes results.	55	141	S: 3. And I was just going to see if it would, if it was the same.
				142	I: Was it?
				143	S: No. There was ... it was different .. ah ... problem.
		144: probing reasoning.	9	144	I: Did that tell you something?
				145	S: No. I was still just going through and seeing what would happen. I didn't, nothing clicked until after, until I got down into –
No.5.	307. x 64. = 19648. **	147: reference to printout.	10	147	I: Alright. So let's go to No.5.
		148;150;No.5: checking problem 4.	48 55 40	148	S: OK. And I was doing the same thing there.
		149;151: clarification.	10	149	I: What were you checking?
				150	S: Just to see if the ah ... sums were equal.
				151	I: You mean if 1228 and what you got in the printout was the same.
				152	S: Yeah.
		153: review of previous statements.	11	153	I: So you're checking problem 4 in No.5?
				154	S: Yeah.
				155	I: And were they the same?

No.6: 2598 – 2358 = 240.

		156	S: No.
157–8: extending from what subject has said.	13	157	I: So ... OK so from what you've done up to No.5 you know that there are mistakes.
		159	S: Yeah.
160: related questions on printout.	13	160	I: What are the mistakes? In which problems?
161: answers related question with no hesitation. Good illustration of logic applied to printout results.	50 55	161	S: 2, 3 and 4.
		162	I: OK, and not in 1.
162: more extension.	13	163	S: Yeah.
164–5: reference to printout.	10	164	I: OK. So now we're going and you're interested in No.6 in finding out what?
166–7: No.6: using a different algorithm (incomplete): finding the difference between the correct and incorrect answer.	47 55	166	S: Ah, what the difference was between ... ah ... the answer that I got and the answer that was on the paper.
168: clarification.	9	168	I: You're talking about problem 2?
		169	S: Yeah.
		170	I: So, this difference was how much?
		171	S: 240.
172; 175–6: probing reasoning.	9	172	I: Did that tell you anything?
		173	S: Ah ... no I was just finding out what the difference would be. I didn't get it until –
		175	I: OK. But I was just wondering if there was reason why you were looking for the difference.

	177-8: a quick look at the different problems shows that this should have obviously not been so.		177	S: Ah, I was going down through these ones to see if maybe the difference would be the same.
No.7. 7096.166 − 7044.62 = 51.546	179;183: reference to printout.		179	I: Ah, like if there was a, OK. So in No.7?
	180;184;No.7;No.8: applying the difference algorithm to compare differences and then look for a possible pattern. Poor reasoning, as mentioned above.	48 55 93	180	S: I was finding out the difference again.
			181	I: Again.
			182	S: In problem 3.
No.8. 19648. − 1228. = 18420.			183	I: Right. And then in No.8?
			184	S: I was finding out the difference.
			185	I: So it's exactly what you said. You were looking for the differences. And then did you compare differences?
			187	S: Sort of.
			188	I: You looked at them?
			189	S: Yeah.
			190	I: Like the difference in No.6, No.7 and No.8.
	191-2: poor reasoning, as stated.		191	S: Yeah. I was seeing if they would, like I thought maybe there was a possibility that they would all equal the same number.
	193: repetition.	4	193	I: OK. In other words that the difference was the same in each case.
	194: Shelley has been dying to explain how she got this answer.		194	S: Yeah. Yeah. But it wasn't. And then in the next one ... step 9 (i.e., No.9) ...
			196	I: Yeah.
			197	S: I went through ...

No.9a. 82.6 x 0.91 = 75.166 **	198-9: reference to printout.	10	198	I: That looks interesting. What have you done in No.9? You've made a change. What did you change?
			200	S: OK. Here I just -
No.9b. 75.166 x 85 = 6389.11 **	201: interruption in attempt to clarify.	2 9	201	I: In problem 3, heh?
	202-4; No.9a: modifying algorithm of No.3, testing certain keys. Faulty algorithm in that question specifies one key is not functioning but Shelley omitted two keys.	49 82	202	S: I thought maybe they missed a number like in here maybe it wasn't in the decimals. So I ... I ... I forgot about 85 and I put in the 0.91.
	205; repeating what subject said	4	205	I: Left off the 85.
			206	S: Yeah.
	207-8: extending from subject's statements.	13	207	I: So you thought maybe, everything before the decimal wasn't registering.
	209-12: explains reasoning with illustration of deductions.	55 54 40	209	S: Yeah, or like there was one number, like maybe then were just using 1. Then I figured, then I realized that it couldn't be 1 because it would have been 82.6. So then I went 9, like 0.91. To see if it would equal that (i.e., the answer in problem 3).
			213	I: OK, would you say that again Shelley.
			214	S: Ah ... OK. I was gonna -
	215: reference to printout for clarification.	10	215	I: So we're talking about No. 9a, heh?
			216	S: Yeah.
	217: repeating what subject said previously.	4	217	I: And you got 82.6 times 0.91.
	218-20: good explanation of reasoning.	65	218	S: Yeah. And I ... I took 0.91 because I thought, like I was going to take 0.01 but if it would have been that it would have been that number again with just the decimal moved.

No.9a. 82.6 x 0.91 = 75.166 **	221: clarification.	9	221	I: It would have been which number again?
			222	S: It would have been 82.6.
	223: extending from statements.	13	223	I: Right. Because 1 times 6 is, yeah.
No.9b. 75.166 x 85. = 6389.11 **	224-5;No.9a: reasoning explained. Analyzes results. Misinterprets the problem in removing two keys. Chooses the candidate 85 randomly.	48 55 92	224	S: And then I realized that couldn't be done so I went 0.91. And that didn't work.
	226-7: question that interrupts train of the discussion.	2	226	I: Ah, so there was one thing you didn't try then. You didn't try 00.01.
			228	S: Yeah.
	229;231: repetition of previous statements by subject.	4	229	I: Because you realized that wouldn't work by just looking at it.
			230	S: Yeah.
			231	I: So then you tried 0.91
			232	S: Yeah.
	233: reference to printout.	10	233	I: And you got this answer.
	234-5: evident analysis of results	55	234	S: And it didn't even come close to the answer that they have down there.
			236	I: OK.
	237;241-3;247-50;No.9b: a change to the algorithm of No.9a so	48 49	237	S: So then I multiplied 85 on to, to see ...
	still an incomplete process. Error of eliminating two keys.	92	238	I: So what have you left off this second part of No.9b?
			239	S: Ah ... 0.91.
	240: probing reasoning.	9	240	I: Was there a reason for that?

No.9b. 75.166 x 85. = 6389.11		241	S: Well I just ... ah ... I didn't want to see, like I knew it wouldn't work with 91 on so I thought maybe they weren't using the decimal place.
244: directing subject to think of relative size.	12	244	I: Alright, then when you say this was a small number, heh, 75.
		245	S: Yeah.
		246	I: Then did you think maybe ... what?
		247	S: I thought it, I knew that couldn't be it like just ... so I went on to see if it was 85. I knew that the decimals ... I don't know ... I went on to see if 85 would work. I just realized that that wasn't it.
251: repeating what subject said	4	251	I: That 0.91 wasn't it.
		252	S: Yeah.
		253	I: So when you multiplied by 85 you got 6389.11. Did that tell you anything?
255-6: subject applies former algorithm but does not develop it any further.		255	S: Ah ... I don't know. It was just kinda ... a different answer. It was ...
257: previous direction again but with different result.	12	257	I: You didn't get the same answer, heh. And it's too big.
		258	S: Yeah.
259-60: continuing direction of line 257.		259	I: Does that tell you anything? Did you look at whether it was too small or too big or anything like that?
261-2: explains reasoning quite frankly.	65	261	S: Yeah, I was looking at that too. And then I ... I realized that I really didn't know what I was doing, so I went on to problem 4.
263: extending from subject's calculations.	13 10	263	I: In No.10 you're doing problem 4, heh?

No.10. 307. x 4. = 1228. **			265	S: Yeah.
	266: probing reasoning.	9	266	I: Tell me what your idea was in there. How you got that idea.
	267-9;271-3;No.10: continues algorithm; gets a bright idea.	48 41 49	267	S: Well I was just taking the last number, see I got the idea here that maybe it was just the last decimal number that they were multiplying.
	270: repeating what subject said with reference to printout.	4	270	I: In problem 3, that maybe it was the last one?
			271	S: Like I thought maybe it was the last number that they were multiplying it by. But then I realized that it wasn't that one but I just decided I'd try that one anyways.
	274: clarification.	9	274	I: In problem 4.
No.11. 82. x 85.91 = 7044.62 **	275-7;No.11;No.12: finds the error and repeats the successful algorithm on other problems as checks.	40 48 54	275	S: Yeah. So I went 307 times 4, and I got the answer. And then I realized it must have been 6. So I went up and I did the rest of them with a 6.
No.12. 131. + 538. + 934. + 728. = 2331. ** 2331. + 27. = 2358. **	278;280: reference to results.	10	278	I: So in No.10 you're leaving off the 6.
			279	S: Yeah.
			280	I: And you get 1228.
			281	S: Yeah.
	282: extending from results.	13	282	I: And that agrees with the mistake, heh?
			283	S: Yeah.
	284: probing reasoning.	9	284	I: So then you figured it was which key?
			285	S: The 6.
			286	I: So then you go back.

No.11. 82. x 85.91 = 7044.62 **			287	S: And I went 82.
	288: reference to printout.	10	288	I: In problem what? In No.11, which problem are you checking?
No.12. 131. + 538. + 934. + 728. = 2331. ** 2331. + 27. = 2358. **			289	S: 3.
			290	I: And tell me what you did.
			291	S: I took ... OK, I went 82 times 85.91.
			292	I: You left off ...
	293: evidence of analysis.	55	293	S: The 6. And it came up with the same answer that they had here.
			294	I: So it agreed.
			295	S: Yeah.
			296	I: It looks like you're onto something. Doesn't it?
	297-8: explains reasoning.	65	297	S: Then I went up here and I left out 267 and I added all the other ones together.
	299: reference to printout.	10	299	I: Yes. This is No.12 now. Go on.
	300-2: deductive reasoning good when she represents 267 as 27.	40	300	S: And then it came, it didn't come quite to that number. So I added 27, I just left the 6 out. And that, it came up with the same answer (i.e., as in problem 2).
	303-4: probing reasoning.	9	303	I: Very nice. So in other words you did all the numbers that don't have a 6 first. Was that what you were planning?
			305	S: Yeah.
	306: extension.	13	306	I: And then you said I'm gonna deal with the one that has the 6.

307-8: explains reasoning behind 27 entry instead of 267. Deductive reasoning already noted.		307	S: And then I just figured well if they're not using the 6 then probably just went to 7.
309: related question to test reasoning.	5	309	I: OK, can I ask you a question? Why didn't you use 207?
310-11: deductive reasoning previously noted.		310	S: Because I figured if they, if they weren't using, like if the 6 wasn't working, I wouldn't go as 0. I would just go 27.
312: repeats question of line 309.	4	312	I: Did it cross your mind that you might have to put a 0 in there?
		313	S: No.
314: extending for subject.	13	314	I: Just that nothing would go in.
315-9: good explanation of reasoning.	65 40	315	S: Yeah, well the calculator when, like when you press, like if I went like that and I didn't press that one hard enough and I went to that one it would be the same thing as if the 6 wasn't working (i.e., illustrating what she means on the keyboard of the calculator).
320;323-4;326-7: clarifying.	9	320	I: You're showing me now on the calculator you press the 2 and you registered that.
		322	S: Yeah.
		323	I: You went to press the 6 but you didn't press it. So you're saying what?
		325	S: It doesn't work.
		326	I: So you're using the display of the calculator to illustrate to me what 267 would be.
		328	S: Yeah.
		329	I: OK. You didn't fall into a trap a lot of kids fell into.

		330	S: What was that?
331-6: explaining related problem.	1	331	I: A lot of them started working with this 0. You know what I'm very interested in? I don't know whether I have it yet. I'm interested in ... like ... do you think that you stumbled by chance in No.10, do you think you stumbled just by chance that it was the 6?
337-8: indication of previously presumed random selecting.		337	S: More or less, yeah. I was just kind of fiddling around with the numbers seeing what would happen.
339: clarifying.	9	339	I: If what?
		340	S: If I just left off one number.
341-2: trying to establish randomness of selected keys.	9	341	I: Right. But you couldn't ... there was no reason why you left the 6 off?
		343	S: No.
		344	I: You just tried it?
		345	S: Just trying the last number. And then -
346-7: reference to results.	10	346	I: Like you had tried in the one before, heh. You had tried leaving off the 85; it didn't work.
		348	S: Yeah.
349-50: reviewing subject's calculations.	11	349	I: So you tried leaving off 0.91. That didn't work. So you were moving around trying them.
		351	S: Yeah. If 4 didn't work I was gonna use the 6. Like 307 times 6 and see if that came up with the same answer.
353: clarification.	9	353	I: You'd leave off the 4 then?

		354	S: Yeah.
355: probing reasoning but with some extension.	13	355	I: So when you hit it, then you realized what was going on.
		356	S: Yeah. As soon as I got that printoiut I realized that it was the 6 and then just went on.
358-9: probing continued.	9	358	I: It's very interesting how you got it. All the way up to No.10, would you say that you were stumped?
360-3: randomness firmly established.	94	360	S: Yeah, I was (laughing). I really didn't know what I was doing. It was just kinda by chance that I came across ... I was ... all of a sudden it clicked that maybe it was the last number and ... that was causing the problems in there.
		364	I: So you really were searching for ideas, heh?
12:38		365	S: Yeah.

Bruce Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:19				1	I: Alright, what I want you to do is read the problem over and see if you understand what it's asking you.
				3	B: OK (starts reading the problem aloud).
				4	I: No, no. You don't have to read it out loud.
		05: questions conditions of problem.	20	5	B: Like which input key or which typing key?
		06-08: affective statement. Explanation of a condition of the question.	3 1	6	I: That's a good question. The input keys. In other words, keys that you have to press to get information into the calculator not keys the calculator has to use to give you answers.
		09-10: continues questioning conditions.		9	B: OK. So you want me to work it through this (i.e., the printing calculator)?
		11: questioning understanding.	9	11	I: Now, do you understand really what it's doing.
		12-13: restates problem	21	12	B: Well, it's trying to see, well, like if, maybe the 2 key doesn't work or something.
				14	I: Right. You got it. OK. So what I'll do now is I'll leave you alone to work on it. And if you think you've got the answer let me know, and if you're really stuck –
		17-18: another good condition specifying question.	20	17	B: Which, all these problems? Is it the same key for all these problems?
				19	I: Yes, that's what I wanted to see if you understood too. That's right.
				21	B: OK.
					CALCULATOR OPERATED (No1), (2 minutes).
					Like the answer I printed through here –
				23	I: Yeah.

No.1. 41.7 + 5079. = 5120.7 **	24-7;No.1: applies a checking procedure to problem 1. Shows deductive reasoning in dealing with calculated result.	47 40 55	24 B: Corresponds to the answer that one that you put on there (i.e., the given problem). Then that one indicates to me that none of the keys are amuck. That answer, if that answer is right then that means that the thing is not mucked up.
			28 I: OK.
	29: requires reassurance from the investigator.	84	29 B: So now I go on to problem 2, heh?
			30 I: You got the idea.
			CALCULATOR OPERATED (No.2), (8 minutes).
No.2. 131. + 267. + 538. + 934. + 728. = 2598. ** 2598. = 3326. **	31-2;No.2: makes a good deduction based on results of the printout and then questions the conditions of the original problem based on No.2. Hits the equal sign twice.	40 20 100	31 B: When your key doesn't work what is the effect? Does it print 0 or does it count as nothing or ...
			33 I: I can't tell you that; you have to figure that out. There's ways.
	35-36: deductive reasoning.	40	35 B: So the key might print or might not do anything, or might print a different number.
	37-38: encouragement.	3	37 I: You can figure it out. Like if you ask me that question you're well on the way to understanding what's happening.
			38 B: So it can do one of two things: print a different number which would mean it mucks things up.
	40: works a hint from the investigator.	8	40 I: I'll tell you right now it doesn't print a different number.
	41-42: seeks clarification of conditions (i.e., wants more hints).	21	41 B: It doesn't print a different number. Does it change the value - would it change the value of it (thinking aloud to himself)
	43: clarification.	9	43 I: What do you mean by the value?
			44 B: If I hit the 3 would it print or like, no that's OK. So that means it wouldn't work ... so OK ...

CALCULATOR OPERATED (Nos. 2 to 8), (12 minutes).

Comment		Line	Transcript
46: asking for clarification of untaped portion.	9	46	I: What did you just tell me about the process of elimination?
		47	B: Like, what do you mean by that?
		48	I: Well, you said –
49-53: still having trouble with an understanding of what malfunction means. Can see the deductive reasoning in operation.	83 40	49	B: Like you go through all the numbers in the calculator, like, if I did ah, process of elimination, I'm saying if I hit, OK, if I typed out-131 and I hit the 1 key, only the 3 would be printed because the I, if that key was not working then that number would not show up or would not register.
54;56: extending reasoning of the subject.	13	54	I: So you'd get it that way.
		55	B: Uh hum.
		56	I: So you'd keep on trying keys until –
57-58: describing his reasoning and developing an algorithm to deal with malfunction. There is a clear strategy developing.	65 40 47	57	B: OK. Whenever I would show up in any of these numbers I'd just skip it, or whenever 2 shows up I'd skip that –
59-61: rewording what subject said.	4	59	I: That's the way you'd check. To see which one, that's what you mean by the process of elimination. OK. Do you, well that might be a method; a way of doing it. Any other questions?
		62	B: About that? No.
63-64: reference to previous discussion. Direction given.	12	63	I: No, not necessarily about that. You just asked me something a minute ago. What was it?
		65	B: I don't know.
66: repetition.	4	66	I: Oh, is it the same key in each problem?

			67	B: Yeah.
	68;70-1: this information is implied in the question itself.	82	68	I: Yes.
			69	B: It is.
			70	I: If you get, if it's not working in a problem, it'll be the same key in each problem.
			72	B: Well, OK, what, it doesn't work ... the keys are all working in that problem.
	74: clear direction.	12	74	I: So then that tells you something about those particular keys.
	75-76: uses the direction.	40	75	B: Those are all working. Ah, so that helps me, I see. So that means I have a choice of ... 2, 3 or 6 or an 8.
	77: clarification.	9	77	I: You're looking at problem 1, heh?
	78-81: putting problems into his own words; analyzes calculator results.	21 35	78	B: 2, 3, 6 or an 8. OK. So if all those numbers work, and that works out, then it has to be 2, or 3 or 6 or 8 that doesn't work (talking aloud to himself). So then I'll just try those numbers.
	86-87: reference to printout.	10	82	I: Alright. Let me just, let's stop for a minute, maybe we can go over part of what you did (advancing and removing the printout). And then I'll leave you to work on it. Let's just see what you did, I'll give these different numbers, and if you can, try to explain to me what you're doing. Like in No.1 what are you actually doing?
No.1. 41.7 + 5079. = 5120.7	88-9;No.1: reaffirms checking procedure stated. Uses checking procedure and then develops a difference strategy.	47	88	B: That one's problem 1. I'm checking to see if the answer, what the difference in the answers is. To see -
	90-91: clarification.	9	90	I: In other words the answer on my, on the printing calculator and the answer in the problem.

			92	B: To see -
	93: clear interruption to question reasoning.	2	93	I: And what was the difference?
	94-95: evidence of analysis.	55	94	B: The difference on that one wasn't anything cause that all worked.
No.2. 131. +	96: reference to printout.	10	96	I: That well worked. What are you doing in this (i.e., No.2)?
267. +	97;99;No.2: checking problem 2.	48	97	B: Well that was just checking it out to see what the -
538. +				
934. +				
728. =	98: clarification.	9	98	I: Checking what out?
2598. **				
2598. -			99	B: The answer to this.
3326. **				
	100: clarification.	9	100	I: The answer to which problem?
2958. *	101-2: identifies error.	54	101	B: 2. And I don't know what I did. I hit the equals button there and printed that and I don't know what happened there.
2598. -				
2358. =				
240. **	103-4: review of printout.	10	103	I: Holy cow, heh. In other words the first time you hit the equals button you got 2598, right.
	105;No.2: exploratory manipulation.	22	105	B: Then I decided to see what would happen -
	106: completing what subject says.	2	106	I: If you pressed the equals again.
			107	B: Yeah.
			108	I: Ah, do you know why?
			109	B: Why?
	110: reference to printout. Explanation of what the calculator actually does.	10 1	110	I: You see the last thing you did before the equals was add 728.
			111	B: OK.

119-20: calculator misentry.	100	118	I: So you discovered something neat there. What are you doing in this second part of No. 2? Where you have a - the single star there means you misprinted, heh?
		121	B: Yeah.
122: related question.	5	122	I: OK. So you misprinted 2958. So you really wanted which one?
		123	B: I wanted 2598.
124: reference to printout.	10	124	I: And what were you doing here?
125-6: developing a difference process already mentioned.		125	B: I was finding what the difference was between the two answers was which was 240.
127: probing reasoning.	9	127	I: OK, Is there a particular reason why you were interested?
128-32: analyzes printout in terms of place value. 129: affective comment.	65 55 66	128	B: No, I was just checking that cause I figured, well I don't know it was kind of dumb, but I was thinking there might, well OK. If it was say 2, and there was no 2's in the one column, 2 in the second, I think that well, it might work out that way. That there would show up -
133-4: seeking clarification.	9	133	I: OK. You're looking at problem 2 now. And explain it to me again.
135-6;138: deductive reasoning evident in the use of a place value technique for finding the malfunction.	40	135	B: I say there was no 2's in that column then there would not show a difference, I don't think.
137: repeating subject.	4	137	I: A difference in the units?
		138	B: A difference in the answer. Unless I -
139-40: interruption: related question to probe reasoning.	2 5	139	I: Is there a difference in the answers in that particular column?

141: clear answer to related question.	50	141	B: Yeah there is a difference by 240.
		142	I: Yeah, but, watch this, use your idea, you just told me.
		143	B: I know.
1[illegible]lear direction.	12	144	I: There's a 0 there in 240.
145-6: doesn't know what his algorithm is producing.	92	145	B: Yeah, I know. OK. And if saying that there were, I was looking at 2's in that column to see if ... I don't know.
		147	I: It's OK, just tell me the idea, yeah.
		148	B; But if there were no 2's in that column and there was no difference in it, then it would show 2 in there, right. And if there was, OK, say two 2's in that column, then it would show 40, the difference.
		152	I:Yeah.
		153	B: If there was a 2 in that column it would show 240 the difference total. See what I'm driving at?
155: asking for clarification.	9	155	I: No. Say it again.
156;158-61: algorithm now becoming clearer. Tries linking the difference to what the bad key can be. Deductive reasoning applied to printout.	40 55	156	B: If there was one 2 in that column, like there's that 2.
		157	I: In the 267.
		158	B: No just say if it was anywhere. OK. In the hundreds column, and there was 2's in the tens column, that would make 40, and if there was no 2's in the ones column then the difference in that would make it 240.
		162	I: Is that what you were thinking of?
		163	B: Yeah, that's what I was thinking of.

	164-9: reviewing what subject said.	11	164	I: In other words you're looking from the difference to spot which key might be wrong. Like if they differ by 4, two 2's makes 4. If they differ by like, you're saying 240 that might be 1 and two 2's, so you're trying to locate, OK, I like that idea, it's very interesting, I haven't seen that before.
	170: identifies error.	54	170	B: But it doesn't work. Cause there's only one 2.
No.3. 11. + 167. * 267. + 58. + 94. + 728. = 1158. **	171-2: reference to printout.		171	I: OK. You know, you sort of gave up on it but I see what you're getting at. Now what are you doing in No.3?
	173;175-6;No.3: trying to find the bad key by eliminating a possibly bad key as determined in No.1. Algorithm developed and tried unsuccessfully. Note misentry.	47 92 33 100	173	B: That one, I ran through problem 2 again.
			174	I: No you haven't.
			175	B: OK. That one I was trying the process of elimination. Like removing the 3 from it.
			177	I: OK, so let me just see what happens.
	178-80;184: explains reasoning.	40	178	B: I was saying, basing it that if the 3 key didn't work and I typed 131 I'd get 11 instead of 131. And then 267 worked. 538 -
	181: reference to calculation.	10	181	I: But, what happened - oh! 167 is a misprint.
			182	B: Yeah.
			183	I: Then you put 267. Go on.
			184	B: Then if I typed out 538 I'd get 58 instead of 538.
	185-6: affective statement.	3	185	I: Yeah, see you're right on. This is very good what you're doing.

No.3. 11. + 167. * 267. + 58. + 94. + 728. = 1158. **			187	B: 94 instead of 934. 728 I'd get that. Then I'd get 158 (really means 1158).
	189: related question.		189	I: So No.3 is really a test of which key?
	190: too simple a question.	50	190	B: Testing key number 3.
	191: probing reasoning.	9	191	I: Is there a particular reason that you suspected key 3?
	192-3: based on this statement, there is random selection of candidate keys.	94	192	B: No, I was just, I just noticed it, looked like a likely number, so I did it.
No.4. 82.6 x 85.91 = 7096.166 ** 7096.166 - 7044.62 = 51.546 **	194: reference to printout.	10	194	I: OK. Just by picking it. Alright, No.4, what are you doing?
	195: repetition.	86	195	B: I'm running it through again. To see -
	196-7: interruption.	2	196	I: Oh, just a sec, sorry to interrupt you, do you get anything from this 1158?
	198: not evaluating results.	96	198	B: Ah, not directly, no.
	199: direction.	12	199	I: It's not the answer, heh?
	200: appears to be postrationalizing.		200	B: No. But there is what, 1200 difference.
	201-2;204-5: probing reasoning.	9	201	I: OK. Does that mean anything? It's actually 1100, no 1200, you're right. Does that tell you anything?
			203	B: No, it doesn't. It only means I'm missing 1200 somewhere.
			204	I: OK. When you're missing 1200, you don't have 1200 enough, does that tell you anything?
			206	B: ... not really, no.
			207	I: OK.

			208	B: I'm missing 1200 ...
No.4. 82.6 x 85.91 = 7096.166 **	209: reference to printout.	10	209	I: If it doesn't, it doesn't. In No.4 what are you doing?
7096.166 − 7044.62 = 51.546 **	210;No.4: checked problem 3 and applied the difference algorithm again.	48	210	B: I tried to find out what the difference was.
	211: clarification.	9	211	I: So, of which problem?
			212	B: Problem 3.
	213-4: review of method described	11	213	I: So you multiplied it out. And you made a subtraction. Now does this difference tell you anything?
	215-7: deductive reasoning applied to results.	40 55	215	B: Ahm ... 51.546 ... tells me that ah ... there has to be a difference in the multiplication or the decimal point, decimal end of it I think.
			218	I: Why? Explain that idea a little more, it sounds very interesting.
	220: explains reasoning.	65	220	B: Like, OK, the answer, according to this is 7044.62.
			221	I: In the problem 3.
	222-8: his explanation is muddled with the difference idea combined with the effect of place value. This blocks him from seeing directly which key is malfunctioning. He has the answer but is not certain.	95	222	B: Printed here. And actually I got 7096.166. So that means there's 51.546 difference, so that means there has to be ... .546 difference somewhere, that well, I don't know, how it'll work out, between the keys that are malfunctioning. So that I don't know, it strikes me that something has to be within the 6 or the 9 or the 1 to cause it to do that. And I think it has to be the 6.
			229	I: Why?
			230	B: Well, because −

	231-3: trying to determine if the expressed reasoning was made during calculations.	9	231	I: OK. Let's just go back. Something has to be in the 6, the 9 or the 1. In other words, in the decimal places. Is this an idea you have right now?
	234-5: admits to postrationalizing.		234	B: Yeah, I think so, because, looking at this one, all those work, and so my options are 2, 3 -
			236	I: Looking at problem 1.
	237-9: spots the potential bad key from previous work and not from his difference strategy. Finds the answer with some uncertainty.	40 50	237	B: My options are 2, 3, 6 and 8, and since that's the only one, 6, I presume 6 is the only one of those three, so I think that 6 would be the one that's not working.
	240: related question.	5	240	I: OK. So what would you do then?
			241	B: For what?
No.5a. 307. x 64. = 19648. **	242-3: reference to printout.	10	242	I: OK, Let's go back and look at No.5. What are you doing in No.5?
	244-5;No.5a: check of problem 4.	48	244	B: I'm just checking out the actual answer against, cross-checking it against the one on the paper.
No.5b. 7. x 64. = 448. **	246-7: reference to printout.	10	246	I: Problem 4. And you got 19648. Now what did you do in here? Right after 19648 (i.e., No.5b).
No.5c. 64. x 30. = 1920. **			248	B: I ... mucked up quite badly.
	249: countersuggestion.	6	249	I: No, I don't think so.
	250;No.5b: repeats previous work of eliminating the 3.	48 92	250	B: I was thinking that if the 3 didn't work ...
	251: from No.3 he should know it's not the 3.	93	251	I: Yes ... so it's another 3 test.
	252-4: checking reasonableness of his answer. Uses previous concept in lines 178-80.	40 31	252	B: If the 3 didn't work when I typed in 0 nothing would show up and when I typed in 7 it would just print 7. Then 7 times 64 was what? 448.

No.5a. 307. x 64. = 19648. **	255-6;258-9: probing reasoning with reference to calculations.	10 9	255	I: OK. So you're really making another 3 test. What are you doing in - you got 448. Did that tell you anything, the 448?
			257	B: No, it didn't.
No.5b. 7. x 64. = 448. **			258	I: Alright. Then let's go on. 64 times 30. Is there a reason why you did that?
No.5c. 64. x 30. = 1920. **	260;No.5c: making a test of the 7 key by eliminating it from 307. Note that the order of multiplication has been reversed.	48 92	260	B: I was testing the 7.
			261	I: So you left out -
	261: interruption.	2	262	B: 7.
	263: reference to printout.	10	263	I: OK. Now you switched the order in which you multiplied.
			264	B: Did that have any relevance on it?
	265: clear direction.	12	265	I: Don't think so. What do you think?
			266	B: No, don't think so.
			267	I: It's just numbers, heh?
	268: getting into present rather than past exploration of ideas.		268	B: Well, it would have a difference if I switched the position of the numbers around.
No.6. 3. + 267. + 538. + 934. + 728. = 2470. **	270-1: related question. Extending from subject's calculations.	5 13	270	I: But you didn't do that. So really you've left off the 7, you're testing the 7. Is it the 7?
	272-4: answers related question. Demonstrates deductive reasoning. Means 1228 instead of 1128.	50 40 100	272	B: No, ah, well it's not the 7. Cause you're saying if the 7 didn't work and I did 30 times 64 I'd get 1128 but I got 1920 unless something's wrong in here.
	275: reference to printout.	10	275	I: No, no our calculator is good. Now what about in No.6?
	276;No.6: repeats algorithm.	48 92	276	B: I was doing a 1 test.

No.6. 3. + 267. + 538. + 934. + 728. = 2470. **	277: clarification.	9	277	I: So you're testing the number 1. So it's really problem 2.
	278-81: there is no 1 key in problem 1 so he is not using what he observed previously. No reasoning other that size for key selection.	96 94	278	B: Oh, problem 2, OK. So now I'm starting to take in order ... the numbers so I did that. And the only ones that I came across were in 131, so I left those out, and so I just ran it through the rest of the way. And I got 2470..
	282-3: too much direction.	12	282	I: If you compare to what you got in No.2 you're getting closer aren't you (from 3326 to 2470), heh?
			284	B: Yeah.
	285: questioning reasoning.	5	285	I: Did that mean anything to you?
			286	B: No, it didn't, not directly.
	287-8: reviewing.	11	287	I: As far as you're concerned, you left the ones out, you didn't get the answer you wanted. Do you go on right after that?
			289	B: Yes I did.
No.7. 131. + 67. + 538. + 934. + 78. + 1748. = 3496. **	290: reference to printout.	10	290	I: So in No.7?
	291;No.7: using the same method as previously on problem 2.	48 92	291	B: I'm doing a 2 test.
	292: reference to printout.	10	292	I: So you left the 2 out here. You left it off 267, 278.
			293	B: 728.
			294	I: Did it work out?
	295: continuing interviewer's direction from lines 282-3.		295	B: I was getting closer.
			296	I: Yeah, 728, right.
			297	B: I was getting closer but I was still what, ah, I went over.

No.7. 131. + 67. + 538. + 934. + 78. + 1748. = 3496. **	298: related calculator query.	5	298	I: Ah, but do you see why?
			299	B: No, no.
	300: calculator error specified	100	300	I: You hit the plus one time too many.
			301	B: I did?
	302-3: review of calculations.	11	302	I: Yeah, watch. 131, 67, 538, 934, ... you should have said equals right here.
			304	B: Ah.
	305-7: questioning reasoning based on calculations.	10	305	I: It'll do it. So in No.7 really it's the 1748 you're interested in. Did you notice that, that 3496 looked kind of big or that you had added 1748?
No.8. 11. + 167. ** 267. + 58. + 94. + 728. = 1158. **	308-9; No.8: repetition of previous calculations. Not analyzing calculated results.	86 96	308	B: No, I didn't really notice. No.8 is just a recheck of No.3. So I did it over.
			310	I: No.3. Is there ... if you did the same thing again, heh?
			311	B: Oh yeah.
			312	I: You're hitting sort of equals, you're hitting the plus at the wrong time. Now from the talk that we've just had do you have any other ideas you want to try now?
			315	B: Not off hand.
			316	I: What were you just telling me before?
			317	B: About elimination?
	318: reference to previous statements.	4	318	I: Yeah, there was one key you were suspicious of.
			319	B: Ah, the 6, because of problem 3 and that ...

			320	I: Yeah, OK. So why don't you try something with that.
			321	B: (Talking to himself while operating the calculator)
				CALCULATOR OPERATED (No.9).
No.9. 82. x 85.91 = 7044.62 **	No.9: uses elimination method successfully on problem 4.	48 50		82 point, so it would be point 0, times 85.91. 7044.62. So it is the 6 key in that one. So now I check through .... you got problem 2. Sorry.
	325: review of calculations.	11	325	I: OK. So you've worked out what's wrong with problem 4. OK.
			326	B: Problem 3, yeah.
			327	I: Problem 3, you're right.
			328	B: So now I go through the rest of these.
				CALCULATOR OPERATED (No.10).
No.10. 131. + 27. + 538. + 934. + 728. = 2358. **	No.10: got problem 2 and not only used elimination but avoided 207 trap.	48 50	329	I: Now you hit the equals, heh Bruce? So you got that one too.
	330: recognizes solution.	54	330	B: So that's 6.
				CALCULATOR OPERATED (No.11).
				So according to this it's the 6 key that's out of order.
No.11. 307. x 4. = 1228. **	331: knows he has the answer.	54	332	I: And you proved that nicely too. You see to be honest with you - let's just go back over these and you'll tell me what you did. I think I almost know what you did. What are you testing in No.9?
	334-5: reference to printout.	10		
	336: explains reasoning.	65	336	B: I'm testing the 6 key.
	337-8: clarification.	9	337	I: And you're leaving it off. So this is a check of which problem?

No.10. 131. +			339	B: Problem 3.
27. +	340: probing reasoning.	9	340	I: And what do you notice?
538. +				
934. +				
728. =	341: checks for correctness.	55	341	B: That the answer works out.
2358. **	342-3: reference to printout.	10	342	I: On both the printout and the problem. And what did you do in No.10?
No.11. 307. x	344;No.10: repeats algorithm.	48	344	B: I left out the 6 again.
4. =				
1228. **			345	I: And you made, the only number that has 6 –
			346	B: Was 267.
	347: reference to calculations.	10	347	I: So you made that 267: what did you make that?
			348	B: I made that to 27.
	349-50: reference to printout.	10	349	I: And that worked out to be the ... you got the wrong answer twice. And then in No.11, you're checking which problem?
	351;No.11: repeats algorithm	48	351	B: Problem 4, And I did 307 times 4.
	352-9: review.	11	352	I: And got the same wrong answer. What I can see is that you know what to do. What I mean by that is you've started eliminating in here, like if you look at No.3 and particularly I liked No.5, you left off the 3. You were cooking, heh. You almost got it. By chance, what would have happened if you'd left the 6 off? You'd get it right away. In your own words, one last thing I want you to tell me, why were you suspicious of the 6?

360 B: Well because when I ran over problem 1 and you told me that all those keys work so that told me that I had the numbers 2, 3, 6 and 8 left that had possibilities. Then I went to problem 2, and I don't know, it was just a feeling that the 0.62 and 0.546 and so I thought it had to be one of those three because the decimal point really got out of whack. So I didn't think it would be the 1 that was causing the problem because 1 times anything is still the same, so I didn't think of that off-hand. So I don't know, I thought it would be the 6. Because another thing was the 1 key worked, because of the 41.7 –

371 I: In problem 1, yeah.

372: identifies previous error. 54

372 B: And the 9 key worked. So that left only 6.

373 I: In problem 3. And you sort of feel that your mistake is on the decimal side (i.e., to the right of the decimal). Like 0.6 or 0.91. And again, why did you think it was on the decimal side?

3:54

377 B: I just felt that it was quite a bit of difference between the two. Like it was 0.166 and 0.62 so there's almost a hundred decimal points. Whatever terminology you care to use.

Clive Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:13				1	I: What I want you to do is read the problem over and see if you understand it. Then if you do understand it I'm going to leave you alone to work on it and I'm going to go there (i.e., up front to my desk) and if you think you have the answer or if you are having really a lot of trouble, if you've been trying it and you can't get it, just give me a call.
		07: questions conditions of problem.	20	7	C: So I just add up all these and then ...
		08;11: clarification.	9	8	I: Well read what it says. Did you read it?
				9	C: Well, no.
					PAUSE (20s).
				10	I: Do you think you understand it?
				11	C: Yeah, I think so.
		12: repeating question.	4	12	I: What's the problem ask you to do?
				13	C: Add up those ... well saying that one of these keys is not working right.
		15–16: probing understanding and giving a hint.	1 5	15	I: And from the problem you should be able to find, - there's only one key that isn't working, and what you try to figure out -
		17: continues questioning conditions.	20	17	C: You see all these to figure out which key is -
					I: Well that's what you have to try to figure out, what you want to do, whatever ideas you get you just use the calculator, don't worry about it. And then afterwards if you think you've got which key it is then we'll go over it.
					CALCULATOR OPERATED (Nos. 1 to 14). (11 minutes).
					So you've been working on it for a while so I think we'll just go over what you've got. Now what did you just ask me?

	24: misinterprets the question.	82	24 C: I thought that you had to find the missing number for each one.
	25: probing reasoning.		25 I: Did you think it was a different number in each one?
			26 C: Yeah. Because like the one missing in there was 4, for that one and ...
No.1. 41.7 + 5079. = 5120.7 ***	31-32: reference to printout.	10	28 I: OK, let's go over it and see what you've got (advancing and removing the printout). You got a lot of work here anyways. What I want you to do is try your best to explain to me what you did in each one. So what were you doing in this No.1? This is problem 1 and I'm going to call this printout No.1.
	33:No.1: check of problem 1.	47	31 C: Well, I just added that one up.
	34: repeating what subject said.	4	34 I: You added problem 1 up. And what happened?
	35: evidence of analysis.	55	35 C: Nothing was wrong with it.
	36-37: probing reasoning.	5	36 I: You got the same answer again. So did you conclude anything from that, did you get any ideas from that?
	38: deductive reasoning but error in that misinterprets the problem as meaning the bad key not bad in certain problems.	40 82	38 C: The key must have been working in that.
	39;41: clarification.		39 I: Which key?
		9	40 C: The one that wasn't working, I guess.
			41 I: Must have been working for that one?
			42 C: Yeah.
	43-44: assisting by explaining the problem.	1	43 I: No. If it's not working, it's not like it's going to be working in some and not working in others (i.e., problems).
			45 C: Oh, oh well ...

			46 I: You follow?
	47: doesn't understand an explained aspect of the problem.	83	47 C: No.
No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	49: reference to printout.	10	48 I: OK. Let's just leave that one a minute and go on to No.2 here. What did you do in No.2?
	50;No.2: check of problem 2.	48	50 C: I just added that one up too.
	51: probing reasoning.	9	51 I: And what happened there?
	52;61: checks for correctness but otherwise does not analyze results.	55 96	52 C: I just got the wrong answer but I went on to the last ...
	53-54: reference to calculations but clear direction given.	10 12	53 I: Well, you did something though. You got 2598 and here you have 2358. Right? So what does that tell you?
	55: deductive reasoning.	40	55 C: That one of the keys was missed over.
	56-57: probing reasoning with related question.	5	56 I: Or something's going wrong there. Did you have any ideas what might be going wrong there?
			58 C: No.
			59 I: Just that something is going wrong. So what did you do after that?
			61 C: I just went on to the next one.
No.3. 82.6 x 85.91 = 7096.166 **	62: reference to printout.	10	62 I: So in No.3 you're testing problem 3. And what happened there?
			63 C: I got the wrong answer.
	64: clarifying between the two answers.	9	64 I: Well, the answer you're getting on the printout, this is the right answer.
			66 C: Well, yeah.

	67-68: partly directing.	12	67 I: So that one, the one in the problem is wrong. But you're getting different answers. Does that tell you anything?
	69: deductive reasoning.	40	69 C: Well, one of the keys wasn't working there again.
	70,72: clarification.	9	70 I: You still don't know which one?
	71: knows he has the answer.	64 50	71 C: It was number ... number 6 wasn't working right.
			72 I: In this one here?
			73 C: For that one.
	74-76: reference to printout for clarification.	10	74 I: In problem 3. Did you figure that out though in this part (i.e., in No.3)? Did you know yet that the 6 wasn't working in here, in No.3?
			77 C: No.
	78: extending from subject's statements.	13	78 I: You just saw it didn't match.
			79 C: Yeah.
	80-81: probing reasoning with direction.	12	80 I: Did you look for any reasons why it didn't match? Or you just saw it didn't match?
	82: follows given direction.		82 C: I just saw it.
No.4: 307. x 64. = 19648. **	84: reference to calculations.	10	83 I: So you didn't worry about it after that. You just figured it didn't work. Then in No.4 what did you do?
	85-6;No.4: just a routine check of problem 4.	48	85 C: I just ... times the number, see what it would come out in this one.
	87-88: clarification.	9	87 I: In other words you checked problem 4. And what did you notice? What happened?

No.4. 307. x 64. = 19648. **	89: starts paying attention to amount answer is off: the familiar difference strategy? Analyzes the result.	55	89	C: Well the answer was a lot off there.
		5	90	I: So like when you saw a lot off -
			91	C: This one was, yeah.
	92-93: rewording what the subject said.	4	92	I: Problem 4 was way off. So you're not just looking at whether it's off or not but how much it's off?
			94	C: Uh huh.
	95: clarification.	9	95	I: When you saw it was a lot off, did that mean anything to you?
	96-98: deductive reasoning.	40	96	C: Yeah, it must have skipped one of the top digits or one of the bottom one (i.e., 307 considered as the top number and 64 as the bottom one). For this one.
	99-101: probing reasoning.	9	99	I: For problem 4. Can you tell me why you think it just have skipped them? I don't know, maybe you're right, heh? Why do you think that it must have skipped one?
	102-3: fails to recall the key that he actually found was wrong. Will be established in future lines.	100	102	C: Well that answer would have been ..., like if you had had the 4 taken away, then it would have answered right.
			104	I: If it had 4 taken away?
			105	C: Yeah.
	106: clarification.	9	106	I: In problem 4?
			107	C: If I skipped the 4.
	108: extends from previous statement by the subject.	13	108	I: In other words if you just had the 6?
			109	C: 6 times, yeah, 307.

No.5. 30. x 64. = 1920. **	110: reference to printout.		110 I: Yeah. Let's just see what happens here, OK. What did you do in in No.5? Looks like you've got an idea here.
	107;112-3;No.5;No.6: changes approach and starts the familiar elimination strategy applying the algorithm twice. Successfully in No.6. Uses relative size (difference) as a guide to which digit to eliminate.	47 40 55	112 C: Which one's that? Oh. I was just seeing what they come to: 64 times 30 to see if it was 7 that was skipped out.
			114 I: And you got 1920. Did that tell you anything?
			115 C: I was sort of close I guess then.
			116 I: Yeah, but they aren't equal, heh?
	114: probing reasoning based on calculations.	10	117 C: No.
	115: indication of successive approximations strategy.	55	118 I: Well, what do you get from that? When they aren't equal?
	118: related question.	5	
	119: deductive reasoning.	40	119 C: It's not 7 that was missed.
No.6. 307. x 4. = 1228. **	120: reference to printout.	10	120 I: So then you went on and in No. 6 what did you do?
			131 C: I did 307 times 4.
			122 I: So what did you skip there?
			123 C: I skipped the 6.
			124 I: And what happened?
	125-6: realizes he has the right answer.	54	125 C: It was off ... well it wasn't ... oh, what am I doing. Well it was right, yeah.
	127-8: clarifying the timing behind the reasoning.	9	127 I: Did you think up 'til now that it was off? Like was this the first time you noticed that the two numbers are the same?
			128 C: No, I just got mixed up.

No.6.	307. x 4. = 1228. **	130-1: probing reasoning by directing to printout.	10	130	I: That's OK. What does this tell you now, when you get 1228 in both?
		132: deductive reasoning. Note reference is only to problem 4.	40	132	C: That in problem 4 they missed the 6.
		133-4: back to find the reasoning when protocol first began.	11	133	I: Now, did you think it was different keys in each problem that might be wrong?
				135	C: Yeah.
				136	I: Why?
		137: disregards information given in the original question.	82	137	C: I thought that it just be one key (note: the question does specify that one key is malfunctioning, but means that it is the same key in each problem) that's missing in each problem.
		140-1: extends reasoning of subject.	13	140	I: Like when they say which key isn't working that it's only one key but it could be a different key in each one?
				142	C: Yeah.
No.7.	89. RM	143-4: reference to calculations.	10	143	I: Let's go on. What did you do in No.7? You put something in, oh you recalled something from memory.
		145;No.7: calculator misentry.	100	145	C: Something happened there, I don't know what happened.
		146-9: explanation of what may have happened.	I	146	I: Well maybe you just pressed the button. You remember I had starred 89 in there (i.e., at the beginning of the session the interviewer showed Clive how to use the memory with the number 89 and this was still in the memory register).
				150	C: Yeah, that was it.
No.8.	85. x 822. = 69870. **	151-2: reference to calculations.	10	151	I: So you recalled it, heh? So what are you doing in this one here, in No.8? 85 times 822.
		153;155: explaining reasoning.	65		
		Elimination algorithm repeated.	48	153	C: I was just seeing –

No.8. 85 x 822 = 69370.	154: interruption that disturbs flow of subject's thinking.	2	154	I: So you're working with problem 3. And what were you seeing?
			155	C: I was going to see what just those two would come to.
	156: clarification.	9	156	I: Which two?
			157	C: 82 times by 85.
	158: contradiction based on printout.	6	158	I: OK, but that's not what you've done.
			159	C: Oh ... getting mixed up here ...
	160-1: clarification using ideas pieced together from previous statements.	9	160	I: Well, it is 82 you're interested in isn't it? So did you make a mistake in here when you put in 822?
	162;164;No.8: calculator misentry and a modification of the previous algorithm to cover two keys	95 49	162	C: Yeah, I must have. I didn't see that.
			163	I: Did you want to put in 82?
			164	C: Yeah, I just want to put in 82 times 85.
	165: repetition of previous attempts at clarification.	4	165	I: It's not 822 - it should have been 82.
			166	C: Yeah.
	167: using results to probe reasoning.	10	167	I: Alright. When you got 69870 ... what did you notice about that?
	168: using relative size. Should have found error based on this. Was not analyzing result	96	168	C: I was way off there ...
			169	I: So when it was off what did you do?
				PAUSE (10s).
			170	C: Well I didn't know I had the 2 there (i.e., in 822) may ...
			172	I: That's why it was way off, heh!

			173	C: Yeah.
	174-6: probing reasoning, but with direction.	9 12	174	I: OK. What is - like when you see that it's way off - like 69870 - ah ... how do you think like - do you think you just go on to another one or do you try to look at why it's way off?
	177: follows direction.		177	C: Yeah. Why it's way off.
	178: related question.	5	178	I: Did you see any reasons why it might be way off?
			179	C: I must have not looked at that one - didn't see the 2 there.
No.9. 82.6 x 85.6 * 85.9 = 7095.34 **	182-3: reference to printout.	10	180	I: OK. So it's just a mistake ... No.9 - see in these you don't have to worry so much about mistakes - know what I mean ... you are just trying. So what are you doing in No. 9? Which problem are you checking?
			184	C: Same one.
	185: clarification.	9	185	I: Problem 3?
			186	C: Yeah.
	187-8;No.9: calculator misentry.	100	187	I: I see a single star (beside 85.6) means like you didn't want 85.6 -
			189	C: No (agreeing).
	190: clarification.	9	190	I: Is it 85.9 you wanted?
			191	C: Yeah.
	192: probing reasoning.	5	192	I: So you're idea in problem 3 is to do what?
			193	C: Just trying to ... to take away the 1 (i.e., in 85.91) to see if it would make a difference.

No.9. 82.6 x 85.6 # 85.9 = 7095.34 **	193-4;No.9: using previous algorithm that was successful in No.7 on problem 3 but testing a different key (i.e., 1). This is a continuation of the error in understanding the problem.	48 92	195	I: And did it? PAUSE (5s).
			196	C: Yeah, well it came ... it came a bit closer.
	196: explaining reasoning. Still interested in relative size.	65 55	197	I: Well what did you have before? ... all the way back up in No.3, heh? ... so when you left - was there any reason why you picked the number 1?
	197-9: probing reasoning with direction.	12	200	C: I was just sort of randomly choosing them.
	200: guessing rather than using previous results. 201-4: probing reasoning.	96 94 9	201	I: OK ... did you realize at this point that the 6 did not work in problem 4? PAUSE (5s). Did you realize after No.6 that it was the 6 that wasn't working in problem 4?
			205	C: Yeah.
			206	I: You knew that?
			207	C: Yeah.
	208: reference to calculations.	10	208	I: So here we go now in No.10 what are you trying to do? Checking problem 3 again?
			210	C: Yeah.
			211	I: And what are you doing now?
			212	C: I just timesed it out to see what it would come to ...
			213	I: Without ah ... what change have you made in this problem?

				PAUSE (5s).
No.10. 82.6 x 5.91 = 488.166 **	212;214;No.10: checking problem 3 and eliminating the 8 in 85.91. Applying the algorithm again.	48 92	214	C: Well I took out the 8 – yeah.
	215-7: contradicting the implied ideas in the calculations.	6	215	I: Without the 8 in 85.91 ... OK, let me ask you something Clive ... why wouldn't you take out the 8 also in 82.6? ... if it's one key that isn't working, right?
	218: misunderstands original conditions of the problem. Didn't identify error.	82 95	218	C: Oh! ... I didn't notice that.
	219-20: extending from statements and calculations of subject.	13	219	I: So what you were doing is you're working on not, just one key but one number at a time.
			221	C: Yeah.
	222: probing reasoning.	5	222	I: So you got 488.166 – did that tell you anything?
				PAUSE (5s).
			223	C: Well that – that was ... ah ...
				PAUSE (10s).
	224: trying to remove direction		224	I: Or didn't it?
				PAUSE (3s).
	225: extent of analysis of calculated results indicated.	56 96	225	C: It didn't really say that much, no ... thought that was wrong.
No.11. 82.6 x 85.1 = 7029.26 **	226: reference to printout.	10	225	I: You saw that's wrong so in No.11 what's your idea now?
	227;No.11: uses algorithm again; applied to test 9 key malfunction.	48 92	227	C: I took away the 9 to see if it would do any difference.
			228	I: Ah – did it?
			229	C: It came out pretty close – but ...

No.11. 82.6 x 85.1 = 7029.26 **	229: not analyzing printout give successive approximations as he needs more than 7029.26. 95.91 has to be increased as 7029.26 is less than 7044.62, the given incorrect answer.	55 96	230	I: In fact now you're below the number ... in problem 3 ... and then what's cooking in No.12? PAUSE (3s).
			232	C: I took away the 9 and the 2 - and the 1.
No.12. 82.6 x 85. = 7021. **	230-1: reference to printout. 232: repeats algorithm of No.8. Error in understanding original conditions of problem.	10 48 82	233	I: So you're working with two (different) keys ... but it says up there (in the question) that only one key -
			235	C: Yeah - I know.
	236-8: reference to printout.	10	236	I: That's OK, you're trying something just a little bit different ... now we come to No. - ah, you got 7021 - did that tell you anything?
	239: notes relative size.	55	239	C: Just ... came pretty close to getting it.
No.13. 82. x 85.91 = 7044.62 **	240-1: reference to printout and noted inhibition.	10 97	240	I: Then we come to No.13 - and you're still sticking to problem 3 - you've decided it's problem 3 or bust ... so in problem 3 (i.e., No.13) what are you changing?
	243;245;247;No.13: applies algorithm successfully and knows he has the answer.	48 54	243	C: I just took away the 6 -
			244	I: Ah -
			245	C: From 82.6.
			246	I: And what happened?
			247	C: It worked out.
	248-9: review of calculations.	11	248	I: Nice, you've got it. So which problems have you found the error in?
			250	C: This one and that one.

	251: related question.	5	251 I: 3 and 4, heh? And in both cases which number was it?
	253: error in recalling previous work.	95	253 C: One was 4 and one was 6.
	254-6: review of previous work.	11	254 I: Let's go back and take a look ... where did you get the answer - you got the answer in No.6, right ... what key did you leave off in that one?
			PAUSE (5s).
			257 C: Which one was that (referring to the printout)?
	258-60;262-3: review continued.		258 I: OK, let's look at problem 4, heh. You checked it (i.e., problem 4) in No.6 and you got 1228 in both cases. So which key did you leave off?
			261 C: The 6.
			262 I: It was the 6 in problem 4, heh? Go to No.13 - which key did you leave off?
			PAUSE (3s).
			264 C: The 6.
No.14. 6. S1 4. S1	265: reference to printout.	10	265 I: 6 again. What's your idea in No.14?
			266 C: I just -
	267-8: clarification.	9	267 I: So up to this point you thought it was the 6 and the 4 - two different keys?
			269 C: Yeah.
			270 I: Now why did you think it was the 4? Any reason for that?
			PAUSE (5s).

			You just thought that, heh?
		272	C: Yeah.
273-5: extending based on the subject's present and past statements.	13	273	I: Like what I think - if I understand it - I think what happened is you took off the 6 but in your mind you thought it was the 4 you had taken off.
		276	C: Just wrote down the wrong one.
No. 14: calculator operation error in that one value can be held in memory at a time.	85	277	I: Why do you store 6 and store 4 (i.e., in No. 14)?
278; No. 14: exploration.	22	278	C: I just wanted memory - to keep in memory -
279-80: extending continued.	13	279	I: Are those the 6 and the 4 like that you ah ... that you're thinking don't work? Is that why you're storing them?
		281	C: Yeah.
282-4: related question.	5	282	I: Now let's go on. So you've really worked out what's wrong in problems 3 and 4. Do you think right now - with me - that you could figure out what's the other problem that's wrong?
		285	C: Ah, 2 (i.e., referring to problem 2).
		286	I: Do you think you could figure out what's wrong in problem 2?
			PAUSE (10s).
287: finds the final error.	54	287	C: I think it's the 6 right there (i.e., in 267).
288-9: related question repeated.	4	288	I: OK ... if it is the 6 ... could you try that problem again and see if you could get 2358 ... in the calculator?
		290	C: Yeah.
		291	I: OK. Go ahead. See what you can do.

294: questions conditions.	2	294	I: Would that be [illegible] then [illegible] number 267)?
		295	I: That's what I want to see - you try a couple of things and see what's going to happen. You want me to leave you with it a while?
		296	C: Yeah.
			CALCULATOR OPERATED (Nos. 15 and 16). (1 minute).
297-8: repeating subject's previous question.	4	297	I: So we're going on - it only took you about a minute ... so you finished off by just asking me is the 6 a 0. Is it?
		299	C: No.
		300	I: What is it? ... what should you do?
301-2: answers related question.	50	301	C: Well, it would just be ... it just skipped it then - it would be 27.
303-5: reference to printout.	10	303	I: It didn't enter the 6 at all ... so we're just going to go over what you got here - cause I was watching you. So in No.15 what change did you make?
306-7: No.15: redoes previous work. Error in repetition.	86	306	C: Well I just added up that sum - just added up all the numbers here (i.e., in problem 2).
308-9: contradicting the approach taken by the subject.	6	308	I: Just to see what it would become - but you had already done that one time here -
		310	C: Yeah -
311-31: some direction.	12	311	I: In No.2 ... but that's OK, so you just did it again. I thought in No.15 you'd entered 0 (i.e., made 267 into 207) but you didn't ... so you realized -

No.15. 131. +
267. +
538. +
934. +
728. =
2598. **

	314-5: continues direction.	50	314 C: Yeah I realized it - it wouldn't print 0 if it wasn't working at all.
No.1[illegible] 131. + 27. + 538. + 934. + 728. = 2358. **	316-7: reference to printout.	10	316 I: So you didn't bother even entering it. And then in No.16 ... you went and what did you do?
	318;No.16: modifies algorithm so that it is successful.	49 54	318 C: Well, I just skipped the 6. Instead of 267 I just put down 27.
			319 I: And it worked out ... so No. 16 is just a check?
12:41			320 C: Yeah.

Ryan Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:14				1	I: If you think you understand it I'm going to leave you with it.
					PAUSE (20s).
					Do you think you understand it?
				3	R: Yeah.
				4	I: I'll leave you for a while on it.
					CALCULATOR OPERATED (Nos. 1 to 13).
		05-06: clarifying a previous point raised in the pretaping.	9		We'll just go over part of it and see what happens. You think you're stuck?
				7	R: Yeah.
				8	I: Why (advancing and removing the printout)?
		09-10: shows deductive reasoning and develop a checking procedure.	40 55	9	R: I got down ... and I noticed that ... ah ... problem 1 it worked out ... so these numbers would have to be correct.
		11: clarification.	9	11	I: Which numbers?
		12-13: distinguishes between input and output keys.	55	12	R: These wouldn't be - these don't have to be (i.e., the numbers in 5120.7); they're the answer.
		14: repetition of subject.	4	14	I: So in problem 1 ... 5120.7 - that's the answer.
		15: continues explanation.		15	R: Which leaves 2, 3, 6 and 8.
		16: clarification.	5	16	I: Why does it leave 2, 3, 6 and 8?
		17: answers related question	50	17	R: Cause the others are in here (i.e., he points).
		18: probes reasoning using related questions.	5	18	I: 4, 1, 5, 7, 0 and 9. And they all work. How do you know that?
				19	R: Cause they worked out to the correct answer.

Printout	Analysis	Code	Line	Transcript
No.1. 41.7 + 5079. = 5120.7 **	20,22: relating to printout.	10	20	I: So you tried - in No. 1 - the problem 1 again?
			21	R: Yeah...
			22	I: And the answer you got here (i.e., in No.1) -
	23: explains reasoning based on printout.	55	23	R: Was the same as here (i.e., that given in the problem).
	24-28: review what subject said	11	24	I: So the answer on the printout is the same as in the problem and that - you just told me what that meant. From that you could tell which keys are working and which ones are not, but you still haven't got it down to one key. In No.2 what are you doing?
No.2. 131. + 267. + 538. = 936. ** 936. + 934. + 728. = 2598. ** 2598. M+ 82.6 x 85.91 = 7096.166 **	27-28: reference to printout.	10		
	29-32: No.2: continues checking procedure and analyzes results.	48 55	29	R: Ah ... I ... added the numbers that they had here to find out what answer I had gotten, to see if this one (i.e., problem 2) was OK ... and it didn't work out ... that didn't tell me which number might -
	33-34: reference to printout.	10	33	I: What did you get instead of the 2358 in the problem - what did you get on the printout?
			35	R: 2598.
			36	I: You did this addition a little funny - like you added the first two -
	38: No.2: calculator misentry; identifies error and shows understanding of calculator operation by modifying.	100 54 49	38	R: I ... meant to press plus but I pressed equals instead.
	39-42: reference to printout and explaining.	1.	39	I: But it'll work - doesn't it? ... all you're doing then is carrying over the - you're adding - finding out what two numbers add to ... and then adding two more numbers to that ... no, one more -
			43	R: Three.

No.2. 131. + 267. + 538. = 936. ** 936. + 934. + 728. = 2598. ** 2598. M+ 82.6 x 85.91 = 7096.166 **	44-45: continued explanation on the operation of the calculator.		44 I: Two more numbers to that - 934 and 728 ... so you did the five ... you added three numbers first -
			46 R: Three ... and then two to the sum of that.
	47: reference to calculation	10	47 I: What did you do here where you've got M+?
	48-49: forgets his idea.	95	48 R: ... I don't know - I can't remember now ... I was going to something but then ...
	50;52: explanation of calculator operation and symbols.	1	50 I: M stands for memory.
			51 R: Memory ...
			52 I: So were you going to do something with the memory?
			53 R: ... I don't know what it is ... I was going to do something but ... I couldn't ... I was going to do something but then I couldn't remember ...
	56: directing.	12	56 I: How to do it?
			57 R: Yeah ... no ... it wouldn't work out on it ... I thought it through and it wouldn't work out - I can't remember what it was ...
	60: clarification.	9	60 I: So this is an idea that's lost now?
			61 R: Yeah.
	62-64: reference to printout.	10	62 I: You can't remember ... what you were trying to do ... what did you do in the second part - the last part of No.2? Right in here - 82.6 times -
			65 R: I was doing problem 3 -
			66 I: Checked that (i.e., the problem).

65;67;No.2: routine check of problem 3; deductive reasoning illustrated.	48 40 55	67	R: Checked this and it -
68-69: probing reasoning.	9	68	I: Like Ryan when you saw that this one didn't agree, heh, did that tell you anything?
		70	R: Ah ...
			PAUSE (7s).
71;73: illustrates deductive reasoning.	40		It told you that one of the keys is in this ...
72: clarification.	9	72	I: What one of the keys?
		73	R: One of the keys that isn't working. The same in problem 3.
74: repeating what the subject had previously said.	4	74	I: Now what did you tell me the keys could be?
75: answers related question.	50	75	R: 2, 3, 6 and 8 ... and 3 isn't there ...
76: probing reasoning.	9	76	I: Then what does that tell you?
77-80: some confusion but good deducing continued.	40	77	R: 3 couldn't be ... 3 would have to be working ... 3 wouldn't have to be working but in this problem it's not used ... it still couldn't be working ... or no ... there's only one key ... so 3 would have to be working.
81-85: repeating what subject said and probing reasoning but with direction.	4 12	81	I: So from problem 3 you eliminated 3 as one of the ones that wouldn't be working - in other words it should be working ... did you - from problem 2 did you learn anything? ... like when you saw 2598 and the numbers you added up, did you learn anything from that?
			PAUSE (4s).
86-88: deductive reasoning continued.	40	86	R: They're all in there ... all the numbers are in there ... 2, 3, 6 and 8 ... are all in problem 2 ... so it could be any of them.

No.3. 307. x 64. = 19648. **	89-90: reference to printout.	10	89	I: So you still haven't reduced any of those ones. What did you do in No.3?
	91;No.3: checks problem 4.	48	91	R: I went to check this -
			92	I: Problem 4 -
	93: shows analysis of results.	55	93	R: And it came out wrong.
	94-95: reference to printout.	10	94	I: It came out to 19648 on the printout and it's what in the problem?
			96	R: 1228.
	97: probing reasoning.	9	97	I: Does that tell you anything?
	98;100-2: good illustration of deductive reasoning. Identifies error.	40 54	98	R: ... 8 or the 2 ... 8 and 2 couldn't be the ones ...
			99	I: Why?
			100	R: Cause there's only one key - and I forgot that when I went on to ... work it out later on - I forgot that there was only one key.
	103-4: clarification by repetition.	4	103	I: You mean you forgot there was only one key that could be wrong later on -
			104	R: Yeah, and I -
			105	I: You thought there could be more than 1 -
	106-7: explains reasoning; identifies error.	65 54	106	R: And I went to taking out the 3 and that - now I see that I should have -
	108-9: clarification.	9	108	I: But you realize that just now, heh? Not while you were doing all this work?

				110	R: I noticed it down here ... when I was working this out –
		111: clarification.	9	111	I: When you were working problem 2 out?
		112-7: explains reasoning.	65	112	R: I was working problem 2 out again down here ... I just looked at this and I noticed 3 couldn't ... would have to be working ... cause it ... I was having problems with number 2 so I read this over again (i.e., the question) and there's only one key ... and then I noticed that 3 ... I think it was my next one –
No.4.	7. x 54. * 64. = 448. **	118-9: reference to printout.	10	118	I: Let's take a look at that ... No.4 ... OK, in No.4 – you just told me what ... taking the 3 out?
		120-1; No.4: developing an elimination process for testing which key is malfunctioning.	47	120	R: That the 3 wouldn't be working which would leave it to 7 (i.e., in 307).
		122-3: clarification with reference to printout.	9	122	I: And you multiplied – instead of 307 what did you change it to?
				124	R: 7 times 64.
		125-7; No.4: calculator misentry.	100	125	I: The single star means you printed ... 54 when you wanted 64 – so you did what I told you (i.e., printed errors rather than erasing them) ... when you got this 448 ... and it's –
				128	R: 1228.
		129: probing reasoning.	5	129	I: Does that tell you anything?
		130: deductive logic applied to calculated results.	40 35	130	R: 3 would have to be working.
		131-2: repeating subject.	4	131	I: It couldn't be the one that's creating the error. Now let's go down to – did you realize that –
				133	R: Yeah.

		134-5: review.	11	134	I: So after printing No.4 you knew it wasn't the 3 that's not working.
		136: recalls former problem; interrupted.	53 2	136	R: And then I went back to 3 in –
No.5.	307. + 4. = 311. **	137: reference to printout.	10	137	I: Let's take a look at No.5. In No.5 what are you doing?
		138;140-2;No.5: explains reasoning. Calculator misentry. Redoes No.4 using elimination procedure.	100 48 54	138	R: Ah ... I took out the 6 ... yeah, I took out the 6 ...
		139: hint.	8	139	I: And look what you did Ryan ... can you see something in there ... look at this problem ...
				141	R: Oh! Oh I added it – that was it, and I noticed that ... cause the next one (i.e., No.6) I multiplied it.
		143-4: reference to printout but extending subject's ideas.	10 13	143	I: So when you added it – so in No.5, you're working to see if which key doesn't work?
				145	R: ... ah, 6.
		146-7: reviewing subject's calculations based on printout.	11	146	I: And you added by mistake and got 311 – you realized though that you should have multiplied?
				148	R: Yeah – in the next one –
No.6.	307. = 307. **	149: reference to printout.	10	149	I: In No.6 ... you're printing 307 equals 307 –
	307. x 4. = 1228. **	150;153-4;No.6: identifies error; applies algorithm successfully to 6 key; knows he has the answer; calculator misentry. Shows deductive reasoning.	48 54 40 100	150	R: I goofed up again and pressed the equals.
				151	I: So then you just did it again in No.6. Now look, something interesting is happening.
				153	R: It worked out ..., which means the one that I took out would have to be the missing key.
		155: simple related problem.	5	155	I: Or the key that's not working. And which key did you take out?

				156	R: The 6.
		157-8: review of subject's statements.	11	157	I: So then you proved in this one that problem 4's error is created because the 6 doesn't work. Now –
No.7.	82. x 85.91 = 7044.62 **	159;No.7: check of problem 3 using successful algorithm. Knows he has the answer.	48 54	159	R: I went to check problem 3 –
				160	I: No. 7 you checked problem 3 by doing what?
		160: reference to printout.	10	161	R: Taking out the 6.
		162: clarification.	9	162	I: In which number?
				163	R: The top.
		164: trying to determine if the subject was analyzing the results.	5	164	I: 82.6. And did you create what you should have got?
				165	R: Yeah ... and then –
		166-7: review and extension of Nos. 6 and 7.	13	166	I: So you corrected the errors – you found out how the errors were produced in problem 4 and problem 3.
				168	R: And then I went to problem 2.
		169-70: review of what subject originally said in line 6.		169	I: And you're saying it's in problem 2 you're having your trouble?
		171: repeats previous process		171	R: Yeah, I took out the 6.
No.8.	131. + 207. + 538. + 934. + + 728. = 2538. **	172-3: review of calculations in printout.	11	172	I: Let's just take a look at what you did in No.8. You took out the 6 and what ... did you put in there instead?
				174	R: ... zero.
		175: repeating what subject said.	4	175	I: So you're replacing the 6 with a 0. And what happened?
		176: identifies error.	54	176	R: Oh no ... I see what's wrong. I should have –

			177	I: What's wrong?
	178-81: gets bright idea; changes existing algorithm so it becomes successful.	41 49	178	R: I should have - I noticed now how it probably worked. ... 131 plus 27 - if the 6 isn't there you're still - you're not going to put in the 0 because you're absent-mindedly not knowing it.
	182-7: reviewing previous discussion; probing reasoning.	11	182	I: I want you to try that now - your idea you just told me about. CALCULATOR OPERATED (No.14), (40s). You got it (advancing and removing the printout). Can I ask you a question? You realized now what you should have done ... like that you should have made it 27 instead of 267 ... let's go back - there's something in here that triggered that - what made you realize that just now?
	188-9;191-3: explains reasoning behind the bright idea.	65	188	R: ... I noticed that ... if you were ... punching in ... 267 ... and not knowing the 7 was not working ...
	190: correction.	6	190	I: The 6 was not working -
			191	R: The 6 was not working ... you wouldn't put a 0 in ... or the calculator wouldn't put a 0 in for you and it would only go to 27.
No.9. 131. + 207. + 538. + 934. + 728. = 2538. **	194-7: repetition and review.	4	194	I: In other words if the key isn't working it's not going to replace it with another key ... 0 ... it's just not going to anything. And you realized that where - when we were just doing No.8 here?
			198	R: Yeah.
	199: reference to printout.	10	199	I: OK, what were you doing in No.9?
			200	R: It didn't work - I ...

No.9.	131. + 207. + 538. + 934. + 728. = 2338. **	201;No.9: repeats previous work.	86	201	I: It's really the same thing ... except for this plus, heh?
		202-3;205-7;No.9: explains reasoning behind printout. Sees double + as a possible error (it actually isn't as the + key does not have a constant function.	54 55	202	R: Yeah ... I noticed ... I pressed it once and it went again. I looked at it -
		204: clarification.	9	204	I: You pressed what once?
				205	R: Plus ... I pressed it twice (i.e., in No.8) I guess - I didn't really -
		207: clarification.	9	207	I: So is No.9, No.8 again without that extra plus?
		208;210: subject is generally having problems with the calculator.	85	208	R: I thought that might have done something -
				209	I: I understand -
				210	R: Not knowing the calculator -
No.10.	131. + 7. + 538. + 934. + 708. = 2318. **	211-2: reference to printout.	10	211	I: Just checking it (i.e., the work in No.8) over. Now in No.10 what were you doing?
				213	R: ... then I got back to which key isn't working and I -
		214: extension of comments.	13	214	I: You thought it might be more than one -
				215	R: Yeah -
		216: clarification.	9	216	I: So which one -
				217	R: So I took out the 2 (i.e., in No.10).
		218;220-1: reference to print-out to establish that subject attempts to eliminate two keys contrary to instructions.	10	218	I: And you took out the 6 -
				219	R: Yeah -
				220	I: Both of them, heh? You thought it might be two keys not working at the same time?

	222-3; No.10: explains reasoning and based on results (i.e. taking out 6 did not produce result) he modifies algorithm. Note 2 has been removed from two numbers.	65 49 92	222	R: Cause I knew it was the 6 ... so I took out the 2 because it didn't work that way.
	224: reference to printout to probe reasoning.	10	224	I: When you got 2318 ... in No.10 ... did that tell you anything?
	226-8: error in recalling from printout.	95	226	R: Yeah, it told me that it couldn't be 2 or ... it might be the 2 and not the 6 ... no, no it couldn't be the 6 ... no ... I trying to think.
	229: introducing contradiction in review.	11	229	I: Well you've eliminated two of them - the 2 and the 6.
	230-1: makes deductive conclusion.	40	230	R: So it would have to be the 2 ... you'd have to take the 2 back.
	232-3: extending from subject's statements.	13	232	I: In other words it can't be the two together that's causing the problem.
	234: analyzes size of the result during the discussion.	55	234	R: Cause it's too low (i.e., 2318).
	236-7: clarification.	9	236	I: So what are you doing in No. - what do you mean it's too low - the 2318?
			238	R: Yeah.
	239: related question to probe reasoning.	5	239	I: Is lower than?
			240	R: 2358.
	241: extending from subject's comments.	13	241	I: So you're looking at the size of the numbers too.
No.11. 131. + 7. + 538. + 934. + 708. = 2318. **	242-3; No.11: repeating previous work.	86	242	R: Then I looked at it ... and in the next one (i.e., No.11) I added it all over again.
	244: clarification.	9	244	I: In No.11?
			245	R: Yeah, to see if anything ...

No.12. 131. + 307. + 438. *	246-8: extending from subject's statements. Reference to printout.	13 10	246	I: So you really redid problem 10 in case there was something going wrong with the calculator ... what are you doing in No.12?
	249-50;252;254-5;259;No.12: misentry in calculator and pressed star after pressing + so the total was calculated. He identified the error.	100 54	249	R: I had ah ... punched in 131 and then instead of 207 I put 307 in -
			251	I: By mistake -
			252	R: Then I pressed P ... and it gave me 438 ...
	253;256-7: explaining calculator operation.	1	253	I: (Laughing) it added then - it's not supposed to is it?
			254	R: No ... and I noticed that - and I thought about it - I just cleared it -
			256	I: I told you to press P so it would clear and it added so I didn't give you the right instructions (laughing) -
			258	R: I just cleared it ... cause I didn't need that ...
No.13. 131. + 207. + 538. + 934. + 728. = 2538. **	259: reference to printout.	10	259	I: And then in No. 13 what are you doing?
	260;No.13: repetition because he doesn't understand the operation of the print function.	85	260	R: Trying it again.
			261	I: You're really, actually -
			262	R: Trying ... it's the one up here - 9 -
			263	I: And 9 -
			264	R: And 9.
	265: probing reasoning.	5	265	I: Is there a reason why you went back to it again?

	266-8: explains reasoning; says can't solve the problem.	63	266	R: ... I knew it couldn't be the 2 ... and I was just trying to convince myself that ... I was stuck on it ... just to see ... that last one ...
No.14. 131. + 27. + 538. + 934. + 728. = 2358. =	269-70: reviewing printout in reference to earlier discussion.	11	270	I: Now this last one, No.14 — this is the one you just did, when you got an idea in here somewhere ... I forget exactly where.
			271	R: No.9.
	272: clarification.	9	272	I: No.5 — that's where you got the idea —
	273: explains reasoning.	65	273	R: I noticed that it wouldn't work there.
12:49	274-80: review of subject's work.	11	274	I: And so you took your idea of just not — like you were saying not even entering the 6 and just making it 27 — it worked, heh? You see if you want to look at the question it says which key isn't working ... you found the key ... but you were having more trouble — you were not only trying to find the key you were trying to find what caused the mistakes and that is where you ran into a bit of trouble.

Lisa Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:26				1	I: Now you're going to read the problem and if you have any questions you'll ask me right now and then I'll leave you alone to do it.
		03-04: error in understanding the problem; misinterpretation.	92	3	L: OK. It's read over. One of these (i.e., the problems) are wrong, right?
		05-06: explains the problem.	1	5	I: No. You'll try the problems and one of the keys isn't working right. And I want you to find out which one it is. OK?
		07-08: has the answer but is cautious.	95	7	L: I guess so.
					CALCULATOR OPERATED (Nos. 1 to 7), (9 minutes).
					Would it be the 6?
		09-11: questioning subject's certainty.	7	9	I: OK. You think it's the 6. Ah ... let's go over ... OK. We're stopping at 3:36. How come you didn't say to me that you had it or that you thought it was the 6?
				12	L: Well I was ... see if I was right.
				13	I: It is the 6.
				14	L: OK (laughing).
		21-22: questions reasoning based on printout.	10	15	I: So you know what you're doing. What I want to do with you is just take this out of the calculator, right (advancing and removing the printout). And go through with you what you did; what ideas you had in different parts. So I'll show you different parts of it and you try to tell me like ... just what you were trying to do. Whatever it was. So we'll look at the first part. We'll call it No.1. And what were you doing in just this part here (i.e., the first three entries)?
				23	L: Ah ... seeing if they had the right answer or the wrong answer.
				24	I: OK. What do they have?

No.1. 41.7 + 5079. = 5120.7 **	23;25;No.1: routine check of problem 1. Analyzes result for correctness.	47 55	25	L: The right answer.
	26-27: related question based on printout.	10	26	I: OK. Now, when you see that, like the two answers 5120.7 for both, does that tell you something?
	28;30-1: demonstrates deductive reasoning.	40	28	L: That everything was working right.
	29: clarification.	9	29	I: What do you mean by everything?
			30	L: Well in the calculator that they used here all the numbers that they punched were working.
	32: related question.	5	32	I: Which numbers were working?
	33: answers related question.	50	33	L: The 4 the 1 the 7, all of them ... on this side.
			34	I: On the left.
			35	L: Yeah.
	36: related question to see if subject distinguishes between input and output malfunction.	5	36	I: Why not on the right?
	37: answers question correctly	50	37	L: Because that's the answer. You don't have to push any buttons.
	38-40: reference to printout.	10	38	I: That's right (laughing). Very good. OK. No.2. Ah, so in this one (i.e., No.1), you know that there are certain keys that are not working right. So which keys could be working wrong then?
	41: repetition of previous correct work.		41	L: Ah ... the 0, the 6 ... that's all ... 3.
			42	I: Yeah.
			43	L: 8 ... 2.
	44: clear direction given.	12	44	I: 3, 6, 8, 2 ... go on. Anything else? Don't think so, heh?
			45	L: Don't think so.

No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	46: reference to printout.	10	46	I: So let's look at what you did in No.2 then.
	47; No.2: explains reasoning; repeats checking process and analyzes result.	48 -55	47	L: See if the answer is right.
			48	I: The answer to which problem?
	48: clarification.	9	49	L: Problem 2.
	50-51: review of previous work.	11	50	I: So all you've done is entered everything that's in problem 2. And came up with the answer.
			52	L: Yeah.
	53-54: probing reasoning with some direction to printout.	9	53	I: Your answer on this ... on the calculator is 2598. The answer in the problem is 2358. What does that tell you?
	55-56: demonstrates deductive reasoning,	40	55	L: Something went wrong (laughing). One of the keys ... one of the numbers in there is wrong.
	57: probing reasoning.	9	57	I: Do you know where it is wrong? ... Yet?
			58	L: No. No.
No.3. 82.6 x 85.91 = 7096.166 **	59-62: reference to printout.	10	59	I: So you just knew that something was wrong and that's all ... the only ideas you got in there? You had no ideas about it? OK. We'll look at No.3. What were you doing in there?
	63-64: strategy: check all of the problems to see if there are any errors.	65	63	L: See if the answer is right. That's what I did first in all of them.
	65: repeating what subject said.	4	65	I: So you're doing problem 3.
			66	L: Yeah.
			67	I: Now when you get this answer (i.e., 7096.166) does it agree with what's there (i.e., in problem 3).

			69	L: No.
	70-72: direction given to subject.	12	70	I: OK. When it doesn't-agree ... does this No.3 on the printout or No.2 on the printout or No.2 and No.3 together tell you anything? Did you look at them together to tell you anything?
			73	L: No.
	74: repeating subject's earlier statements.	4	74	I: You just were checking. All you were interested in is that.
	75 No.3: check of problem 3.	48 55	75	L: To see if it was wrong or right.
	76-77: review.	11	76	I: So that's in this one ... in No.2 you found that problem 2 was wrong in No.3 you found that problem 3 was wrong.
			78	L: And problem 4 was wrong.
No.4. 307. x 64. = 19648. **	79: reference to printout.	10	79	I: And in No.4 you ...
			80	L: Found 4 was wrong.
	81-82: probes reasoning.	9	81.	I: OK. Alright. So you found No.4. You get any ideas from the answer you get in here (i.e., 19648 in No.4)?
	83-4 No.5: gets a bright idea.	41	83	L: I took this answer and divided it by that (starting to explain No.5).
	85-86: clarification.	9	85	I: OK. In No.5 ... when you say and this answer divided by that ... which numbers do you mean?
	87: explains reasoning as indicated on printout. Error in calculator entry.	65 100	87	L: Ah ... 7044.62 divided by 85.6 ... no ... 85.91.
			88	I: OK. You've got a single star there ... which means that you printed in the wrong number and ... so instead of dividing by 85.6 you meant to divide by 85.91. Now when you divided, what did you get as an answer?

			92	L: 82.
	93: probing reasoning.	5	93	I: Did that tell you anything?
			94	L: Uh hum.
			95	I: What?
			96	L: 6.
	97: clarification.	9	97	I: What about the 6?
	98-9; No.5: uses deductive reasoning to determine which key is not functioning. Gets the answer. Strategy: by reversing the operation of multiplication, 82.6 can be checked.	55 54 47	98	L: If you punched it in it wouldn't have registered because you got ... you got the answer you would have if you punched in 82.
			100	I: Ah ... OK. So. All you're doing in No.5 then is reversing the multiplication by division ... to see if one of the original numbers you multiplied by is wrong.
	100-2: extends reasoning of subject.	13	103	L: Uh hum.
			104	I: You found what one of them was?
			105	L: Yeah.
	106: clarification.	9	106	I: Which one?
	107-8: explains calculator result and sees solution.	55 54	107	L: The 82.6. Well it was supposed to be 82.6 but it was ... ended up being 82.
No.6. 1228. / 307. = 4. **	109; reference to printout.	10	109	I: In No.6 what are you doing?
	110; No.6: applies the algorithm of No.5 a second time to determine that the 6 is again the key that is malfunctioning.	48 47	110	L: Dividing 1228 by -
			111	I: So this is really which problem you're working on?
	111: clarification.	9	112	L: Problem 4.

			113	I: OK. And what happened there?
	114: evidence of analysis.		114	L: You just get 4.
	115: probing reasoning.	5	115	I: So what does that tell you?
	116: deductive reasoning.	40	116	L: That ... either never punched in the 6 or it didn't work.
No.7. 131. + 267. * 207. + 538. + 934. + 728. = 2538. **	117-9: reference to printout.	10	117	I: OK. But once again it's the 6, heh? OK. So you found in both of of these that it's the 6 that isn't working. Now in No.7, what's happening?
	120-1: misinterprets the problem. No.7: calculator misentry.	82 100	120	L: It didn't ... well ... I tried not to punch the 6 but it didn't work out.
	122: clarification.	9	122	I: When you say you tried to punch the 6, what do you mean?
	123: evidence of analysis.	55	123	L: Yeah, like ... I put 207 instead.
	124-6: reference to calculations.	10	124	I: Oh, I see, so this single star once again, ah ... printing error. Right, you made a mistake there. so when you're putting 207, you replaced the 6 with a 0.
			127	L: Uh hum.
			128	I: And you got 2538.
			129	L: Which is wrong.
	130: review of subject's statements.	11	130	I: So it still doesn't agree.
	131: gets bright idea during discussion.	41	131	L: Yeah. It might work if you put in 27.
			132	I: OK. Why don't you try that.

CALCULATOR OPERATING (No.8).

	133: identifies solution.	54	133	L: It worked.
	134-6: review of previous calculations to probe reasoning.	11	134	I: So you got the problem. It's the 6 key. Can you see ... let's go back to what you did here in No.7. When you print 207, what were you doing?
	137-9: explains her reasoning; identifies error.	65 54	137	L: I was just taking the 6 out and it wasn't really ... the person would have been doing it wouldn't know the 6 wasn't working so they'd just put in 2 and 7; they wouldn't ...
	140-1: explaining how the problem works.	1	140	I: So really when you put a 0 in there in No.7, you're holding the place. See?
No.8. 131. + 27. + 538. + 934. + 728. = 2358. ≈			142	L: Yeah.
	143-5: reference to printout.	10	143	I: But your idea in this last one you just did, in No.8, you realized that you can't hold the place, there's no 6. You just did what you told me. What did you replace in No.8?
	No.8: uses her bright idea to test problem 2 and see if she can get the wrong answer with a functioning calculator.	48	146	L: 267.
			147	I: With ...
3:44			148	L: 27.
			149	I: And it worked out, heh?

Ruth Problem 4

Time/ No.	Printout	Comment	Code)	Line
1226		01: clarification.	9	1 I: OK. I've shown you the problem. Do you think you understand ?
		02: does not understand.	83	2 R: Not really.
		03: clarification.	9	3 I: What don't you think you understand about it?
				4 R: Ah ... I don't know (laughing).
		05-08: explanation.	1	5 I: OK. All they're saying is some calculators make mistakes and they're saying these four problems were done by the same calculator. OK. And from what answers you get in here (i.e the problems) there's one key that is not working.
				9 R: OK. OK.
				10 I: Alright?
				11 R: Ahm.
				12 I: And I want you to find out which one it is.
				13 R: OK.
				14 I: So I'll leave you to it.
				CALCULATOR OPERATED (Nos. 1 to 5), (14 minutes).
				OK. So you've been working on it for about 15 minutes. I think, what we'll do is we'll just go over what you have, OK? And then ... ah ... from that we'll get some ideas of what's happening. Cause you think you have the answer, do you?
		18: questioning certainty.	7	
		19: expresses uncertainty about having the answer.	95	19 R: Well, I'm not sure.
		20: clarification.		20 I: You're not sure, you may have it?
				21 R: Yeah.

	24: repetition to determine understanding.	4	22 I: We'll go over it. And we'll see. I'll just take out what you've got in there (advancing and removing the printout). OK, just off hand, we'll go over this. What key do you think is not working?
	25,27: subject repeats uncertainty		25 R: The 6 I think ... I don't know for sure.
			26 I: Alright. We'll see if you've got it here. Ah, it is the 6.
			27 R: Is it? Oh, OK.
No.1. 41.7 + 5079. = 5120.7 **	28-29: reference to printout.	10	28 I: But I just want to see how you figured it out. So we'll call this No.1. Can you tell me what you did in there?
	30-1;No.1: checked problem 1.	47	30 R: OK. I punched two numbers here to see if it equalled it on paper. Just checked it out.
	32: clarification.	9	32 I: Checked which problem out?
			33 R: Problem 1.
			34 I: OK. And what happened?
	35: analyzes result.	55	35 R: The same answer.
	36-37: repeating what subject said	4	36 I: So ... ah ... you got the same answer on the calculator as in the problem, right?
			38 R: Uh hum.
	39: questioning reasoning.	9	39 I: Did you make any conclusions from that?
	40-41: illustration of deductive reasoning. Also, begin to see a strategy emerging.	40	40 R: That ... OK ... the key that isn't working couldn't be the 4, 1, 7, 5, 0 or 9.
			42 I: Why?

	43: explains reasoning.	65	43	R: Because it got the answer.
	44-46: repeating what subject said and extending.	4	44	I: So if you get the answer there ... if the answer you get on the calculator and the answer in the problem are the same then you think all those keys must be working.
			47	R: Uh hum.
	48: compliment.	3	48	I: OK. I think that's very logical.
			49	R: Whatever.
No.2. 131. +	50-51: reference to printout.		50	I: Alright. So let's go on to No.2 and you can tell me in your own words what you were trying to do there?
267. +				
538. +				
934. +	52: repeats checking procedure and analyzes result for correctness.	48 55	52	R: I just worked it out to see if it came out with the same answer.
728. =				
2598. **	53;55;57: clarification.	9	53	I: Which one did you work out?
			54	R: Problem 2.
			55	I: And what happened?
			56	R: It didn't.
			57	I: What did it give you?
			58	R: 2598.
			59	I: And what was the problem 2 answer on the paper?
			60	R: 2358.
	61-62: questioning reasoning of the subject.	9	61	I: OK. When you saw the two answers weren't the same, heh, did that give you any ideas?

No.2. 131. +
267. +
538. +
934. +
728. +
2598. **

63: deductive reasoning.	40	63 R: Yeah, that something had to be added wrong or whatever.
64-5;68: questions reasoning.	[illegible]	64 I: OK ... now ... ah, did you have any ideas yet what it might be that wasn't working?
66-7;69: extends reasoning of lines 71-2. Shows a new line of deductive reasoning. She compares solution (i.e.,2598) with known results (i.e., 2358).	40 55	66 R: Well ... it had to be one of those numbers in the tens or hundreds place. 68 I: Why? 69 R: Because the ones place adds up right. You know, to get the 8.
70-1;73-5: extends subjects reasoning. Probes subject's reasoning based on generated results. Asks related question.	9 5	70 I: Ah, alright, ... so ... in other words you got an 8 given in the problem. 72 R: Ah hum. 73 I: And an 8 given here (i.e., in No.2). So then you don't think it's the units. OK. Do you have any candidates yet? Do you have any ideas what it might be? 76 R: Uh hum.
77: clarification.	9	77 I: From that problem, from problem 2?
78: answers previous related problem.	50	78 R: 6 or 3 or 2.
79: contradiction to see if she can identify her error.	6	79 I: Ah, you're sure 2 is one?
80: intuitive flash.	41	80 R: Oh!
81-83: related question based on line 79.	5	81 I: OK. What I'm getting at, in problem 1 right, you've got a 2 in the answer. But does that mean that the 2 could still be wrong?
84: identifies her previous error.	41	84 R: Yeah. Because you didn't punch it in with the calculator.

	85-86: repetition.	4	85 I: Right. That the calculator gave it out. OK. I wanted to see if you saw that. Say again which ones you think could be wrong.
			87 R: The 6, 3 and 2.
	88: clarification.	9	88 I: So those are the ones you were thinking of from then on?
			89 R: Yeah.
No.3. 82.6 x 85.91 = 7096.166 **	90: reference to printout.	10	90 I: OK. So that was No.2. Now in No.3 what are you doing?
	91;No.3: deductive reasoning illustrated. Repeats checking procedure on problem 3.	48 40	91 R: Well I just worked out the problem and the answer –
	92;94: clarification.	9	92 I: Which problem is that?
			93 R: Problem 3.
			94 I: And what happened?
	95;98-99: deductive reasoning illustrated in a different direction. Compares results.	40 55	95 R: It didn't work out. Like a different answer.
	96: probing reasoning.	9	96 I: OK. You got an answer here that differs from the answer given in the problem. OK. Now did you get any ideas from that?
	98: put together with lines		98 R: Yeah. That it couldn't be the 3. Cause there's no 3 in that equation.
	100-1: repetition of what subject said and a probe of reasoning.	4 9	100 I: Ah! I see what you're saying. Problem 3 has no 3 in any of the numbers. So then, what numbers now are you left with?
	101: answers question		101 R: 6 and 2.
	98: in combination with lines 40-1 and 66 indicates an eliminating procedure that links Nos. 1, 2 and 3.	47 40	102 I: OK. And ... did you then go on to check problem 4?
	102: reference to printout.	10	103 R: Yeah.

	104-5: clarification.	9	104 I: Did you get any more ideas in here? Besides that it couldn't be the 3?
			106 R: Not really.
	107: extending from what subject has said.	13	107 I: OK. So you're sort of working by the process of elimination, heh?
			108 R: Uh hum.
No.4. 307. x 64. = 19648. **	109: reference to printout.	10	109 I: What's happening in No.4?
	110: routine checks; repeats procedure.	48	110 R: I just worked out the problem.
	111: questions reasoning but with direction.	12	111 I: And you got 19648. Did you conclude anything from this one?
	112-4: deduces which key it is by combining the results of all printouts Nos. 1 to 4. Sees common feature in 3 incorrect problems.	40 55	112 R: Well, that it had to be the 6. Cause that had a 6 and that had a 6 and that had a 6 (i.e., all the problems that had wrong answers on the sheet).
	115: related question to probe reasoning.	5	115 I: Why couldn't it be a 2?
	116: illustrates reasoning in her answer to related question.	50	116 R: Because it (i.e., problem 4) didn't have a 2 in the equation.
	117: clarification.	9	117 I: In problem 4.
			118 R: Yeah.
	119: questioning reasoning.	9	119 I: OK. So what change did you make then?
No.5. 131. + 297. + 538. + 934. + 728. = 2628. **	120-1; No.5: Ruth does not stop after finding the key. She retraces her steps to check her work by trying to find the mal-function. Error in that the 9 appears to be chosen randomly.	55 47 94	120 R: Well I ... tried to make the 6 a 9 but it didn't work out. Like you know? (See No.5 which was not discussed).
			122 I: OK. So you still haven't found out what they did that was wrong but you know it's the 6.
	122: reviewing previous work. 124: was told this in line 26.	11	124 R: It's the 6.

125-7: affective statement.	3	125 I: OK. I think your reasoning so far is good. I want to leave you just to try to figure out how you could get those numbers there (i.e., in problems 2, 3 and 4).
		128 R: Oh, OK.
129-31: explaining the question.	1	129 I: The question really doesn't ask that. The question says which key isn't working (i.e., it doesn't ask to find what is the error in each problem).
		132 R: Which key isn't working.
133-5: giving the subject some direction though she already was looking for the malfunction in No.5.	5	133 I: And you found out which one. Now I'm sort of asking you a second question; could you tell me what they did wrong in these problems?
		136 R: Oh, OK.
137-9: repeating question.	4	137 I: In other words could you do something with this calculator to show me how they got those (i.e., the answers in problems 2, 3 and 4).
		140 R: Yes.
		141 I: Right.
		CALCULATOR OPERATED (Nos. 6 to 8). (2 minutes).
142: reference to printout.	10	What have you got?
143-4: explains reasoning; says she has difficulty with problem 2. Analyzes result. Repeats substituting process three times.	50 48 55	143 R: OK. For this ... for the ... third and fourth problem I get worked out with leaving the 6 out. And the second one it didn't.
145: reference to printout.	10	145 I: Let's just advance this for a minute (advancing and removing the printout). Cause you understand what you're doing. So in this in No.6; what did you do there?

No.6. 131. + 267. * 207. + 538. + 934. + 728. = 2538. **	148;No.6: tries replacing the 6 with a 0. This is not the same error as in No.5 but represents an understanding error. 267 is also a calculator misentry. 149: reference to printout. 151-3: review of calculations.	48 92 100 10 11	148 R: I took out the 6 ... made it 207. 149 I: But now, let's look at what you've got here. You got 131, 267? 150 R: But I took it out the point (i.e., note the single star). 151 I: Oh I see. That's just an error and you printed it. And then you went 207. So you took it (i.e., the 6) out and replaced it with a 0. 154 R: Uh hum.
	155: extending reasoning.	12	155 I: And it didn't work out. 156 R: Right.
No.7. 82. x 85.91 = 7044.62 **	157-9: reference to printout and questioning but with counter-example.	12	157 I: OK. Now let's take a look ... we'll skip ahead. And let's look at No.7. Now, here you got 82 point ... 6 and you left the 6 off. Did you replace it with a 0 in there?
	160: explains reasoning and shows she is checking conditions. Analyzes printout. 161: extending reasoning.	40 55 13	160 R: No, cause it won't make any difference, the 0. 161 I: OK. And you got the answer they got, heh? 162 R: Uh hum.
No.8. 307 . x 4. = 1228. **	163-5: questioning reasoning but assuming too much direction in the interview. General review.	11	163 I: Alright. So in No.7 you were able to duplicate the error in the problem, No.8 ... No.7 you duplicated the error in problem 3. In this one (i.e., No.8) you duplicated the problem in?
	No.7: applies the idea she used in No.6 that was not working but becomes successful. 194: interruption. 168: analyzes result and uses deductive logic.	49 2 55 40	166 R: In No. - 167 I: 4. How did you do it? 168 R: Just took the 6 out. 4 and not 64.

		169-70: related question with some direction.	12	169 I: OK. So you see what you're doing wrong here? Or what you're not doing in here in No.6 that you've done in No.7 and No.8?
No.9.	131. + 27. + 538. + 934. + 728. = 2358, **	171; No.9: gets bright idea; sees error in No.5. Uses her idea to find the error in problem 2. Changes existing algorithm, successfully.	41 54 49	171 R: Oh! Make it 27. 172 I: OK. So let's try ... ah ... make what 27? 173 R: That ... 267, 27.
		174: questions certainty.	7	174 I: And you think it'll work then?
		175: uncertain.	95	175 R: I don't know ... might.
				176 I: Alright. So try it and that'll be the last step. CALCULATOR OPERATED (No.9).
		177: questioning certainty again	7	Got it?
		178: certain.	64	178 R: Yeah.
		179-80: review of work.	11	179 I: So you figured the problem out. All you had to do really was go back and look at how you had done 7 and 8 —
				181 R: Yeah.
12:45				182 I: And you have found that. OK. And that's excellent.

Tonia Problem 4.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
12:12				01	I: So what I want you to do is read this problem ... and if you understand it I'm going to leave you alone to work on it for awhile ... and I'm going to go down there (to the desk in the front of the room) ... and when you think you have it solved ... or say you can't get any further - say you're stumped - then you'll let me know.
				07	T: OK.
				08	I: Just read it over and see if you think you understand it.
					PAUSE (10s).
		09: questions certainty.	7		So you think you understand it?
				10	T: Yeah, I understand it. May take me a while to get it done - yeah I understand it.
				12	I: So then I'm going to leave you with it ... and if you think you you get it - you'll let me know ... and if you have a lot of trouble - let me know. So just take your time.
					CALCULATOR OPERATED (Nos. 1 to 3). (4 minutes).
		15: repeating subject's ideas that were untaped.	4		Tell me what you just asked me again ... you don't know how to -
		16-17: mathematics error.	85	16	T: I can't get my decimals to line up - I don't know how I should punch it in to do that.
		18-23: explaining calculator operations.	1	18	I: You don't have to line them up ... it (i.e., the calculator) will just move it (i.e., the decimal) around depending on where it is. So they don't have to line up. You compare them - like there's one decimal (i.e., in 82.6), there's two (i.e., in 85.91), there is three (i.e., in 7096.166) - you can tell how many places there are by how far to the left the decimal is -
		24: identifies error.	54	24	T: Oh yeah, in multiplying they don't have to line up - I forgot.

CALCULATOR OPERATED (Nos. 4 and 5), (4 minutes).

	25: repeating untaped statement.	4	25	I: You think you've got it figured out - you said?
	26-27: explains reasoning.	65	26	T: Yeah. It's not on the calculator though ... how I figured it out was just in my head.
			28	I: You did?
			29	T: Yeah ... like I used it -
	30: interruption and repeating the original question.	2 4	30	I: What key do you think it is that isn't working?
	31: knows she has the answer.	54	31	T: The 6.
			32	I: OK, you're right. Let's just talk a little bit about how you figured it out in your head. But I think you'll find that you used a bit of this (i.e., the printout). So we'll number these (i.e., the printouts) and you're going to explain to me what you've done each time ... and then you're going to explain to me what you mean by figured it out in your head ... so in No.1 what happened - what did you do?
No.1. 5079. + 50. * 41.7 = 5120.7 **	37-38: reference to printout.	10		
			39	T: I figured out the problem here -
			40	I: So it was -
	41;43-4;No.1: explains reasoning. Makes routine check of problem 1 and compares results.	65 47 55	41	T: I just wanted to find the right answer.
	42: clarification.	9	42	I: To problem 1?
			43	T: Yeah - what I found out by doing this was ... ... the right answer ...
			45	I: Yes.
	46: deductive reasoning.	40	46	T: So it can't be any of these keys that is wrong.

No.1. 5079. + 50. * 41.7 = 5120.7 **	47: related question.	5	47	I: So which one - which one(s) of the keys can't it be then?
			48	T: ... can it be?
	49: repetition.	4	49	I: Can't it be.
	50: deductive reasoning demonstrated. Note the numbers chosen are those that are entered and not printed.	40 55	50	T: I don't think it can be 4, 1, 7, 5, 0, 9 or 7.
	51: reference to printout.		51	I: OK, what's the single star in No.1?
	52-3; No.1: calculator misentry.	10 100	52	T: Oh, I made a mistake - I was going to copy this one (i.e., 5079) again.
No.2. 131. + 267. + 538. + 934. + 728. = 2598. **	57-58: reference to printout. 59;61: routine check of problem 2. Note the developing difference strategy.	10 43 55 47	54	I: So you did like I told you (i.e., rather than clear incorrect entries, students were asked to print these by hitting the P key, and then starting again). So you got something out of No.1, from when the two answers matched you got something ... what about in No.2 - what did you do?
			59	T: Well I did the question -
	60: clarification.	9	60	I: In problem?
			61	T: Problem 2. And I found out that the answer was off by 240 ...
			62	I: OK. So -
	63-64: explains reasoning. Note, she makes a specific point of making mental computations.	65 55	63	T: So what I did was I ... I went through and I added it up in my head -
	65: repeating subject's words.	4	65	I: Added what up in your head?
	66: uses place value on problem 2 also. Shows deductions.	47	66	T: ... each column.
	67: clarification.	9	67	I: Like units, tens and hundreds?

68-69: continues explanation.		68	T: Yeah ... ... and then I got 8 and then I think I had a remainder of 2. I'm not sure ...
70: repeating subject's words.	4	70	I: Anyways ... in the first column - in the units column you got 8.
		71	T: Uh huh.
72;74: probing reasoning.	9	72	I: What did that tell you? ... anything?
73: strategy: compares calculated and given units columns and deduces which keys are candidates	40	73	T: That none of these (i.e., entries in the units column) was wrong?
		74	I: Why? ... like I agree with you ... but why?
75-77: continues logic of 73.	65	75	T: (Laughing) because they added up right. Like if there was something wrong with them I don't think it would have added up right.
78: related question to probe reasoning.	5	78	I: How do you know it's added up right - that's what I'm getting at.
79-80: repeating what she said in 75-77.		79	T: ... because it's got that - that answer in the ones column is right.
		81	I: ... and also on the calculator (i.e., printout) you've got an 8 there (i.e., in 2598).
		83	T: Yeah.
84-86: reviewing the logic developed by the subject and extending from statements and calculations.	11 13	84	I: That's what you mean. So because you get an 8 you know it's not one of these numbers. Does that - in other words not a 1, 7, 8 or 4 - does that ... agree with what you told me in problem 1?
87-88: combines results of printouts to form conclusions.	40	87	T: Yeah ... except that it gives me two more numbers that aren't wrong.
89: clarification.	9	89	I: Which ones?
90: answers related question.	50	90	T: 8 and 4 - ah, 8.

		91	I: 8. So it gives you –
		92	T: One more –
		93	I: Another one. Good.
94: continues place value strategy.	48	94	T: And then I added this one (i.e., tens column) ...
95: related question that directs subject.	5 12	95	I: And did it come out to what you have here?
96: answers related question.	50	96	T: No it didn't.
97–99: clear direction given to the course of the discussion.	12	97	I: Now ... you added up the tens (i.e., column in problem 2) ... and ... it came out on your printout as 9 (i.e., 2598) ... and there (i.e., in 2358) it's a 5 ... did that tell you anything?
100: forgetting.	95	100	T: I can't remember how I did this now ...
		101	I: It's OK.
			PAUSE (10s).
102: continuation of the idea in line 94.		102	T: It comes out to be 19 ...
103–6: reviews previous work to clear confusion.	11	103	I: But in the problem – in problem 2 it says 5 (i.e., 2358) ... does that tell you anything?
			PAUSE (10s).
			OK. Let's go back, when you see that you get an 8 and 8 in the units column (i.e., in 2358 and 2598) then you know that ...
107: same as former conclusion.		107	T: None of these numbers can be the one that is not working.
108: previous repetition again.		108	I: Which numbers?
		109	T: 1, 7, 8 or 4.

	110-1: review of work with probe of reasoning.	11	110	I: So when you come back and in the tens column you get a 5 there and a 9 there -
	112-5: remembers what she forgot and uses deductive logic to find the malfunctioning key.	54 40 55	112	T: Oh, I remember! I remember! ... I got the wrong answer so what I did was I checked with the 4, the 8, the 7 and the 1, 5 and the 0 and the 9, and I seen [sic] which one of those numbers was not in one of those, in one of those lists.
	116-8: repeats and extends what subject said previously.	4 13	116	I: Well, let's look at the numbers you have ... you got a 3, a 6, and a 2 (i.e., in the tens column) ... now look at what you've got in here and what you've got in problem 1 ... what numbers?
	119: answers related question.	50	119	T: 4, 1, 7, ...
	120-1: extension of subject's ideas.	13	120	I: So I - I still don't think you know if it's a 3, a 6 or a 2 yet. ... you follow?
			122	T: Uh huh.
	123: probing reasoning.	9	123	I: Did you do anything more with it?
	124: continued recall problems.	95	124	T: ... how did I do that?
No.3. 82.6 x 85.91 = 7096.166 **	125-6: reference to printout.	10	125	I: What more did you do with it? ... alright, we'll come back to it. Take a look at No.3 ... now in No.3 you - what did you do?
	127-9;No.3: routine check of problem 3 and deductive reasoning applied to results.	48 55 40	127	T: Well I did the question ... problem 3, and I got a different answer ... oh, so I knew that one of these had to be wrong - one of these numbers.
	130: clarification.	9	130	I: Which ones?
	131-4: illustrates a combining of the results from different checks and thus deductive reasoning.	40 55	131	T: 8, 2, 6 ... 5, 9 or 1 had to be wrong ... had to be one of the answers that was wrong ... so the 8 was in this column (i.e., in problem 2) which we know was not wrong ... the 5 is up here (i.e., in problem 1) which I know is not wrong.

	135: repeats what subject has said.	4	135	I: The 5 is in problem 1.
	136-8: continues the logic of 131-4.		136	T: Problem 1. 9 is in problem 1 ... the 1 is in problem 1, the 2 is, isn't and the 6 isn't either so ... now I know that the key could be either 2 or 6.
No.4. 85.91 x 82.6 = 7096.166 **	139-42: related question on calculator commutativity to test knowledge. Reference to printout.	5 10	139	I: Very good. We'll come back to that. Now what did you do in this part here in No.4? You've turned the numbers around - heh, Tonia? ... you see? In problem 3 you've turned it around from 82.6 to 85.91 (i.e., as the first factor) -
	143-4;146;No.4: redoes No.6 to get the decimals to line up.	85	143	T: That's because I ... I forgot that we didn't have to line up the decimals.
	145: repetition of subject's idea to establish error.	4	145	I: In No.4 you thought the decimals had to be lined up in a row.
			146	T: Yeah, but they wouldn't line up for me ...
No.5. 307. x 64. = 19648. **	147-8: reference to printout.	10	147	I: And that's what you asked me about. So in No.5 now what did you do?
	149-53;No.5: explains reasoning. Gets confused in her explanation. Continues deductions of 137-8.	65 48 55 40	149	T: ... I did problem 4 - and I got another different answer ... so I knew that it could be ... a 3, 0, 7, 6 or a 4. So what I did was I checked back to my other problem 1 ... my columns 1 in problem 2 ... and I checked my numbers ... and ah ... a 3 ... it can be either the 3, the 6 or the 2 ... PAUSE (5s).
	154-5: review of subject's work with a hint given.	11 8	154	I: You're multiplying these numbers (i.e., 307 and 64) and you're getting the wrong answer ... is there any 2 in there?
	156-9: final deductive reasoning completed to give the solution.	40	156	T: No ... so it can't be the 2. And the 6 is in here (i.e., 64) ... so it can possibly be the 6 .. the 4 is in problem 1 which I know wasn't wrong ... the 3 isn't in any of the problems that I found wasn't wrong.

160: contradiction.	6	160	I: Oh yes it is – it's in problem 2.
		161	T: Yeah, but that's in the one that could be wrong.
162-3: Tonia didn't need this hint as she had previously deduced this conclusion (i.e., lines 158-9).		162	I: So it could be the 3 that's causing ... if you go to problem 3 you might notice something ...
164: continues explanation encompassing hint.	40	164	T: The 6 is in there and the 3 isn't.
		165	I: So then ...
166: knows she has the answer.	54	166	T: It's the 6.
167: questioning certainty.	7	167	I: Why is it the 6?
168-70: condenses all previous explanations.	65	168	T: Because you have a 6 in problems 2, 3 and 4, not in 1 which worked out ... and the 2 in problems 2, 3 and 4 didn't ... and the other numbers check out that are in all those problems.
		171	I: In other words – you found the key that's wrong ... and I like the way you did it –
173: anticipates the next related question.		173	T: Don't ask me now to fix it (laughing).
174-80: second part of question given as a related question.	5	174	I: That's what I was going to ask you – that's the next part ... can you go back – and this is what I'd like you to do for the second part – you just try it in the little bit of time that is left – can you go back and see what was happening with the 6 in problems 2, 3 and 4. Could you try to get these numbers: 2358, this number (7044,62) and the answers. See if there's a way of doing it – now that you know it's wrong.
		181	T: That took me a while to understand.
182: complimentary statement.	3	182	I: No it didn't – you did a good job on it.

					CALCULATOR OPERATED (Nos. 6 and 7), (2 minutes).
					You don't see how that can be?
No.6.	13. + 267. + 538. + 934. + 728. = 2480. **	184;186-7;No.7: misinterprets problem and uses substitution. Interprets malfunction as a 0 replacing 6 (see lines 269-70 for confirmation).	82 49 92	184	T: I added problem 2 up and instead of 267 I used 207 -
				185	I: So what have you done then?
				186	T: I said that the 6 wasn't working at all and that there was no number punching out from it.
		188: hint given.	8	188	I: You said there was a 0 coming out.
No.7.	131. + 207. + 538. + 934. + 728. = 2538. **			189	T: No I - OK, yeah, OK.
		190: reference to calculation to continue question.	1	190	I: And you got a different answer - you still didn't get the one that's there (2358) ... so you're going to have to come up with a different idea.
		193-4: questions reasonableness of answer based on size of subtracted number and results produced. Good analysis of the results of calculation.	60 55 40	193	T: But I subtracted, I used a smaller number than 6 - I used 0 - and I got a larger answer than there's in ...
				195	I: Yeah, that's telling you something ... that's good.
					CALCULATOR OPERATED (Nos. 8 to 10), (2 minutes).
		379-80: reference to printout.	10		What we're going to do now is I'm going to take a look at what you did after this No.5. We're going just over (advancing and removing the printout) the things you tried. So in this one ... in No.6 - which problem were you trying to do in there?
		200-2;No.6: routine check of problem 2. Error in repeating No.2. Calculator misentry.	86 100	200	T: I was doing problem 2 ... and I figured out ... the answer ... to the question if the number 6 was working ... and I got the answer.
				203	I: But ah ... you've got 13 here on the top ... in there (i.e., the problem) it's 131.

No.6. 13. + 267. + 538. + 934. + 728. = 2480. **	204;No.6: error not recognized.	95	204	T: I must have forgot to push the 1 again.
			205	I: So in No.6 you made an error ... but you didn't realize ...
	206: good illustration of deductive reasoning during interview.	40	206	T: I don't think that'll change my ...
	207-9: potentially negative comment. Reference to printout.	3 10	207	I: We'll just see. In No.7 - this was the one you were talking to me about - you called me over for ... yeah you did - you showed that to me -
No.7. 131. + 207. + 538. + 934. + 728. = 2538. **			210	T: Oh.
			211	I: Remember?
			212	T: Yeah.
			213	I: Now what did you do in there - what did you change?
	214-5;No.7; continued explanation of reasoning (see lines 186-7 for rationale).		214	T: I thought that maybe the 6 wasn't working at all and that you were just getting a 0 ... in its place.
	216-7: repetition of subject's ideas.	4	216	I: So instead of 267 ... you put in 207 and then you were upset by the fact that even though you did that - what happened?
	218: illustrates difference strategy.	55 40	218	T: It got larger than the answer -
	219-22: review of calculations.	11	219	I: In the problem - you got 2538. So even though you made this smaller you still got a larger answer ... now we talked a little bit about that ... did you get any ideas like what you might do next?
			223	T: ... not really -
	224-5: reference to printout.	10	224	I: Oh you've got a nice ... I wonder if that's - let's take a look at No.8 ... and you tell me what you were doing in No.8 ...

No.8.	131. + + 26. * 277. + 538. + 934. + 728. = 2608. **	226-7;230-2;No.8: uses difference strategy to make a sum larger than No.7. Replaces 6 with a 7 (i.e., 267 becomes 277). This algorithm is faulty. Note calculator misentry error.	48 40 55	226	T: I tried to make the number bigger ... make it 277 instead of 267.
				227	I: So the 26 is actually a typing error ... so you put a star (*) by that - and you put in - tried to make it bigger - was there any reason why you tried that?
		227-9: repetition of subject's ~~remarks;~~ questioning reasoning with direction.	4 12	230	T: ... I was confused that when I made it smaller that the number got larger ... and so I tried to make the number larger to see if it would get smaller - I don't know (laughing) -
No.9.	131. + 22. * 257. + 538. + 934. + 728. = 2588. **	233-4: reference to printout.	10	233	I: It's alright ... and then in - it still stayed bigger didn't it - 2608 ... did you get any ideas from that? ... like in No.9 now -
		235;No.9: difference strategy continued.	48	235	T: Just that the number was smaller than 6.
		236-7;No.9: misentry.	100	236	I: So what did you try - or another error in here - 22, heh - don't worry about it. I see 257 ..., is that the change you made?
				238	T: Yeah.
No.10.	131. + 227. + 538. + 934. + 728. = 2558. ** CLEAR 131. +	239: reference to printout.	10	239	I: What did you try to do in there?
		240-1;No.9: continued using faulty algorithm in attempt to get answer by successive approximations. Error: misentry.	48 92 55 100	240	T: I made the 6 a 5 ... and my answer was still ... quite a bit larger (i.e., than 2358) ...
		242-3: probing reasoning and extending subject's remarks in question form.	13	242	I: You got 2588 instead of 2358. Did you get any ideas in here from that?
				244	T: I knew that it was something different going on ... and I could not figure out what it was ... yet.
		246: reference to printout.	10	246	I: In No. 10 you've done what - what's the change you made in No.10?
		247;No.10: continues successive approximation strategy.	48 92	247	T: I made it 227 instead of 267.
				248	I: So still smaller ... now this is the point at which you came to see me, why?

250-2: explains reasoning.	65 40	250	T: What I don't ... I don't know how to show this ... to make sure that I'm right, but I thought maybe instead of the 6 adding that was subtracting from the answer ... like ...
253-4: extending subject's ideas and giving clear direction.	13 12	253	I: In other words ... take off - the 6 there is in the tens place - in other words subtract 60 ... is that what you were thinking of?
		255	T: I'm not -
256-68: explanation of solution.	1	256	I: Alright you didn't get this part - you figured out which key it was but you couldn't figure out this other part & we're going to show you basically what you should do ... you see you've replaced the 6 with a 0 ... what about leaving the 6 out completely ... and instead of 267, 27 ... leave the 6 out completely ... do you follow - now if you had gone and you had tried some of these other problems ... and left your 6 off - like in problem 3 or in problem 4 - you would have gotten it. If you had made it 0 there (i.e., in 64) in problem 4 ... 04, or you had make it 82 (i.e., in 82.6) you would have gotten the answers in here (i.e., given in the problems). So one of the things that happened to you is you got stuck on problem 2 and you weren't ready to go to other problems.
269-70: does not understand all the information given in the original problem.	83	269	T: I thought the 6 was doing different things in problem 2, problem 3 and problem 4 ... was wrong in different ways in each problem.
12:52		271	I: So that would explain why you stuck with problem 2 ...

Nardi Problem 7.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:25 Ex.1.	23. + 55. ** 56. = 79. **	01-08: explanation of P-key.	1	01	I: So first of all what I want to do is I'd like to show you an example of what I'm going to call a sequence. I showed you how to use the P-button on the calculator (i.e., before taping, as Ex.1). That's the first thing I did. Let me make an example of a sequence.
Ex.2.	5. + 2. x 3. = 11. ** 11. + 2. + 3. = 17. ** 17. + 2. x 3. = 23. **				CALCULATOR OPERATING (Ex.2). I could start with 5 say, and add 2 and times by 3, equals, And then add 2 and times 3, equals. The sequence would be made up of the first number you start with ...
				09	M: Yeah.
				10	I: Which would be what?
		11,14,18: answers related questions.	50	11	M: 5.
				12	I: And every other number that has stars beside it ... what would they be?
				14	M: 11, 17 and 23.
		15-17: explanation of terms in the question using related questions.	1	15	I: So the sequence would be 11, 17 and 23. Now in order to make that sequence there are certain constant operations. Things that I do again and again. Can you tell me what the constant operations are?
				18	M: You added 2 and times 3.
		19-23: explaining introduction of problem 7.		19	I: Each time, heh. So in the first one I added 2 and timesed by 3 to 5. Then I added 2 and timesed by 3 to 11. And so on. So what I'd like you to do first ... is just look at the introduction to this question and see if you understand it - don't worry about the question - just look at the introduction. Think you understand it?
				24	M: Yeah.

25-26: continued explanation.		25	I: Let me just make sure that you do. On the left I'm giving you a printout with the constant operations in there.
		27	M: Yeah.
28: related question.	5	28	I: So can you tell me what the sequence would be on the left?
29;33: answers related question.	50	29	M: ... 8, 25, 76, 229, 688.
30-32: explaining spacing importance. Related question.	1 5	30	I: Then all I did was I took the numbers that make the sequence and I pulled them out. Can you see that I've maintained the space between numbers? Can you tell me what the constant operations are?
		33	M: Multiply by 3 and add 1.
34-38: continued explanation.		34	I: So all I've done is I've left the constant operations out. And also you see the first term 8 on the left has no star, and here (on the right) I put a star .... Read the question and see if you understand that. PAUSE (20s). Do you understand what I want you to do?
39-40;43;45: restates problem; questions conditions of problem.	21 20	39	M: I'm not quite sure. You want me to find out what the constant operations are ... by just using these (i.e., the numbers).
41-42: repetition.	4	41	I: Use the numbers that are there and look closely at the whole thing (i.e., the problem).
		43	M: At these things.
44: clarification.	9	44	I: Which things are you pointing at?
		45	M: The ... divide sign and the 3.

	46-50: question explained without giving direction.	1	46	I: So that's a little bit ... that's something you have to see. You saw that. And I'd like you to go from the right to the left. Do you catch on? And you have to use the calculator ... to find out which numbers, the sequence – all the operations would be. Catch on?
			51	M: Yeah.
			52	I: So we're going to leave you to do it for awhile.
				CALCULATOR OPERATED (Nos. 1 to 3), (3 minutes).
No.1. 140. + 4. / 3. = 141.33333 **	53-59: says she got it before taping began. Reference to printout.	64 10		OK, so you think you've got it. And I look at it and I see right away you've got it. And you didn't do very much work either. My lord, you got it almost right off. What I'd like you to do now is I'd like you to explain to me what you did in different steps. And how you figured out what to do. And I'll be interested in that ... because looks like you really know how to figure this one out. So first you tried to do what?
	60-2;64;No.1: explains reasoning illustrating reasoning. Note: printout does not indicate part of strategy (i.e., wanted to subtract 2). Algorithm in formative stage. Sees pattern in question.	47 55 40	60	M: I thought maybe if I – I wanted to subtract 2 I thought I if I add 4, I'm up to 6 ... and then divide by 3 and then subtract ... but that's – the answer's 2 so I thought that might do it.
	63: clarification.	9	63	I: What do you mean the answer's 2? Would you explain that to me.
			64	M: If I want to lose 2.
			65	I: Is that what you want to do? Why do you want to lose –
			66	M: From 140 to 136 –
	67: contradiction.		67	I: 138.
			68	M: Sorry, 138. I wanted to lose the 2. So I thought if I added on 4 to make 6 and divide by 3 –

				70	I: Oh, OK. I got you, yes.
No.1.	140. + 4. / 3. = 141.33333 **	68-9;71-2;No.I: excellent explanation of deductive reasoning and strategy of algorithm. Error of 61-2 classified.	40	71	M: So then you divide by 3 and that's 2 so I thought maybe that would bring it down 2 but it didn't.
		73-74: reference to printout to probe reasoning.	10	73	I: So when you saw this answer of 141.33333, did that tell you anything?
		75: not analyzing how the operations work together.	96	75	M: Not right away, no.
				76	I: You didn't want it, though.
				77	M: No I didn't want that.
		78: probing reasoning.	9	78	I: What did you want?
				79	M: I wanted 138.
				80	I: So when it isn't 138 ...
		81;83: continued support for classification of nonanalysis.		81	M: I tried again.
		82: reference to printout as basis for direction.	12	82	I: If it's bigger than 138, did that tell you anything?
				83	M: No, I didn't notice right off ...
		84-85;87: trying to determine randomness in decision.	9	84	I: All you did is you said I didn't get 138 so I'm going to try something else.
				86	M: Yeah.
				87	I: Did you have any idea of what else you wanted to try?
		88-90: explains changes to algorithm and reasoning. 6 chosen randomly. Analyzes result	48 94 40 55	88	M: Well ... I thought ... if that didn't work ... maybe it's just, I want to add 6 and divide by 3. And that's ... 6 divided by 3 is 2 so I thought ... and then when I saw that answer –

No.2. 140. +
-6. ÷
3. =
142. **

91-93: reference to printout and repetition.	10 4	91	I: OK. Hold on, you're racing into the answer ... and it does too. OK. Can you tell me again in your own words why you added 6 and divided by 3 in No.2?
94-95: strategy described.	65	94	M: Well ... I wanted to subtract 2. I had to find something that left 2 when divided by 3.
96: repetition for clarity.	9	96	I: That left 2 when divided by 3.
97-8; No.2: explains reasoning. Note: she has the order of operations correct here.	40	97	M: Yeah, so something divided by 3 equals 2. And that's 6. So ... I added 6 ... then divided ... switched the properties ...
99: clarification.	9	99	I: Would make it smaller. Is that what you mean?
		100	M: Yeah.
101-2: related question.	5	101	I: OK. Let me ask you something: when it says add 6 and divide by 3, what's it doing first?
103: error in understanding how calculator operates.	85	103	M: It's adding.
104: contradiction.	6	104	I: No it isn't.
		105	M: Sure it is.
		106	I: Oh no it isn't.
		107	M: OK. So it's dividing.
108-12: explains order of operations of calculator using printout No.2. Extending from subject's reasoning.	1 13	108	I: When you do this - add 6 and divide by 3 - what it does ... is it says 3 into 6 two times and then it adds that 2 onto 140. You didn't know that did you? Like you were trying to make 146, weren't you? And like when you saw it go to 142 ... what you realized is that it increased by 2.

No.3. 140. – 6. / 3. 138. **	113-4;No.3: modifies algorithm correctly. Explains reasoning.	49 40	113	M: Yeah. So from there, I decided if I wanted to decrease it by 2 I'd have to do the same thing only subtracting.
	115: reference to printout.	10	115	I: So you tried No.3.
138. – 6. / 3. =			116	M: Yeah.
136. ** 136. – 6. / 3. =	117-9: probing reasoning based on printout.	7	117	I: ... so then what you did is you subtracted 6 and divided by 3 and kept on doing that. How do you know that is the right answer? With the work you've done here ... in No.3.
134. ** 134. – 6. / 3. = 132. **	120: . compares solution with general known results; checks solution.	54	120	M: I matched all the starred ones up.
			121	I: They do. Did you try holding them beside each other, anything, to see if they were the right distance apart?
			123	M: No.
	124-8: probing general understanding.	5	124	I: You just figured it was. Because the distance they're apart is very important. You've got it. I think you saw the answer - well you tell me, what do you think was the big point in your seeing answer? ... what did you need to realize or understand in order to get the answer?
	129: she did not know this as seen in line 103.		129	M: That the ... machine will divide first ...
	130: contradiction.	6	130	I: (Laughing) yeah, but you didn't see that.
			131	M: I know.
			132	I: And you still got the answer didn't you.
3:40.	133: this was her deductive step.	65	133	M: Well then I had to realize that you have to subtract ...

Bruce Problem 7.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:28		01-08: explaining problem by giving an example. Related question.	1 5	01	I: So what I want to do is just give you an example of a sequence. So I'll type in a number, say 23, multiply it by 4, add 2, and hit equals. I'll take the answer that I get there, I'll multiply by 4, add 2, hit equals. I'll take that answer, and multiply by 4, add 2, equals. So if I advance it (i.e., the printout) I've made a sequence. Now what do I mean by a sequence? The actual sequence is the number you start with and every number that has stars from then on. So the sequence would be - what?
Ex.1.	23. x 4. + 2. = 94. ** 94. x 4. + 2. =				
	378. **	09: answers related problem.	50	09	B: 23, 94, 378 and 1514.
	378. x 4. + 2. = 1514. **	10-11;13-5;17-20;22-5: explains the problem and the checking method.	1	10	I: Now to get that sequence there are certain constant operations I've done.
		12: shows understanding.	40	12	B: Multiplication of 4 and addition of 2.
				13	I: Right. Those are the two things I did throughout. OK. So you think you understand that. If I wanted to check, say I did something else, if I wanted to check that I had the right sequence.
				16	B: Huh, huh, OK.
				17	I: I could tear this out (i.e., the printout), put another piece - try it all over again. And move the two pieces of paper side by side and these two should match up. So that'd be a way of checking it.
				21	B: The same numbers and stuff.
				22	I: Everything. And they should be just as far apart ... on the two. Follow? Now what I'd like you to do is just read the introduction of this problem, don't worry about the question. Do you think you understand what it's getting at?
				26	B: Uh huh. And I'm supposed to figure out -
		29: related question to check understanding.	5	27	I: Let's just worry about the introduction. What they've done here is they've given you an example of a printout with operations that are constant: what are the constant operations?

30: answers related question.	50	30	B: Multiplication by 3 and subtraction of 1.
31-33: continued explanation.		31	I: Right. And then what I did was, I've taken the same sequence and just pulled the numbers that formed the sequence out. Now I don't have a star beside the 8.
		34	B: That's cause you're just starting.
		35	I: But see there's the sequence I'm talking about (i.e., on the left of the example). Now here is the question - read the question and see if you think you understand what I'm getting at.
38: questions conditions.	20	38	B: With the sign here and the 3 -
		39	I: Yeah, OK.
40-41: illustrates deductive reasoning.	40	40	B: So that means you're dividing by 3 and doing something else afterwards.
42-44: avoiding any direction or hint.		42	I: Yeah, these are the things you have to see (i.e., avoiding a discussion at this point). I just wanted to know if you understand what I want you to do.
45-47: restating problem in his own words.	21	45	B: So what - OK - if this is the calculator printout, I can assume automatically that behind each of these numbers, that one there will be 140, right there will be 138, well that's just spacing.
48: hint given.	8	48	I: Yeah, that spacing tells you something.
		49	B: So there's ...
		50	I: Do you see what I want you to do? That's what I'm getting at. I would like you now to tell me what is the total printout with the constant operations.
		53	B: I think well - since, OK ...

	54-57: avoiding direction.	1	54	I: I don't want you to tell me about it now. You're going to try it. I just want to see if you understand what I'm getting at. Like you should end up with something like I have here on the top.
			57	B: Starting with 140 and ending with 132.
	58: explanation.	1	58	I: You should have all the things in.
			59	B: OK.
	60-61: rephrasing the question.	4	60	I: In other words, you're trying to spot really the constant operations.
				CALCULATOR OPERATED (Nos. 1 to 9), (5 minutes).
	62: questioning certainty.	7		OK. So you think you've got the problem then Bruce?
			63	B: Yep.
			64	I: Alright, let me see your printout.
			65	B: I did two (i.e., two separate pieces of paper).
	66: clarification.	9	66	I: Which is the one you did first?
			67	B: The long one. First I thought -
No.1. 141. + 140. - 2. x 3. = 134. **	68-69: interruption to organize printout; repetition of subject.	2 4	68	I: Let me get this organized here. So I got No.1. So what do you want to tell me? At first you thought what?
	70-1;73-4;No.1: calculator misentry. Tries subtraction but makes error, identifies it and stops process.	100 47 54	70	B: Well first I thought was to use decimal points, heh. But then I just worked on that ... wasn't getting too far that way.
			72	I: You weren't getting too far with decimal points. In No.1 -
			73	B: I was just doing this first one - er that was the one that you showed me (i.e., the example done).

75-77: clarification.	9	75	I: Oh wait. This is my example OK. Very good, I should have noted that. So that's my example. All the way down to 1514. This is you now.
78-80: noted misentry.		78	B: Starting off with No.1. I made a mistake there (i.e., note the single star beside 140). Second part I hit multiplication. So that section was scrapped.
81;84: clarification.	9	81	I: The whole thing? Like 141 to 134?
82-3;85-6: explains reasoning behind abandoning the printout. Second misentry identification.	65	82	B: Well 141 was when I - like first typed it out and I accidentally hit 1 instead of 0.
	100 54	84	I: So you wanted 140?
		85	B: Yeah. And then I did that sequence and instead of division I hit multiplication which wrecked the sequence.
87-88: probes reasoning.	9	87	I: Oh, OK, you knew right away it had to be divide by 3. How do you know that?
89: answers related problem.	50	89	B: Well ... I didn't really know that I saw it on the page here -
90-92: generalizing.	13	90	I: That's right. Noticed it on the paper - that's what I was getting at. Every one of these - ah - all your sequences you know that the last constant operation is divide by 3?
		93	B: Uh hum.
		94	I: You do. Like you never ever changed that?
		95	B: No.
		96	I: Good. So you noticed that. I didn't know whether you would or not - some people don't. Right away when you hit times by 3 you know it was a mistake.

		99-103: explains reasoning; reviews printout.	55	99	B: Well, not right away - when I just looked back over it I saw that it was a mistake. Actually I did that and I was - part way down after like point-8 cause I thought I went too high with the 2 and hit division instead so after that sequence I thought I checked and found that I had -
		104: clarification.	9	104	I: So at first you didn't realize it?
				105	B: No I didn't.
No.2a.	140. - 1. / 3. = 139.66667 **	106-7: reference to printout.	10	106	I: So let's go to No.2a. Now when you got from 140 to 134 did that tell you anything?
		108-10;No.2a: analyzes No.1 to change subtrahend.	55 48	108	B: Well at first when I hit the multiplication that told me my minus number was too high and No.1 just told me that my minus number was too high. I lowered it -
		111-3: reference to printout with extension of subject's ideas.	10 13	111	I: So you lowered from minus 2 to 1 and you - oh I see - you didn't realize that you didn't have division so you divide by 3 for the second time -
No.2b.	140. - 0.8 / 3. = 139.73333 **	114;No.2b: explains reasoning. Algorithm developed in No.1 and changed in No.2b. Does not analyze printout correctly.	55 48 96 5	114	B: That's right. And the same with the third time as well.
				115	I: You got 139.66667. Did that tell you anything?
		116;118;120: reasoning with reference to printout.	65 40	116	B: It told me it was too high again.
			55	117	I: What do you mean by too high?
				118	B: Like too high - I had to go down.
		119;121: clarification.	9	119	I: Further. To what?
				120	B: Well, too a smaller number.
				121	I: Which number?

No.2b.	140. - 0.8 / 3. = 139.73333 **	122;126: error repeated of picking too small a number (i.e., 0.8). 123: related question. 124: answers related question	86 5 50	122 123 124	B: Then I went to point 8. But I don't know what I was thinking of. I: Which number do you want to come up with next? B: I wanted - I wanted to come up with 138.
				125	I: Right.
				126	B: It was still too high.
		127: reference to printout.	10	127	I: So then you're trying another one, right?
				128	B: I used point 8.
		129-31: review of previous calculations.	11	129	I: In other words, so far what you're working with is still this - always subtraction ... and you're trying to find what you think is the number that you should subtract by.
				132	B: Yes.
		133: return to printout.	10	133	I: OK. I got you. So it's minus point 8 you're trying to subtract.
				134	B: Which was too small and just raised it and then I realized that -
		135: direction given.	12	135	I: OK. Was it even worse than minus 1?
				136	B: Yeah, and then I realized -
No.3.	140. - 2. / 3. = 139.33333 **	137-9: extends subject's reasoning and reference to printout.	13 10	137	I: Because you got a number even bigger again, heh? You got 139.73333 right? I think what I'm gonna do now is call this one, 140 minus 2, printout No.3. So tell me what you're trying to do there?
		140;142;No.3: redoes No.1 with corrections. 141: reference to previous calculations.	48 54 10	140 141	B: Then I tried it with the right number. Like instead of times 3. I: But you had already done division by 3 here, you were OK here.
				142	B: But instead of like multiplication of 2 and division of 3.

	143-4: repetition.	4	143	I: Oh, I got it. This is No.1 over again. Doing it the right way. And did this tell you anything?
	145-6: successive approximations described. Sees pattern in subtrahends.	48 54	145	B: Still too high so I went down again. But I was doing this by decimal points.
	147: probing reasoning based on printout.	5	147	I: Is there any reason why you picked decimal points?
	148: explains reasoning.	5	148	B: I don't know, I just thought it would be easier that way.
	149-50: probing reasoning but with obvious direction.	12	149	I: Well did you think that picking a whole number like 3 would be too big?
			151	B: Yeah I did.
			152	I: Is there any reason for this?
			153	B: No.
No.4. 140. − 2.1 / 139.3 =	154-6: No.4: reference to calculations and reasoning probed.	11	154	I: OK. So we'll go to what we call No.4. Now you're taking away 2.1. That's what we were just talking about. And you saw 139.3. Did that tell you anything?
	157: No.4: repeats algorithm, analyzes result for closeness to answer. Subtrahend clearly chosen at random.	48 55 94	157	B: High again so I decided to get −
			158	I: By too high again you mean the answer is too high.
No.5. 140. − 3. / 3. = 139. =	159-61: No.5: explains reasoning but still using same algorithm and successive approximations.	48 55	159	B: Yeah. So I decided in the next one, No.5, to go − to try it by a whole number ... and I realized it was ... just 1 point too high so I decided to move my point so I decided −
	162-3: review of No.5.	11	162	I: Hold on, we're going a little fast here. In No.5 you're subtracting 3 and −
			163	B: Dividing by 3.

No.5. 140. – 3. / 3. = 139. **	165-7: related question to test calculator knowledge.	5	165	I: Can I ask you a question What is – you got a 139 so what did the calculator really do? ... if you went from 140 to 139, what did it do?
	168: answers related question.	50	168	B: Took off 1.
	169-70: reference to printout to probe subject's reasoning.	5	169	I: OK. Can you see, looking at just this No.5, can you see why when you subtract 3 and you divide by 3 you'll be subtracting 1?
			171	B: ... ... yeah.
			172	I: Why?
	173-4;176: not answering related question. Does not understand calculator operation.	83	173	B: Uh ... ah ... it has something to do with the calculator itself, doesn't it?
			175	I: That's right.
			176	B: Well ah ... I don't know ... ...
	177;180-1: explaining calculator operations through related question.	5	177	I: Which one of these will it do first – subtract 3 or divide by 3?
	178-9;182: still avoiding a question he does not know the answer to. Error in calculator understanding.	85	178	B: Well it should – well, unless it doesn't have AOS, it should subtract by 3?
			180	I: It's got AOS (i.e., the abbreviation used by Texas Instruments to represent Algebraic Operating System).
			182	B: So subtract by 3 first.
	183-4: contradiction and explanation of the answer.	6 1	183	I: No way. If it's got Algebraic Operating System – that's what you mean by AOS, right – it should do divide, it will do the order –
	185: still does not see his error.	95	185	B: No, no.
			186	I: Oh yes.

			187 B: OK, so it'll divide by 3.
	188;190: related question.	5	188 I: Which - what number did it divide by 3 in No.5?
			189 B: ... ... ah ... I don't know, it should have divided by 3.
			190 I: Divided by 3, but what number did it divide by 3?
	191: answers related question.	50	191 B: 3.
			192 I: Right. And what's 3 divided by 3?
			193 B: 1.
	194: probing understanding of the calculator's operation.	9	194 I: And then what did it take away?
			195 B: Subtracted the 1.
	196: clarification.	9	196 I: Right. Do you follow? Did you realize that?
			197 B: No.
No.6. 140. - 4. / 3. = 138.66667 **	198-9: extends from subject's reasoning based on calculations.	13 10	198 I: OK. Let's go on now to No.6. So you realize that you've got to beef up what you've subtracted.
			200 B: Uh huh.
	201: clear direction given.	12	201 I: Cause 139's not low enough. Is that right?
	202-3;No.6: continues algorithm and successive approximations strategy. Indicates deductive reasoning. Not analyzing operations and how they relate.	48 55 40 96	202 B: Yeah. So I took it by 1 - really didn't click that that was 138.66667 and that was too big. So I decided 3.5 -
			204 I: So you thought you weren't big enough subtracting 4.
			205 B: I don't know why but I just didn't think that was, I just thought too small.

No.7.	140. − 3.5 ÷ 3. = 138.83333 **	205-6;No.7: same algorithm but not being developed correctly. Not analyzing calculator results	48 94 55	207	I: So in No.7 you decided to cut the 4.
		207: probing reasoning of No.6 and No.7 but extending from subject's statements.	13	208	B: And I realized that I was too - that I was increasing instead of decreasing.
		208-9: identifies error when he performs some analysis of results.	54 55	210	I: Can I ask you a question though Bruce? Why - how do you know it's subtract?
		212-3: explains reasoning, Does not know he has the answer.	65 95	212	B: I don't know I just thought it had to be subtract since we were going down.
		210-11;214-7: probing reasoning with related question.	5	214	I: The sequence was going down. OK. I'm interested in - like you know when I look at your printout it's all subtract and divide. Which is what you have to do, OK. But I'm just wondering how you would that it has to be subtract?
				218	B: I don't know just - felt it would be.
		219: extending from subject's statements.	13	219	I: Because you saw the sequence going down, like you said.
				220	B: Yeah.
		221;223:reference to results.	10	221	I: In No.7 you subtracted 3.5, you didn't get what you wanted.
				222	B: No.
				223	I: Why aren't you happy with 138.83333?
		224: successive approximation strategy.	55	224	B: Still too high.
		225: clarification.	9	225	I: Too high compared to what?
		226: goes in the right direction but still experiments in choice of subtrahend.	40 94	226	B: 138. I had to get 138.
				227	I: So you've decided now to -
		228;No.8: successful algorithm.	54	228	B: Try it - try it by 5 (i.e., No.8).

No.8. 140. -			229	I: And you're getting pretty close?
5. /				
3. =	230;234-5;No.9: 6 is still used without knowing it is the right subtrahend.	94 48 50	230	B: Uh. So I decided to try it by 6.
138.33333 **			231	I: Now, when - did you realize - like when you saw subtract 5 was 138.33333 did you think it was 6 before you actually tried it with 6?
	231-3: trying to determine if he substituted the 6 randomly.	9		
No.9. 140. -				
6. /				
3. =			234	B: Yeah I did actually cause I thought it had to be a number larger than 5 so I tried 6.
138. **				
138. -				
6. /	236-7: related question.	5	236	I: What I'm wondering, like - do you know what decimal 66667 is as a fraction? What has the calculator done with that?
3. =				
136. **				
136. -			238	B: Ah ... put it in decimal point. Other than that ...
6. /				
3. =	239: hint given.	8	239	I: The calculator rounds -
134. **				
134. -			240	B: Rounds.
6. /				
3. =	241: hint expanded.	8	241	I: This is two-thirds - 0.6666. What's 138.33333?
132. **				
	242: answers problem.	50	242	B: One - third?
	243-5: explains the generation of the printout based on the operations of the calculator.	1	243	I: That's right. So you're going down by thirds when you subtract by 1 more. Like when you subtracted by 5 you had one-third left. You didn't really notice that?
			246	B: No I didn't.
	247-8: questioning certainty.	7	247	I: And then bull's eye, bang, No.9, you subtract by 6 - did you realize you had the pattern then?
			249	B: Uh hum. Well then I finished off the pattern.

No.10.	140. – 6. / 3. = 138. ## 138. – 6. / 3. = 136. ## 136. – 6. / 3. = 134. ## 134. – 6. / 3. = 132. ##	250-2: review based on printout.	10	250	I: In No.9 you're just going down and right away I can see you realized you got 138 so you kept on going. And then in No.10 which is the other piece (i.e., of paper).
				253	B: That was just getting all the extra stuff.
		254-6; No.10: repeating previous calculations.	86	254	I: Really you're duplicating what you've got in No.9 just so you can get it separated by itself – what the actual printout is. And you have the problem.
3:52					

Sherri Problem 7.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:35		01-11: explanation using a calculator. Related question to explain terminology.	1 5	01	I: What I want to do is give you a few examples just to explain to you the type of question we're going to be doing. I can create a sequence with this calculator, for example I could start off with 5, times by 2 and subtract 1.
Ex. 1,	5. x 2. - 1. = 9. ** 9. x 2. - 1. = 17. ** 17. x 2. - 1. = 33. **				CALCULATOR OPERATED (Example 1). Then I could times by 2 again and subtract 1. Times by 2 and subtract 1. Now if you take a look at this ... what I'm - the sequence is actually made up of the numbers that have stars: 9, 17 and 33. So that's the sequence: 5, 9, 17 and 33. The numbers between - which I do again and again - I call constant operations. So can you see what my two constant operations would be in here?
		12: answers related question.	50	12	S: Times 2, minus 1.
		13-17: explanation of checking procedure.	1	13	I: Right. So ... ahm ... I could - let's say for example I want to do this sequence again - I'm not sure if I got it or not - an important thing in checking the sequence like - I'll start again - so we'll start with 5 (using the calculator to reproduce example given) and what did we do?
				18	S: Times 2, minus 1, ... equals.
		19: related question.	5	19	I: And then press it again. Do I have to go one more time?
				20	S: No not necessarily.
		21-22: explanation.	1	21	I: Well I want to get the same one (i.e., printout and sequence) again.
				23	S: Yeah.
		24-28: calculator misentry. Explanation of P-key operation.	100 71	24	I: Whoops! Now if you made that mistake - see I pressed EXP by mistake - you press this P key and all it will do is print it, so I can see what your mistake was, and then do what you want to do. (Note: the P key prints without performing the operation).

		29	S: Yeah.
30-2;34-7;39-41: checking procedure explanation continued.	1	30	I: So ... to check - what I'm trying to show you is this - how can I check that this sequence (i.e., the second one produced) is identical to that one (i.e., the first example)? Take it out -
33: reasons deductively.	40	33	S: And then you check identical stars beside it.
		34	I: Slide it right beside. Now if it's the same one not only should the numbers match up, but the distances between numbers, you follow - like you see there's one , two, three, four steps - one two, three another four.
		38	S: Yeah.
		39	I: Between 5 and 9 there's less room than between 9 and 17. So this business of taking it out and sliding it, I want to show you that so you can check anything. Do you follow what I mean?
		42	S: Yeah.
		43	I: Now I'd like you to read this problem, read the introduction first, and see if you understand just what the introduction says. PAUSE (10s).
		46	S: Yeah, OK, I understand.
47-50: explaining question.	1	47	I: You understand what's happened. All I've done is given you the printout with all the constant operations on the left and then on the right I copied the same printout ... but just with the sequence of numbers.
		51	S: Yeah.

52-55: explanation continued.	1	52 I: And I've made a slight change in that I put a star by the 8 cause that was the first term (i.e., it was not possible to create the problem and get no star printed beside the first term). Like really the star shouldn't be there, right?
		56 S: Yeah.
		57 I: Now read the question that I want you to try to solve and see if you understand that. PAUSE (10s).
59-62: separates conditions of question.	20	59 S: Uh hum. Instead of ... ahm ... the first one you gave the printout with the operations and then the printout without the operations, now you've given the printout without operations and you want the printout with the operations.
63-68: question explanation continued.	1	63 I: Right. I want you to try to fill all the missing steps. And you understand that there are constant operations - things that you do again and again. And that's what you really need to find. And you'll know you have the answer when you can take your piece of paper, put it right beside the one I've given you, and it'll match. Do you follow?
69: questions conditions of the problem.	20	69 S: Yeah. What do these little divide and equals mean?
70: has already asked this question.	84	70 I: Ah ... that tells you something.
		71 S: OK.
72-74: explanation given.	1	72 I: OK. You can look within here and you'll see what I mean - but you've got it right - it's divide by 3, that's the kind of thing you have to watch.
		75 S: OK. So that, that divide is ... that, right?
76: clarification.	9	76 I: What do you mean?

77-9;81: asks for clarification of calculator printout. Wants to know if the operation immediately preceding an entry acts with that entry.	24	77	S: Here, the times equals the 3 (i.e., she is lining operations on the printout diagonally with the numbers below them), the times and the 3 go together, like it's times 3.
		80	I: Right. Yes.
		81	S: And that was minus 5?
82-84: explanation of printout.	1	82	I: You go diagonally, right. So I'm going to leave you with it. And go for a little while for coffee (i.e., leave the room). And then I'm going to come back.
		85	S: OK. Sure.
			CALCULATOR OPERATED (Nos. 1 to 9), (11 minutes).
86: said she got it; subject's certainty questioned.	64 7	86	I: So you're telling me now you got it, heh?
		87	S: Yeah.
89-90: original question reworded	4	88	I: Alright, let me write the time down. This is encouraging to me cause I haven't had anybody get it yet. So you think you got it. What were the constant operations then?
91: has the answer to the question.	54	91	S: Ah, subtract 6 and divide by 3.
92-94: clarification.	9	92	I: Yes. OK, so you're going to explain to me how you got it. Which one did you do first (i.e., there were two separate sheets of printouts)? This stuff here or that stuff there?
		95	S: This stuff.
		96	I: So what I'll do is I'll number them, you're going to explain to me what you're doing.
98: affective statement.	66	98	S: Knowing me I won't be able to.

No.1. 140. + 1. / 3. = 140.33333 **	99: supportive statement. 100: reference to printout.	3 10	99	I: Oh, you'll do a good job. So in this first part here what are you trying (i.e., No.1)?
			101	S: Over here.
	102: clarification.	9	102	I: You're pointing at the example, yeah.
	103-4; No.1: refers to example given. Reversal strategy shown.	55 40 47	103	S: It was times 3 subtract 1. So I tried plus I divided by 3, and it didn't work.
	105-6: probing reasoning with an extension of what the subject said.		105	I: Is there any reason why you picked 3 and 1? Because they were there (i.e., in the example)?
	107: random response.	94	107	S: I'd just see if they'd work.
	108-9; 111: repetition of previous question.	4	108	I: But this 3 and 1, does it go with the 3 and 1 I gave you, did that give you a clue?
			110	S: Yeah, sort of.
			111	I: Wasn't that though, was it?
			112	S: No.
	113-4: related question.	5	113	I: How can you tell that you don't have it? Is there a quick way of telling?
	115: reference to printout.	55	115	S: Well you didn't get the right answer.
	116: repetition.	4	116	I: Well you didn't get 140 did you? Or 138.
	117: indication of successive approximations strategy developing.	55	117	S: I get more than 140 and you want to get lower, want to get 130.
	118-9: probing reasoning with directed question.	12	118	I: But does the fact that you have a decimal part on the end of it, does that tell you right away that you don't have it?
			120	S: Yeah.

No.2. 140.33333 ** 140. x 1. / 3. = 46.666667 **	121-2: reference to printout.	10	121	I: I think so. What's happening now? We'll call this No.2 here, after the first set of stars.
	123;No.2: exploratory manipulation. Random response?	22 94	123	S: I times by 1, just trying different things.
	124-5: clarification.	9	124	I: So you ... ah ... this is different effort altogether separate from the first one.
			126	S: Yeah.
	127: there are similarities between No.1 and No.2. Reviewing.	11	127	I: It's similar though, cause you're using 1 and 3.
	128-30: more indication of successive approximations. Inspects No.1 and makes change. Analyzes result for pattern.	55 49 40	128	S: But I decided maybe it wasn't plus and subtract so I tried multiplying and dividing and ... it got a lower answer but also decimals so I knew that -
	131: repetition.	4	131	I: Much lower, heh?
			132	S: Yeah. 46 and 6 repeating.
No.3. 140. x 6. / 3. = 280. **	133-5;No.3: probing reasoning by reviewing printout. Some direction.	11 10	133	I: Now it's starting to get interesting. Look quickly you got into in this third one now (i.e., No.3) you got into multiply by 6 and divide by 3. Is there a reason why you picked those numbers?
	136-7: reasons from question and picks numbers.	40	136	S: I thought ah ... you know like it was doubled ... and you want it to go down 2.
	138: clarification.	9	138	I: Each time?
	139-40;No.3: repeats algorithm of multiply and divide with data change.		139	S: Yeah, each time. So I thought if you doubled 3 ... that you might get the answer. Like you know I tried -
			141	I: Double 3.
	142-3: analyzes result for closeness to answer.	55	142	S: And you get 6. Tried multiplying by 6 and divide by 3, but you got a higher answer.

No.3.	140. x 6. / 3. = 280. **	144-6: extends logic and repeats subject's words.	4	144	I: Let me see if I understand you: you want to double 3, like you would use 3 before so because you saw 140 to 138 is 2 and 138 to 136 is 2, you thought double 3?
				147	S: Yeah.
		148-9: probing reasoning.	9	148	I: So you were looking from the sequence in here to come up with the 6.
				150	S: Yeah.
		151-2: reference to printout.	10	151	I: That's interesting. And then you divided by 3. Is there a reason why you picked that?
		153;156: indicates subject noted parts of conditions.	20	153	S: It says right there (i.e., in the problem given) divide by 3.
		154-5: reviewing subject's ideas.	11	154	I: Ah, so you know that's the constant operation. So right from the beginning you knew that it was all divide by 3?
				156	S: Well it says right there so it had to half of it –
				157	I: Yeah, but I put that into the problem wondering if people would see it.
				159	S: Well, I saw it (laughing).
		160-1: reference to printout.	10	160	I: Oh yeah, I knew that from what you told me at the beginning that you saw it. OK, so did that (i.e., the operations in No.3) work?
		162-4: indication of successive approximations.	40 55	162	S: You got a higher answer (i.e., 280) so it couldn't be ah ... multiply by 6, because that made it a lot higher, well it made it double.
No.4.	140. x 3. / 3. = 140. **	165-7: questioning reasoning with reference to printout.	10	165	I: OK. So in this No.4 now, you're trying still sticking with divide by 3 but now you're timesing by 3. And do you know what you're doing?

No.4. 140. x 3. / 3. = 140. ##	168: answers related question.	50	168 S: Getting 140 again.
	169: clarification.	5	169 I: Why?
	170-1: identifies error; demonstrates deductive reasoning.	54 40	170 S: Well, because you're multiplying and dividing by the same thing and they cancel each other out (laughing).
No.5. 140. x 2. / 3. = 93.333333 ##	172-4;No.5: reviewing printout and probing reasoning.	10	172 I: Yup (laughing). It's OK though, it's neat, because you see things happen. Alright, so in No.5 you're multiplying by 2 and dividing by 3. Is there a reason why you wanted to do that?
	175-8;No.5: explains reasoning and illustrates deductions. Does not understand what operations do together.	48 55 92	175 S: I just totally forgot about the 6, slipped my mind. And I thought, if that gives you 140 (i.e., in No.4) multiply by 3 and divide by 3, multiply by 2 and divide by 3 and you might get it. You get close to 100, and you get 93, but that wasn't right.
	179-80: probes reasoning by repetition.	4	179 I: Well, am I right - let me see if I understand you; you're multiplying by something less than 3 now.
			181 S: Yeah,
	182: reference to calculation.	10	182 I: And you're picking a whole number - 2.
	183;185: analyzes result for correctness.	55	183 S: Yeah, but it doesn't - decimals -
			184 I: It's OK, as soon as you see decimals -
			185 S: Yeah, it's wrong.
No.6. 140. / 3. = 46.666667 ##	186-7: review of calculations.	10	186 I: You're sort of on the wrong track. Let's go on to No.6. Looks to me like all you did was divide by 3.
	188: identifies error.	54	188 S: I missed a step.
	189;191: clarification.	9	189 I: Did you realize that?
			190 S: Yeah (laughing).

		191 I: So you just went on to another one?
		192 S: Uh hum.
193-4: extending from discussion.	13	193 I: So you knew that you should have two (i.e., operations) in there and you didn't.
195-6;199-202;No.7: returns to process of No.1 where she alters operation. She did not analyze No.1 or she would have seen 46 was too large if adding I produced an answer that's too large.	48 32 11 40 96	195 S: Well ... I did something - I added, what did I add? I meant to add, ah ...
		197 I: From the next step (i.e., No.7) you meant to add 46, is that what you're talking about?
		199 S: Yeah. I added 46, now why do I do that? ... adding it was changing from multiplying, right, and what I did was I divided 3 into 140 and I got 46 so I thought I add 46 to 140 and divide by 3, that number (i.e., 156.33333) but it got higher than 140 -
		203 I: Do you have any idea why?
		204 S: Ahm ...
205: clarification.	9	205 I: This is No.7 now.
		206 S: Because ...
207-9: explaining basis of question.	I	207 I: This is interesting Sherri, you see. Part of this problem you have to know the order of operations in which the calculator works.
210-11: identifies error.	54	210 S: Yeah. And that I did wrong. Because I did adding first and it has to be dividing or multiplying before adding or subtracting.
212: related question.	5	212 I: But did it add first?
		213 S: No.

	214: related question cont'd.		214	I: What did it do first?
	215: indicates calculator knowledge.	31	215	S: Divided.
	216-7;221-3: explaining calculator operations and extending into teaching.	1 413	216	I: So what will it do first is 46 divided by 3 and then it will add to 140.
			218	S: 146, and that's 46.
			219	I: Do you follow what I mean?
			220	S: Yeah.
			221	I: Like you see this - this 155.33333 is actually 15.33333 higher than 140. So that tells you that 46 divided by 3 is 15.33333. Do you follow?
			224	S: Uh huh.
	225: clarification.	9	225	I: Did you realize that when you were doing it?
	226: clear statement on post-analysis.		226	S: No. I just realized it now.
No.8. 140. + 49. / 3. = 156.33333	227-9: reference to printout and definite direction given.	12	227	I: That's OK. Let's take a look at No.8. And I can see you've ah - you're adding more than 46, you're adding 49. Is t -e a reason for that? You're adding 3 more.
	230-2;No.8: experimental manipulation of procedure in No.7. Does not analyze results.	22 96	230	S: No, I don't know - I just didn't really catch on, like I didn't catch on there (i.e., No.7) so I just went onto there (i.e., No.8).
	233-4: trying to establish randomness of choice.	9	233	I: Yeah, OK, you didn't catch on in No.7 so you went on to No.8. Is there a reason why you picked the 49?
			235	S: Well, it was 3 higher than 46.

No.8.	140. + 49. / 3. = 156.33333 **	236: repeating questioning of lines 233-4.	4	236	I: Did you want to make it 3 more for particular reason?
		237: random approach confirmed.	95	237	S: No, just to try something.
No.9.	140. / 2. / 3. = 23.333333 **	238: reference to printout.	10	238	I: OK, then at that point, we go on to the next one (i.e., No.9).
		239-40;No.9: returns to modifying operations. No reason for changing operation given.	48 96	239	S: And I tried dividing by 2 and dividing by 3. I just skipped out of multiplying.
		241-2: related question.	5	241	I: Do you have any idea what dividing by 2 and dividing by 3 really is doing in one step?
		243: error in mathematics.	85	243	S: Is dividing by 5.
				244	I: No.-
		245: corrects error.	54	245	S: 6.
				246	I: Right. So really what you're doing is dividing by 6. And did it work?
		248: successive approximations applied to results.	55	248	S: No. You got a really low answer, 23.
		249;251: probing reasoning with reference to printout.	10	249	I: OK, when you saw that really low answer -
				250	S: I just -
				251	I: You gave up on dividing and dividing?
				252	S: Yeah.
				253	I: Right away (laughing)?
				254	S: Yes (laughing). I tried ... multiplying again -
		255: reference to printout.	10	255	I: So we're doing No.10a.

No.10a.	140. x 1.5 / 3. = 70. **	256;No.10a: returns to procedure of No.5 and determines 2 based on results of No.5 and successive approximations.	48 33 55	256	S: Tried multiplying again by a lower number than 2 - picked 1.5.
		257-8: reference to printout to probe reasoning.	10	257	I: Why did you want to multiply by a number lower than 2? Are we going back - are you talking about now something like No.5?
				259	S: Uh huh.
		260: clarification.	9	260	I: Did you look at No.5 when you did No.10a?
				261	S: No.
		262-3;265: probe of reasoning continued.		262	I: But you seemed to keep track of it right now, like you remember multiplying by 2. Are you sure that you remembered it then?
				264	S: Probably not (laughing).
				265	I: So when you picked 1.5, was it related to 2 or not?
		266-9;271-2: confirmation of the random nature of the algorithm modifications.	94	266	S: Yeah it was - I didn't really look at the answer, I just - when I do problems like that, I just do whatever number pops in my head. Until I catch on to what is going on - like the sequence if I see something that looks good -
				270	I: But now when you were -
				271	S: I stick with it, but if I don't see anything I'll just - when numbers pop in my brain I just sort of use them.
		273: repetition of subject.	4	273	I: So you're saying 1.5 just popped in your brain.
				274	S: Yeah.
		275: reference to calculations.	10	275	I: Alright. What did you do then (i.e., still in No.10a)?
		276: continues reasoning based on printout.	65	276	S: By 3 and I got 70. So that -

No.10a.	140. x 1.5 / 3. = 70. **	277-8: interruption; reference to printout.	2 10	277	I: Can you tell me - just look at this for a minute, just this first part in No.10a - you're multiplying by 1.5 and dividing by 3.
		279: deductive reasoning.	40	279	S: Well, you're just multiplying by one half.
				280	I: Or dividing by 2.
				281	S: Or dividing by 2, yeah.
No.10b.	140. - 1. / 139.66667 **	282-3: reference to calculations and directing from printout.	10 12	282	I: So that's how you get the 70. Now right after that you quit and do something else (i.e., in No.10b). Or are they related?
		284: takes direction given.		284	S: They're tied together more or less. They're tied -
				285	I: How?
		286-7: recalls previous problem; repeats algorithm of changing operation done previously in Nos. 1, 2 and 9.	33 48 92	286	S: More up with No.1 when I added 1 and divided by 3 tried subtracting by 1 and dividing by 3.
		288-9: probing reasoning.	9	288	I: So now it's getting interesting. Is there a particular reason why you've decided to go the subtraction route?
		290-1: No.10b: random selection of operations.	94	290	S: Well, multiplying was not working, dividing was not working and adding wasn't working because -
		292: clarification.	9	292	I: You're talking as the first operation.
		293-6: deductive reasoning.	40	293	S: As the first operation. Because multiplying you always got a less number well most of the time except when I used 6. Ah .... dividing I got ah ... lower number and adding I got 140 and decimal places.
		297-8: questions reasoning.	5	297	I: Did you realize this when you - like is this why you started subtracting?
				299	S: Uh huh.

No.10b. 140. – 1. / 139.66667 **			300 I: Because you realized all that.
			301 S: Yeah.
	302-4: review of calculations after No.10a.	11	302 I: So now you've decided you're going to go to subtraction and looking down ahead quickly you can see you've done a whole bunch of subtractions.
			305 S: Uh huh.
			306 I: Alright –
No.11. 140. – 2. / 3. = 139.33333 **	307-8;310;312-3;No.11: explains thinking illustrating deductive reasoning and use of successive approximations. Repeats process of No.10b.	40 55 48 92	307 S: See what I did was I did 1, like I subtracted by 1 (i.e., No. 10b) and I got 139.6 repeating –
			309 I: Yeah, yeah, right.
			310 S: So I figured it had to be a higher number so I –
	311: clarification	9	311 I: Higher than what?
			312 S: Higher than 1. So I subtracted by 2 and divided by 3 (i.e., No.11).
			314 I: Yes.
	315: reference to printout.		315 S: And I got 139 and 3 repeating –
	316: probing reasoning.	9	316 I: Ah, does that tell you anything?
No.12a. 140. – 2. / 3. = 139. **	317-8;No.12a: discovers pattern in No.11 and repeats process with reasoning used to make changes.	55 48 40	317 S: Yeah, like every time you go up one, you subtract by one more, it goes down 3 in decimal place (i.e., 3 repeating). Right?
			319 I: Yeah, I got you.
			320 S: So when I subtracted 3 and divided by 3 I got 139 (i.e., No.12a)

No.12a. 140. -
2. /
3. =
139. **

No.12b. 140. -
6. /
3. =
138. **
138. -
6. /
3. =
136. **
136. -
6. /
3. =
134. **
134. -
6. /
3. =
132. **

4:02

317-8;320;322;324-7;Nos. 12a and 12b: discovers pattern using deductive reasoning. 55 40
323: clarification. 9

321 I: So it stayed the same. Oh no! It went down by 1.

322 S: Yeah, went down by 1. So to do that -

323 I: Just a sec. So you're saying it's going down a third -

324 S: By a third. So what I did was when I got the answer 139, I doubled 3. Like I doubled 3 because I wanted to go three more down. So that's how I got the answer. And it's subtract 6 and divide by 3.

Shawn Problem 7.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
4:30 Ex.1.	 8. x 4. + 1. = 33. ** 33. x 4. + 1. = 133. ** 133. x 4. + 1. = 533. ** 533. x 4. + 1. = 2133. **	01-06: explaining sequence by means of example and related question.	1 5	01	I: So what I want to do now is I want to give you an example of a sequence; just to show you what I mean by it. We'll start with a number, say 8, and multiply by 4, and add 1, get equals. Then multiply by 4, add 1, equals, multiply by 4, add 1, equals. And do that one more time. Now what I've done here is I've made a sequence. Do you know what numbers form the sequence?
				07	S: 4 and 1.
				08	I: Nope. The numbers you started with, 33, 133 - what else?
				09	S: 533 and 2133.
		10-12: related question on constant operations.	5	10	I: In other words, the number you started with and all the numbers with stars. Now to make that sequence I used two constant operations. What are they?
		13: answers related question.	50	13	S: The 4 and the 1. Multiply by 4 and add 1.
				14	I: So you understand what I mean by a sequence and constant operations. What I'd like you to do now is just look at this introduction part. See if you understand it. PAUSE (20s) Do you follow what's happened?
				18	S: Uh huh.
		19-23: explaining introduction of problem. Related problem.	1 5	19	I: Now on the left (i.e., of the introduction) I've got what I've showed you (i.e., in the example), not the same ones. But a printout that has all the numbers of the sequence and all the constant operations. What would the sequence be in the example on the left (i.e., of the introduction)?
		24: answers related question.	50	24	S: This one. 25, 76, 229 and 688.
		25-26: repetition of previous related question.	4	25	I: With the number you started with too. What would be the constant operations?

	27: answers related question.	50	27	S: Times by 3 and add 1.
	28-30: explanation of introduction continued.		28	I: So what I've done then is on the right I've pulled out the numbers in the sequence and I've left the constant operations out. Do you follow?
			31	S: Uh huh.
			32	I: Now here's the question I want you to do. Read it and see if you understand what I want you to try to do.
	34: restates problem in his own words.	21	34	S: You want me to find ... ah ... what constant operations you used.
			35	I: Right.
			36	S: To get these (i.e., the numbers in the sequence).
Ex.2. 78. + 44. * 45. = 123. **	37-44: explanation of how P-key works.	1	37	I: Now, can I leave you to do it for awhile ...

CALCULATOR OPERATING (Example 2)

Now also what I'm going to show you is how the P-button (i.e., the print function) works. Let's say I wanted to add 78 and 45 and instead of 45 I hit 44. What I'd like you to do in a case like that - like you don't want the 44, heh, hit P, it'll print 44 but it won't add it. Now you hit 45 and you're safe. That's how the P-button works and I'll leave you to do it.

PAUSE (3 s)

You have a little question for me. What's your question?

45 S: Do I use the divided by 3 (i.e., an operation given in the body of the question)

I: Yeah. All you got to do is be able to see it.

	45-6;48;50-1: questions conditions of problem.	20	48	S: Does it have to be times and divide first?
	49: explaining question withou more definition.	1	49	I: It can be up to you. It can also be add or subtract.
			50	S: First. Oh. OK. And it has to have one of each. Add and subtract and ...
	52-53: repeating what interviewer previously said. No direction.	4	52	I: The first one can be add, subtract, multiply or divide. But you spotted what the second one has to be.
	54-56: misinterprets problem; identifies error.	83 34	54	S: Oh, I thought you had to have add or subtract and a multiply or divide. You had to have one of each. So the first one can be anything else.
			57	I: That's right.
				CALCULATOR OPERATED (Nos. 1 to 10), (5 minutes).
	58-60: subject said he got problem before taping began.	64 9		So you think you've got it and what I'd like you to do is explain to me what you did at different steps. Just show me where you think you've got it.
	61-62: thinks he has it; note idea of closeness to answer.	64 55	61	S: Right in here. So that's the number you multiply by and divide by 3 and then it comes close.
No.1. 140. x 2. / 3. = 93.333333 **	63-65: reference to printout.	10	63	I: OK. We're looking at the bottom (i.e., No.10). You don't really have it. You'll see why in a minute. Alright so in No.1 what are you doing?
	66;68;No.1: develops multiplication - division process and analysis for closeness to answer. Key selected at random.	47 55 94	66	S: I was just testing out to see how close it was to ...
			67	I: So you picked - did you pick numbers, any numbers?
	67: trying to establish randomness of selected changes.	9	68	S: Just 2, anything smaller than 3.
	69: questioning reasoning.	9	69	I: Why pick anything smaller than 3 - any reason?

No.1.	140. x 2. / 3. = 93.333333 **	70-71: support for previously coded randomness.		70	S: Yeah, I don't know why it would have to ... seemed like it ... should work out ... it had to be smaller than 3.
		72-73: related problem to test calculator knowledge.	5	72	I: Well if it was 3, let's say you were timesing by 3 and dividing by 3. Do you know what you would get?
		74: answers related question.	50	74	S: You'd, you'd get the same number.
				75	I: OK. So maybe ...
No.2.	140. x 6. / = **	76-77: deductive reasoning illustrated. Contradicts what he does in No.2.	55	76	S: Well I figured the number would be adding instead of taking anything off if it was bigger than 3.
		78-79: probing reasoning by a review of calculations.	11	78	I: That's right. Like you see the sequence is 2 less each time. So you tried 2 and you got 93.333333. Did that tell you anything?
		80-81;No.2: data change to algorithm and analyzing calculation.	48 55 40	80	S: Yeah, it wasn't 2. It had to be bigger. Like 2.5 or something. And then ...
		82-3;No.2: attempt to establish reasoning behind calculations.	9	82	I: OK. So in No.2 you decided to times - you decided to multiply at first. Was there a particular reason for that?
		84: randomness established.	94	84	S: No, it just ... seemed like it should work.
		85-86: reference to printout.	10	85	I: So then you tried 6 and divide by 3 - you notice what's happening in No.2?
		87: answers related question.	50	87	S: No. It's doubling.
				88	I: Why?
		89-90;No.2: algorithm not developed yet.	92	89	S: Because I times it by three times as much as 3 - I thought it should take it off instead of double.
		91-92: related question.	5	91	I: But when you times by 6 and divide by 3 do you know what one thing you do?

	93: answers related question.	50	93	S: Multiply by 2.
No.3. 140. x 3. = 420. **	94-95: reference to printout.	10	94	I: Right. Ah so then in No.3 - did that (i.e., the answer in No.2) tell you anything when you got 280?
	96-97: identifies error; reasons about data change in algorithm based on printout.	54 40 55	96	S: That I was wrong, That it had to be something smaller than 3 for sure.
			98	I: Smaller than 3?
			99	S: For sure.
	100: reference to printout.	10	100	I: So you went back and you tried 3 times 140. Why'd you do that?
	101-2: experimental manipulation but confused explanation.	22	101	S: Just to see what it would be ... and then ... I timesed it by 140 and then ... divided it by ... added -
	103;105: clarification.	9	103	I: OK. You're talking to me about No.4 now?
	104-5: reversing strategy used to determine missing factor from printout.	47 40 55	104	S: I was trying to do reverse in the steps. Trying to do the steps in reverse.
			106	I: Explain that to me.
	107: previously learned method.	32	107	S: Like you know how - flow charts - how you can do them backwards.
			108	I: Yes.
			109	S: I tried to do it like that.
	110: probing reasoning by reference to calculations.	10	110	I: In other words if it said - you said times by 3, why?
			111	S: Well it was dividing by 3 -
	112-3: trying to establish pre-reasoning for calculations.	12	112	I: So you're reversing it. The first number was 140. Are you really looking for the number even before that then?

				114	S: Sort of I was, yeah.
		115: reference to calculation; contradiction.	6	115	I: You're sort of out of the sequence.
				116	S: Yeah.
No.4.	138. x 3. = 414. ## 414 / 140. = 2.9571429 ##	117-20; No.4: review of printout and questioning reasoning.	11	117	I: But you figured to come back into it. So you then tried 140 times 3 and got 420 (i.e., No.3). Oh now you picked 138 in No.4. That's OK. - so what are you trying to really do in No.4, can you explain it to me?
		121: explains reasoning.	65	121	S: I was trying ... I was going to multiply it by -
		122: clarification.	9	122	I: Multiply it by what?
		123-4; No.4: reversing process developed to determine missing factor.	49 40	123	S: The 138 by 4 - or 3 and then divide it by 140. I think it was to see what I would get and how many times it would have to times -
		125: repetition.	4	125	I: So you're reversing divide by 3 by multiplying?
				126	S: Uh huh.
		127-9: extending and directing subject.	12 13	127	I: And then you're saying, if I divide I'll find what I have to multiply by. So the number you get in the answer in No.4, what are you going to do with that?
No.5.	140. x 2.56 / 3. = 119.46667 ##	130-1; No.5: analyzes calculator result and repeats algorithm of No.2 with data change. Rounding error.	55 48 85	130	S: I was gonna take 140 ..., alright - I rounded it (i.e., 2.9571429) off -
		132: clarification.	9	132	I: What was it rounded to? Is this No.5 you're telling me about now?
				133	S: Yeah.
		134: contradiction established.	6	134	I: Is that it rounded? Take a look. Is 2.957 rounded to 2.56?
				135	S: I know. That's where I messed up.

			136	I: Well, did you realize that you had made a mistake?
No.5. 140. x 2.56 / 3. = 119,46667 **	135;137-8;No.5: does not identify error.	95	137	S: No, I figured something shouldn't be right so I tried all over again.
			139	I: But didn't you see what you had rounded –
			140	S: No.
			141	I: OK, so that's why, heh? OK. And now when it didn't work –
No.6. 136. x 3. = 408. ** 408. / 138. = 2,9565217 **	142;No.6: analyses printout for correctness. Does not analyze printout for error cause. Algorithm unchanged.	96	142	S: I started all over again.
		55		
	143: compares No.5 and No.6.	48	143	I: With a different number though in No.6.
		10	144	S: Yeah, 136.
	145: continued reference to printout.	10	145	I: Tried by 3, looks like you're doing the same thing.
			146	S: Uh huh.
			147	I: Ah, and look. Look what's interesting though, heh, Shawn.
	148: successive approximations strategy.	55	148	S: It's a little bit –
	149;151: clarification.	9	149	I: Than what?
			150	S: The first one.
			151	I: Which is the first one?
			152	S: The 2.957.
	153-5: questions reasoning based on comparing calculations.	10	153	I: The answer you got in No.4. So when you see it's not the same number in No.4 and No.6 (i.e., 2.9571429 in No.4 and 2.9565217 in No.6) ...

	156: refers to error in lines 137-8.		156	S: I never realized that.
	157-8: probing reasoning through related problem.	5	157	I: OK. Right now, when you see they're not the same numbers, what does that tell you?
	159;161: reasoning illustrated.	40	159	S: That that's not right.
			160	I: Why not?
	161: using answer without analysis.	96	161	S: I don't know, cause it doesn't work out to the same each time.
	162-4: explanation based on calculations.	1	162	I: You have to multiply by the same number each time, heh? That should tell you that something is not going right at the very end, heh?
			163	S: Yes.
No.7. 140. x 2.96 / 3. 138.13333	166-7;No.7: extending subject's idea and probing reasoning.	13	166	I: We'll go through it slowly. Alright, so you found 2.9565217 and then in No.7 you rounded, heh? Am I right?
			168	S: Yeah.
			169	I: So you're taking 140 ... are you trying ... what are you trying to make in No.7?
			171	S: 138.
			172	I: The next number in the sequence. And you're using the number you found where?
	174;No.7: corrects rounding error and repeats algorithm.	54 55	174	S: Here. I tried rounding it off in No.6.
		48	175	I: No.6, OK. You rounded it to what?
	175: clarification.	9		
			176	S: 2.95.

No.8. 140. x 2.95 / 3. = 137.66667 **	177-9;No.8: analysis of No. 8 and related question.	10	177	I: Should actually be 2.96. So you multiplied by 2.95, divide by 3, and ... we're talking about No.8 now. What you got - did you want that?
	180;No.8: repeats process of No.7 with data change to get closer to answer.	48 55 92	180	S: I was hoping for something closer (i.e., to 138) so I rounded -
No.7. 140. x 2.96 / 3. = 138.13333 **	181-3;No.7: comparing results of No.7 and No.8 as basis for questioning reasoning.	13	181	I: Just a minute Shawn, you did round in No.7 to 2.96, so you rounded right. See in No.7. But you got an answer bigger than 138. Is that why you picked 2.95?
			184	S: Yeah.
	185: reference to printout.	10	185	I: And then you got - what happened there (laughing)?
	186: analyses calculator result with Nos. 7, 8 and 9 illustrating successive approximations.	55 48	186	S: Got too small.
	187: reference to printout.	10	187	I: And so what did you do in No.9?
No.9. 140. x 2.9565 / 3. =	188-9;192;No.9: uses reasoning to increase factor by rounding to more decimal places. Repeats process using data from No.6.		188	S: I went - I tried using a bigger rounding to a - using more numbers.
	190-1: questions reasoning.	9	190	I: Rounding to a smaller decimal place (i.e., actually to more decimal places). And did it work out?
			192	S: Little closer but not ...
			193	I: Close enough. So what did you decide to do then?
	194: contradiction: there is no evidence multiplication by a whole number in No.9 or No.10.		194	S: Using a whole number.
			195	I: And did it work?
			196	S: No.

No.10. 140. x 2.9565217 / 3. = 137.97101 **	197-200: review of previous work.	11	197	I: So let's just go to No.10. And that's where - in No.10 - reading as far as I can see it - No.7, No.8, No.9 and No.10 have a lot in common. They're really working with the same multiplying number. Is that right?
137.97101 x 2.9565217 / 3. =			201	S: Yeah.
135.97143 **	202: suggests countermethod.	6	202	I: Ah ... alright ... did you put this 2.9565217 in memory?
135.97143 x 2.9565217 / 3. =	203: explains reasoning.	.65	203	S: I just punched that one every time -
134.00083 **			204	I: Each time (laughing)? It would have helped you in memory.
134.00083 x 2.9565217 = 396.17636 **			205	S: Yeah. I never thought of that.
396.17636 / 3. = 132.05879 **	206-7; No.10: related question.	5	206	I: So you tried each one. Really what's your sequence - can you look in No.10 and tell me what your sequence is?
	208: answers related question incorrectly.		208	S: It goes ... 140 times 2.95 -
	209-10: related question continued after explanation.	4	209	I: Those are your constant operations. Which numbers make your sequence (i.e., in No.10)?
	211: correct answer.	50	211	S: 140, 137.97101 -
	212-5: hint given.	8	212	I: Then the next one with two stars and so on. Now I think that what we're going to do is I'm going to give you a hint. And I'm going to tell you that the first operation that you've been searching for is subtract.
	216: questions reasonableness of hint.	20	216	S: Subtract. Subtract?
			217	I: Yeah, And I'm going to leave you to work on it a bit more.
	218: questions conditions of problem (really wants to add).	20	218	S: A negative number (i.e., that should be subtracted)?
	219: extending from statements	13	219	I: Because you want to make it add?

No.11. 0.e^x

CLEAR

140. -
6. /
138. **
138. -
6. /
3. =
136. **
136. -
6. /
3. =
134. **
134. -
6. /
3. =
132. **

		220	S: Yeah.
221: repeats hint.	8	221	I: No, I'm telling you it's subtract. OK.
			CALCULATOR OPERATED (No.11).
222: questioning certainty.	7		You think you've got it?
		223	S: I think so.
224-5: answer not encoded.		224	I: Keep going a little bit further, maybe. So you got it within a minute.
			CALCULATOR OPERATING (No.11 cont'd).
226-8: reference to calculations.	10		You got the answer. Now what's very important (advancing and removing the printout), let's go over it. What's this very first thing here (i.e., in No.11)?
229: calculator misentry.	100	229	S: That's just fooling around.
230: clarification.	9	230	I: So it doesn't mean anything?
		231	S: No.
232-5; No.11: questions reasoning with specific reference to calculations.	10	232	I: Just hit that key (i.e., e to the power x)? Alright; you started out - I told you, gave you the hint that it was subtract - and bang you picked subtract 6. It's very important to me - why did you pick subtract 6?
		236	S: Well just ... it had ... well ... was figuring 3 times 2 and too I wanted to see what would happen ... just and 2 had something to with it.
		239	I: Where - OK, I can see where the 3 comes from, the 3 you're talking about is from where?

No.11. 0.e x

CLEAR

140. -
6. /
138. **
138. -
6. /
3. =
136. **
136. -
6. /
3. =
134. **
134. -
6. /
3. =
132. **

236-8;243;248;No.11: uses decreasing pattern and reasoning to develop a subtraction - division process.	40 47 94		
		241	S: From the divided by 3.
		242	I: Where is the 2 coming from?
		243	S: From it goes down 2 every time.
244-7: related question on the calculator's order of operations.	5	244	I: And so you linked those two together somehow and come up with 6. Let me ask you a question, right:.... the way you entered this in the calculator is 140 subtract 6 divide by 3. Can you tell me what the calculator really does?
248: answers related question.	50	248	S: It divides 6 by 3 ...
		249	I: Which makes?
		250	S: 2.
251: explains order of operating.	1	251	I: And then subtracts.
252: did not analyze printout.	96	252	S: I never thought about doing it that way before.
		253	I: But just right now when I talked with you about it.
		254	S: Yeah.
255-7: probing reasoning by related questions.	5	255	I: This is why I think it's a good problem, because that's well hidden. OK, when I gave you the hint that it was subtract ... did it seem to you that that made sense?
		258	S: No it still didn't - I was just ...
		259	I: Why didn't it make sense? Like to subtract just didn't seem right.
261-2: identifies error in reasoning.	54	261	S: No, no ... it confused me ... I thought you'd have to add or make the number bigger before you can divide -

5:05

263 I: And make it smaller.

264 S: Yeah.

Wayne Problem 8.

Time/ No.	Printout	Comment	Code	Line	Protocol Text
3:42				01	I: OK. So I want you to read the problem you're going to try on this (i.e., the calculator) and then any questions you have you ask me. OK. I'm going to be up there (i.e., at the front of the room). I want to leave you alone for awhile to work it out. If you think you've got the problem, let me know. You'll know when you get it when you can work it all out on the calculator and show it to me.
					CALCULATOR OPERATED (No.1), (2 minutes).
				07	W: Mr. Leesinsky?
		08: questions certainty.	7	08	I: Have you got it?
		09: understanding problem.	83	09	W: Yeah. I don't understand what we're supposed to do.
				10	I: You don't understand what we're supposed to do?
		11: misinterprets.	82	11	W: Find the easiest way to do it, right?
		12-14: explaining question.	1	12	I: No, it's telling you the 7 and the 8 keys don't work. So you've got to figure out a way of finding this sum without using 7's or 8's.
					CALCULATOR OPERATED (Nos. 2 to 5b), (9 minutes).
		15: states difficulty.	63	15	W: Mr. Leesinsky. I can't get this problem.
				16	I: We're going to take a look at what you printed and you're going to explain to me what you did. If we go over that maybe you're going to get some ideas. We'll just see what you got (advancing and removing the printout). So in No.1 you really didn't understand the problem so what did you do?
No.1.	1028. + 882. + 274. = 2184. **	19-20; reference to printout.	10		
		21-2;No.1: reorders addends to make addition easier. After asking for and receiving direction he modifies the process into an exploratory check of the addition.	47 49 22	21	W: I just added these two because they looked like they went together with the 8 and the 2.

No.2. 294. + + 902. + 1026. + 2. = 2224. **	23-27: review and reference to calculations.	10 11	23	I: So you just added the numbers up. So in No.1 you just added them up because you didn't realize what the question was about. So it was here that I explained to you what you weren't allowed to use. So what did you do in No.2? Can you explain to me what you were doing?
	28: explains reasoning.	65	28	W: I was trying to do it without the 7's and 8's.
	29: reference to work.	10	29	I: So you put in 294.
	30;No.2: strategy: change addends by addition.	47 40	30	W: I added 20 on it.
	31-32: reference to printout.	10	31	I: You added 20 on to it. OK. Then you hit plus. And two plusses. Why two plusses?
	33: misentry.	100	33	W: Well my finger just hit ...
	34: related question on calculator knowledge.	5	34	I: Ah, so what did it do when you hit the two plusses? Why did it give you 902? Heh?
	36: error in understanding calculator operation.	85	36	W: I don't know.
	37-38: reference to result. Error by investigator in that the subject entered 902.	10	37	I: OK. When it gave you 902, what did you do? Well, why did you put 1026 in?
	39: continues algorithm of line 30.		39	W: Because I couldn't put 8 there so I added 1026 plus 2.
	41-42: question on calculator result.	10	40	I: OK. So the 1026 plus 2 is 1028. So that's how you made that number. Then you added all of them up together and got 2224. So, was that what you wanted?
			43	W: No.
			44	I: So what did you do?
			45	W: So I tried it again.

	46-48: review of No.2. Related question on incomplete operation.	11 5	46	I: So you tried another one. So basically what you've got a good ... instead of 274 you're representing 294. Twenty more. So how you going to get it back to 274?
	49: identifies error. Forgot to complete process.	54	49	W: Supposed to take twenty away after that.
	50: clarification.	9	50	I: But you didn't do it, heh?
			51	W: No.
No.3. 264. + 10. + 600. + 282. = 1156. ** 1026. + 2. = 1028. **	52-53: reference to printout.	10	52	I: OK. So what happened in No.3 now? Ah, so you got ....explain to me what you did.
	45;54-5;No.3: repeats process of No.2 but makes new error by forgetting given condition.	48 82	54	W: I put 264 plus 10, that's 274. Then I put 600 plus 282 and that's what ...
	56: reference to error.	6	56	I: Ah, but what's ... you used an 8 didn't you?
	57: did not identify error.	96	57	W: For what? It's supposed to be 882.
			58	I: Yeah ...
			59	W: Oh!
	60: already answered this question for the subject in line 56.		60	I: So what did you do that was wrong?
	61: uses previously given answer.		61	W: I forgot I wasn't supposed to use an 8.
	62-63: reference to results.	10	62	I: Alright. And you added all these together and you got 1156. And you put ... what did you do here?
			64	W: Added 1026 plus 2.
			65	I: To get?
			66	W: 1028.

No.4. 1026. d+ 2. + 900. − 12. + 264. + 10. = 2190. **	67–68: review of general strategy.	11	67	I: So what you've done then is to split each of the numbers, ah, what did you do?
	69;72-3;No.4: continues idea of No.3.	33 48	69	W: I added the first two and then I added the second one.
	70–71: probing reasoning.	92 5	70	I: But what did you do to each one of the numbers when you couldn't use 7's and 8's?
			72	W: Well I made them so I could. I took away so much that it would be without a 7 or 8.
	74–77: refers to calculations.	10	74	I: Yeah, OK. I like what you've done. Now in No.4 you're onto a different idea. This is an idea you had already; 1026 + 2. You've made 1028. What's the idea in here (i.e., in the calculations)? What did you do there? Where you got 900 − 12 ... ...
	78;80: identifies error. Mental calculation incorrect.	54 100	78	W: That was to get 882. That's not right.
	79: clarification.	9	79	I: And why isn't it right?
			80	W: I just figured it was 12.
	81–82: reference to results.	10	81	I: So subtracting 12 is wrong? What should you have done? ... Like I think the idea is good. 900 subtract 12, right. You thought that would make what?
			84	W: 882.
	85: simple related problem.	5	85	I: OK, it didn't. But could you have made 882 by subtracting? How?
			86	W: Subtract 18.

No.5a. 264. + 10. + 1026. + 2. + 600. + 262. + 20. = 2184. ##	87–95: review of previous work. Reference to work in No.5a.		95	I: Yeah, very good. So maybe if you had subtracted 18 there it would have been better. OK, what did you do here, in this part? 264 and 10, that's an idea you already had there, right? And then you added those altogether. OK, look, I think basically you've got it. Look, you've got it right here (i.e., in No.5a). 264 and 10, this and this, this and this and you add it and you got it. What's wrong? You've got, don't you? I think you've got it; unless I'm wrong. Let's see. Let's take a look at No.5a. In No. 5a you got 264 and 10. What is that?
			96	W: 274.
	97;99: clarification.	9	97	I: That's your first number. And then what did you do?
			98	W: 1026 plus 2.
			99	I: What's that?
			100	W: 1028.
			101	I: Which is?
			102	W: One thousand twenty-eight.
	103–4: reference to results.	10	103	I: That's one of your other numbers. Right? So you got that. And now what did you do in here?
			105	W: 262 plus 20.
	106: explanation.	1	106	I: Ah ... this part too you see. All three numbers.
			107	W: Oh, plus 600.
			108	I: And that made what?
			109	W: 882.

	110-11: contradicts subject.	6	110	I: And then you added them up and got 2184. You solved the problem and didn't know it.
No.5b. 1000. + 1100. + 264. + 10. = 2374. ** 2374. M+	112-3: compares correct answer to incorrect one without knowing which is which.	95	112	W: Well, I didn't get this one (i.e., he got 2184 instead of 2374 of No.5b which he thought was right).
	114-6: reference to results.	10	114	I: OK, but, right in here you've got the problem solved. Don't you? Like I mean you've worked all the numbers. Added them up, ah ..., what were you trying to do in No.5b?
	No.5a: algorithm modified so all operations are addition but note mental component of operations.	49	117	W: I was trying to add 274 plus 882 and then I was trying to add it to 1000.
			119	I: OK, this 1000 ...
	117-8; No.5b: modifies process but makes mental errors. Uses inverse operations on different addends.	49 40 100	120	W: Or no ... I took the 28 from the thousand and put it with the 882. And if you put 28 with that, you get ... 1110.
	122-4: review of operations. Related question.	11 5	122	I: So you're taking 28 off that (i.e., 1028) putting it with that. So this really is those last two numbers. Oh, I like it, why didn't it work out though?
			125	W: I don't know.
	126-8: explains error.	1	126	I: Well (using the calculator to demonstrate) it's 28 onto 882 (i.e., 910) so your idea was right. You took the 28 and you moved it over but where you said 1100 you should have said what?
			129	W: 910.
	130-1: probing reasoning with direction.	12	130	I: And you would have got it. So did this confuse you when you got 2374? Why did it confuse you?
	132: explains statement in lines 112-3. Assumes No.5b is correct (i.e., not relating to result of No.1).	96	132	W: Cause I thought that I'd got it right.
			133	I: You thought that what you'd done was right. But if you had done it right, what answer would you get at the end?

			135	W: 1028.
			136	I: No. If all your numbers were right, and you were adding them, ... what should you get at the end?
			138	W: 2184.
			139	I: Right, so when you didn't get that ...
			140	W: I knew I was wrong.
	141-2: Reference to results.	10	141	I: Yeah, but the reason you didn't get it was because you punched in in 1100 instead of 910. Why did you stick it in the memory?
4:00	143: misentry error.	100	143	W: I just bumped the button.

Sherri Problem 8

Time/ No.	Printout	Comments	Code	Line	Protocol Text
3:34				01	I: What I'd like to do is leave you alone with the question; you work it out and if you think you've got the answer you call me or if you're really stuck call me.
				04	S: OK.
					CALCULATOR OPERATED (Nos. 1a and 1b), (2 minutes).
		04: questions conditions of problem.	20		I can't use the 7 or 8 button at all?
		05-06: clarification.	9	05	I: OK. You're asking me if you can use the 7 or 8 button at all. Explain to me what you mean.
		07-08: explains reasoning with an example.	20	07	S: Like, OK. Say I did something like ... 200 subtract 70. And then it would like ... 274 subtract 70?
		09-11: explaining conditions.	1	09	I: No, then you'd be using the 7 button. So you can't do that. However, like what you did here ... let's just stop for a minute (advancing the printout).
No.1a.	274. + 882. + 1028. = 2184. **	12-3;17-8;No.1a: exploratory check to find what the answer should be.	22	12	S: I was just trying to figure out what the problem was like so I added them altogether and I divided by 7.
		14: review of printout.	11	14	I: OK. So in No.1a you punched in the numbers that were there.
		15: explains calculator actions.	65	15	S: Yeah and then I added them and divided the sum by seven.
				16	I: OK. Let me just ask you. Why did you punch the numbers in?
				17	S: I don't know. I just wanted to see what the sum would turn out to be.
		19: repetition.		19	I: So you know what your answer should be.
				20	S: Yeah.
		21: probing reasoning.	[illegible]	21	I: OK. And then why did you divide by seven (i.e., in No.1b)?

No.1b. 2184 / 7 = 312,	22-3;No.1b: gets bright idea to divide sum into whole number quotients and use addition process.	55 47 41	22	S: Well, just to see like ... without the seven what you would need to add ... youknow ... how many times ... sort of.
			24	I: OK. Alright. I got you now. You keep on going. And you understand.
				CALCULATOR OPERATED (Nos. 2 and 3). (2 minutes).
				Yes Sherri?
	26-29: refers to process of No.1b and repeats it with data change. Questions correctness of method before she states she has the answer.	30 55 48	26	S: Would that be possible? OK. I did use the seven over here (i.e., in No.1b) to get that. But then what I do is add that number seven times. And you get the answer. But it's still using seven, isn't it?
			30	I: No.
	31: states success.	64	31	S: No. I got the answer.
			32	I: OK. Let's do it this way. That's a solution.
			33	S: Yeah.
	34-5;37-40: reviewing calculations.	11	34	I: OK. We're going to accept that (advancing and removing the printout). Now what you've done is you've found the answer ...
			36	S: Uh hum.
			37	I: And then worked backwards by figuring out what you could add. Could you do this ... could you do something with the numbers without knowing what they add to? Could you work it out that way? Do you follow what I'm saying?
			41	S: You could divide each other number by a different number and see what ...
	43-44: asking for alternate solution.	5	43	I: OK. You work on that. But what I'd like to do is work with the numbers that you have.

	45: reference to previous calculations.	33	45	S: Like these ones here?
			46	I: Yeah ... alright.
	47;49: questions conditions of problem.	20	47	S: You mean like in solving the answer?
			48	I: Uh hum.
			49	S: So you don't want me doing any dividing?
			50	I: No ... you're allowed but what I mean is ... what you did is ... you added all the numbers up ... you know what?
			52	S: Hum ...
			53	I: Let's just accept that as a solution.
			54	S: I could try it the other way if you want.
	57: reference to printout.	10	55	I: No. Then I'm telling you how to do it. That's an answer. Now what I want to do is just go over with you basically what you did. So we went over No.1. Now let's go over No.2. What did you do here?
No.2. 2184. / 8. = 273. ** 312. +	58-9;No.2: applies same algorithm but changes method slightly. Note: finds whole number divisors twice	49 55	58	S: Well, I took the ... sum of all those three numbers and divided by 8 to see what I'd get.
	60: repetition of subject.	4	60	I: You found that sum in No.1?
			61	S: Uh hum.
	62: reviewing calculations.	10	62	I: And you divided by 8.
			63	S: Yeah and then -
	64: questions reasoning.	0	64	I: Why by 8

	65: reasoning illustrated.	65	65	S: Well I wanted to see if it ... like had any 7's or 8's in it.
	66;68: clarification.	9	66	I: What had any 7's or 8's in it?
			67	S: The answer.
			68	I: After you divided?
			69	S: Uh hum.
			70	I: OK.
			71	S: Because ...
			72	I: Did it?
			73	S: Yeah, it did.
	74;76: clarification.	9	74	I: So what did ...
	75: analyzes printout and determines error.	55	75	S: So I couldn't use that one.
No.2. 2184. / 8. = 273. **			76	I: You couldn't use what? 273?
			77	S: Uh hum.
312. +	78-79: reference to No.1. Comparison of No.1 and No.2.	13	78	I: But ... what did you do back in No.1? You divided by 7 and you got 312, so that was OK.
No.3. 312. + 312. + 312. + 312. + 312. + 312. + 312. = 2184. **	80-1;No.3: uses process referred to in lines 22-23. Solves problem.	47 33	80	S: Yeah. So I used that one then. I added it seven times. And I got the answer.
	82-3;No.2: reference to calculations.	10	82	I: Well, what happened in this part right up here? This 312 in No.2. What happened there?
	84-5;No.2: calculator error. Did not hit P key.	54 100	84	S: Uhm ... what I did was put 312 and then put plus and then I put 321 (laughing).

		86	I: So you cleared the whole thing.
		87	S: Yeah.
88-89: explaining calculator operations. Reference to calculations.	I 10	88	I: What you could have done was pressed the clear entry, the CI key. So in No.3, what have you done?
90-91: explains reasoning.	65	90	S: Well I added the 312 together 7 times and I got two thousand one hundred and eighty-four.
92-93: reviews method,	11	92	I: OK. So what you did really is to find out what the answer should ahead of time ... and then just rig ...
		94	S: Yeah.
		95	I: So that it ends up that way. And you don't worry too much about these numbers.

3:45

Earl Problem 8.

Line/ No.	Printout	Comment	Code	Line	Protocol Text.
3:49				01	I: So I'm gonna start you off at 3:49 and I'm gonna leave you by yourself and you're gonna work and here is the problem. Try to do all the work on here (i.e., the calculator) even the ideas that are no good, just let them go.
				05	E: OK.
					CALCULATOR OPERATED (Nos. 1 to 6), (6 minutes).
		06-7;09: uncertain about answer.	95		I think that's it. Is that what you want? Like this? Cause I don't really understand that too much.
				08	I: OK. Let's go over that. You got it!
				09	E: No, it's wrong. I got ... I got one too many digits in there.
				10	I: I don't think so.
				11	E: Yeah ... oh yeah, that's right.
				12	I: OK. So what we're going to do now is I need to go over with you what you did.
				14	E: OK.
No.1.	274. + 882. = 1156. ** CLEAR CLEAR CLEAR CLEAR CLEAR CLEAR CLEAR CLEAR CLEAR CLEAR	15-17: explaining discussion procedure.	3	15	I: You're going to try to explain to me what you did. And if you made a mistake just tell me you made a mistake. Don't worry about it, how it looks or anything. That's foolish, right.
				18	E: Yeah.
		19-20: reference to printout.	10	19	I: In No.1 - boy - a bunch of clearing in here (i.e., the printout had clear printed about ten times in a succession).
		21: affective statement.	66	21	E: I was angry at myself.
		22: clarification.	9	22	I: Why?
		23: misreads problem.	81	23	E: Well, I thought that the second plus sign was an equals in there.

No.1. 274. +	24: repetition.	4	24	I: Oh, I see, you thought that was an equation.
882. =			25	E: Yeah.
1156. **				
CLEAR	26: reference to printout.	10	26	I: Alright. So in the first one what did you do in No.1?
CLEAR	27-8;30-1;No.1: routine check.	22	27	E: Well I thought that was 274 plus 882 equals 1028 so I was just checking to see if that was right or not. And ah ...
CLEAR	29: clarification.	9	29	I: You thought what? Say it again.
CLEAR			30	E: I thought it was supposed to be the first two added together to equal ah ...
CLEAR	32-33: extending and repeating what subject said.	13 4	32	I: Oh I see, you thought it was 1028. So you had this plus sign as an equation. And it didn't equal ...
CLEAR	34: analyzes calculator result; identifies error.	55 54	34	E: So I knew something was wrong.
CLEAR			35	I: OK.
No.2a. 1028. -	36-7;39-40;No.2a: strategy: eliminate nonfunctioning key. Mental operations too.	47	36	E: So then I started thinking in No.2 that ah, get rid of the 7 and how that works out.
204. =	38: clarification.	9	38	I: OK. So how did you do that?
824. **			39	E: So I was thinking if 1028 and you get rid of the 70 well that'd be 204.
	41-42: extending from subject's statements.	13	41	I: In other words when they said the 7 and 8 keys on the calculator don't work, you thought instead of a 7 there should be a 0?
	43: explains reasoning.	65	43	E: Well I was thinking that as an example, right. How that would turn out.
	45: repetition.	4	45	I: OK. So just tell me again why you picked 1028 subtract 204.

No.2a. 1028. = 204. = 824. ##	46-8;50-2;No.2a: strategy: replace addend with two addends that don't use nonfunctioning keys.	47	46	E: Well, OK, I thought the problem was first of things, equals 1028. And I thought well if the 7 key doesn't work, let's say the 0 key does so try it that way.
	49: repetition.	4	49	I: In other words when the person pressed the 7 that they got 0?
			50	E: No, that, ah ... instead of using the 7 you use the 0 and then when you get the sum at the bottom, right, you add those two together, you get 1028. That's what I was thinking of.
			53	I: Oh boy.
			54	E: I know.
	55-56: clarification; affective statement.	3	55	I: No, I'm not saying it's wrong. I'm just trying to follow what you're saying.
	57;63: self-deprecating.	66	57	E: It's weird.
	58-59: reviewing previous discussion.	11	58	I: So you're picking subtract 204 instead of 274, and what are you hoping to get?
	60-61: explains reasoning.	65	60	E: Getting ... trying to get an answer without a 7 or an 8 in it that will add with 204 to equal 1028.
			62	I: Ah, OK. I got you. So what you're looking for ...
			63	E: My mind works in weird ways.
	64-66: repeats subject's reasoning.	4	64	I: That's OK. I'm interested in how your mind works. So what you're looking for here is some number that you could add back with 204 and get 1028.
			67	E: Right.
	68: reference to printout.	10	68	I: That's in No.2a. And then in No.2b?

No.2b.	1028. - 294. = 734. **	69-72;74;No.2b: modifies algorithm so that 824 (which has an 8), becomes 734, which, unfortunately, has a 7.	49 55 54	69	E: Well in No.2b I was thinking well the other way around, instead of subtracting 7 you'd get it above 8 ... so use 294 and see if that would work out better with no 7's or 8's if you said add the two together to equal.
		73: repetition of previous statments.	4	73	I: So in No.2a you ran into an 8 there, heh?
				74	E: Yeah. I was trying to get rid of the 8 in No.2b.
		75-77: reference to printout.	10	75	I: Oh, I see. So in the first part you're trying to get rid of the 8 by making it slightly bigger than it was there and what happened in No.2b?
		78: identifies error.	54	78	E: In No.2b I got 7 in there.
		79: review of error.	11	79	I: But in Nos. 2a and 2b you are assuming that there was an equality.
		80: affective statement.	66	80	E: Yeah, I know.
		81: clarification and repetition of previous statement in line 21.	4	81	I: OK. And then all these clears are out of frustration.
				82	E: Yeah, aggrevation, yeah.
No.3.	274. + 882. + 1028. = 2184. **	83: reference to calculation.	10	83	I: What happened in No.3?
		84-5;No.3: identifies error and makes routine check.	54 47	84	E: OK. Did like ... I did at first. Just check to see if that was alright.
		86;88: review of what subject did.	11	86	I: So you want to know the answer.
				87	E: Yeah.
				88	I: In other words, if the keys were working what the answer would be.
				90	E: Right.
		90: simple related question.		90	I: So you got - the answer should be

No.3. 274. + 882. + 1028. = 2184. **	91-2;94-5;97;No.4: explains reasoning. Modifies original algorithm by performing inverse operations to addends. Does not meet conditions of problem.	49 82	91	E: 2184. Then, I was thinking well, to get rid of the 7 and to get rid of an 8.
			93	I: Yes.
No.4. 294. + 862. + 1028. = 2184. **			94	E: Take 20 from the 882 because that would leave you with 862 so you'd only have one 8.
			96	I: OK.
			97	E: Take the 20 you subtracted and add it to the 274.
	98;100;102-3: review of work by subject.	11	98	I: So that becomes 294.
			99	E: Yeah.
			100	I: So you got rid of the 7 by decreasing this one (i.e., 882).
			101	E: Right.
			102	I: You still ended up with the same two numbers because this one (i.e., 294) is twenty more and ...
			104	E: That's (i.e., 862) twenty less.
	105: related question.	5	105	I: You're still running into what problem?
	106-7: answers related question. Describes changes in algorithm. Deductive reasoning illustrated. Identifies error.	40 54	106	E: Well the problem with the 8. I was just trying to go one digit at a time.
	108: extends subject's ideas.	13	108	I: OK. So you're working with the tens place now.
			109	E: Right.
			110	I: OK.
	111: knows he has the answer.	54	111	E: And then I got the answer so that was right. Then ...

		112: questioning certainty.	7	112	I: OK. When you got the answer you knew that what you were doing ...
				113	E: Was right.
No.5.	194. + 962. + 1028. = 2184. **	114: reference to printout.	10	114	I: So then you went onto No.5.
		115-6;No.5: repeats algorithm.	48	115	E: OK. In No.5 I was thinking well increase the 800 by 100 ... like you take off the 294. To get rid of that.
		117: review of calculations.	11	117	I: So you're getting rid of the 8 you couldn't get rid of in No.4.
				118	E: Right.
No.6.	194. + 961. + 1029. = 2184. **	119-22: review of printouts Nos. 5 and 6.		119	I: And you did that by not only working with the 7. First like in No.4 you worked with the 7, heh, and you left the 8 alone. Now in No.5 you're working with the 7 and the 8; but you still left this one (i.e., the 8 in 1028) in, heh?
		123: explains reasoning.	65	123	E: Because I wanted to take things one step at a time.
		124: clarification.	9	124	I: OK. So 1028 you didn't change in No.5?
				125	E: Right.
		126: reference to printout.	10	126	I: But then in No.6 ...
4:05		127;No.6: repeats algorithm successfully.	48	127	E: In No.6 - yeah - I subtracted 1 from 962 and added to the 1028.

Ruth Problem 8.

Line/ No. 3:28	Printout	Comment	Code	Line	Protocol Text
				01	I: Do you understand the question?
		02: restates problem in her own words.	21	02	R: Yup, you can't use the 7 or 8.
					CALCULATOR OPERATED (Nos. 1 to 5), (7 minutes).
				03	I: How are you doing Ruth?
				04	R: OK.
		05: questioning certainty.	7	05	I: Think you got it? So you got it! Why didn't you let me know?
				06	R: I just figured ...
No.1:	137. x CLEAR	07-12: reference to printout.	10	07	I: Oh you devil. OK. We're just going to go over what you did. OK. So basically what I want to go over now with you is what you did in each step (advancing and removing the printout). To see if I can get a picture ... OK. So this first one (i.e., No.1), what is that?
		13-4;16;No.1: explains reasoning: strategy is to split first number into two factors to eliminate the 7. Identifies error.	65 47 54	13	R: OK. For this one, like I was going to divide it by 2, so I didn't have the 7 in it.
		15: repetition.	4	15	I: You were going to divide this by 2.
				16	R: So I didn't get the 7 but I ...
		17-19: review of subject's ideas.	11	17	I: (Laughing) oh, I see, you wanted to get around the middle 7 and then when you divided by 2 you get into this 7 on the end. OK. So in No.1 you were going to divide by 2.
				20	R: Yeah.
		21-22: related question.	5	21	I: And then what would you have done? Say it worked and you didn't get any 7's.
		23-25: explains reasoning. Plan is to continue factoring.	65	23	R: Then I would have times by 2 and then to get that (i.e., 274). Then I would have added and divided that (i.e., 882) by 2 which is 441 and times by 2.

		26-28: review of subject's strategy.	11	26	I: Oh, I see, you ... so like ... you would have split them in halves and then doubled them to come back. OK. Excellent. But then when you had that (i.e., 137) it didn't work.
No.2.	441. x 2. = 882. **	29;No.2: applies halving strategy to second addend (note: 441 calculated mentally). Checking procedure followed.	48 47	29	R: Yeah. OK, Well then I do that like you know in half?
				30	I: OK. No.2 you did what you were telling me.
				31	R: Yeah.
		32;No.2: review of calculations.	11	32	I: You split the middle one (i.e., 882) in two.
				33	R: Uh hum.
		34: probing reasoning.	9	34	I: And what you do? Just check to see if you get 882?
				35	R: Yeah.
No.3.	441. x 2. + 514. x 2. + 264. + 10. = 274. **	36: reference to printout.	10	36	I: OK. I got you now. This is nice. Now in No.3 what are you doing?
		37;39;44;46;48;No.3: repeats algorithm.	49	37	R: Just figured like divided that (i.e., 882) by 2, right?
				38	I: Yeah.
				39	R: Then divide that one, like timesing by 2, OK? That and then I ...
		40-41: clarification.	9	40	I: OK, just hold on, when you say to get that, what's the very first one, 441 times 2?
				42	R: That's 882.
		43: reference to printout.	10	43	I: OK. I got you. And then what you do in the next step?
				44	R: OK, divided 1028, you know.
				45	I: So you're working with the third number, OK. And you ...

No.3. 441. x 2. + 514. x 2. + 264. + 10. = 274. **			46	R: Times by 2 to get ... you know.
			47	I: OK.
			48	R: And then went 164 plus 10.
	49: contradiction.	6	49	I: Ah no, not 164.
			50	R: Or 264 (laughing).
	51: clarification with reference to printout.	9	51	I: So this ... so you represented the 882 by multiplication --1028.
			52	R: By multiplication.
	53-4;No.3: very good question. The printout was duplicated by the investigator and an answer of 2184 was obtained.	5	53	I: And this one (i.e., 274) by addition and how come you only get 274 (i.e., as the starred answer)?
			55	R: Well because it only added those two.
			56	I: Ah ... I see what you're talking about. OK. So now, OK. How did you get it in this one then (i.e., No.4 has the same keyed entries and operations and a starred answer of 2184)?
No.4. 441. x 2. + 514. x 2. + 264. + 10. = 2184. **	59;No.4: identifies the error made by "the calculator". 60: error by the investigator.	54	59	R: It's exactly the same thing but I got the answer instead of 274.
			60	I: No it's not, because you used the equals at the end.
			61	R: Yeah.
	62;64;66: analyzing calculator results.	1	62	I: Once you use that equals sign ...
			63	R: Uh hum.
			64	I: It's not doing anything else.
			65	R: Yeah.

No.3. 441. x 2. + 514. x 2. + 264. + 10. = 274. **			66	I: In here (i.e., No.3) the equal was just adding 10, right.
	67-8;70: explaining reasoning.	65	67	R: Yeah, but see, I put all that in before it and that I added - like you know.
			69	I: OK, So why is it doing that? 441 times 2 ...
			70	R: See I have just one equals there (i.e., in No.4) too.
No.4. 441. x 2. + 514. x 2. + 264. + 10. = 2184. **	71-74: reviewing printout by redoing it.	11	71	I: ... OK. Let's go back. We're going to investigate No.4. And the way we're going to do it is we're going to punch everything you punched back into the calculator. OK. Now let's start with the clear. Cause once you clear - everything is cleared, right. 441...
			75	R: Times 2, plus ...
			76	I: Plus 514 ...
	77: deductive reasoning illustrated.	40	77	R: Would it be because it wouldn't remember the steps ahead?
			78	I: That's what I think. Let me just see. What are we doing? Times 2, ...
			80	R: Times 2 plus 264 ...
			81	I: Plus 264 ...
			82	R: Plus 10 .
	83-87: subject has confidence to repeat original work. Related question.	5	83	I: Plus 10 (presses equals on the calculator). OK. Now. We have to watch it very closely. There's a reason for it. .... There's a reason. Gee that's a good one. OK. So let me ask you something: when it didn't work in No.3, why did you try the same thing over in No.4 if it didn't work in No.3?
	88-9:No.4: has analyzed results and identifies calculator's error	55 54	88	R: OK, well, those two, you know equal that, so it would have to be a higher number.

	90-2;95: related question and probing reasoning.	9	90	I: So you knew that something was not working in here. What I don't understand is how you could have enough confidence to go back and try the same thing all over again.
			93	R: Because that and that equals the number you have on the paper, you know?
	95: continues reasoning of lines 88-9.		95	I: So what you're saying is you knew it had to work?
			96	R: Well, kind of.
			97	I: In other words, even if the calculator didn't give you ... you knew that it should work. OK. Let's ... I'm going to investigate this one further in No.3. I don't see why it didn't give you 2184. There's a very good reason for that but I can't see it. It should. Cause I think when you hit the clear you see, what you could say is like ... don't see why it shouldn't work. There's got to be a logical reason though? You got all the same numbers.
			104	R: Yeah.
No.5. 260. + 14. + 441. + CLEAR 260. + 14. + 441. + 441. + 514. + 514. + 2184. = 4368. **	105-8: reference to No.5.	10	105	I: The same operations in the same order. Right. Everything's the same. You hit clear there and you hit clear there. So what's the matter? OK. Let's go to No.5. Let's just leave that. In No.5 what were you doing?
	109;117;No.5: modifies algorithm and changes all operations to addition to avoid order of operations. Misentry error (i.e., 2184).	49 100	109	R: I don't know ... instead of using 264 and 10 I used 260 and 14.
			110	I: Did you know down here (i.e., No.4) you had the answer already?
			111	R: Yeah, because I worked it out in my head.
	110: questioning certainty. 111: reference to mental operations.	7 40	112	I: (Laughing) OK. You added these mentally?
			113	R: Yeah,
	114: probing reasoning.	5	114	I: OK. So why did you keep on going after you found the answer?

No.5.	260. + 14. + 441. +	115;117: checking by another method.	47 55	115	R: I don't know, to see ... just to try it again. I don't know.
	CLEAR	116: repetition of subject.	4	116	I: Try it a different way or?
	260. + 14. +			117	R: You see instead of times by 2 I just added them all.
	441. + 441. + 514. + 514. +	118-20: review.	11	118	I: Ah, instead of adding by 2 you just added the same thing twice. Except for this one (i.e., 274) which you couldn't do. Why couldn't you add this one in half each time?
	2184. =			121	R: Because you ended up with a 7.
3:45	4368. **			122	I: (Laughing) that's right.

Lisa Problem 8.

Time/No.	Printout	Comment	Code	Line	Protocol Text
3:48				01	I: Try this problem. You read it. If you have any trouble and you can't do it, you let me know, I'll be up there (at the front of the room). And if you think you have the answer you let me know. OK. I think you'll find it a lot easier if I just leave you by yourself for a while.
		06;09: questions conditions of problem.	20	06	L: You mean I just ...
				07	I: You just try the problem; see what you can do. Read it. Tell me if you think you understand it or not.
				09	L: You mean the 7 or 8 keys on here don't work?
		10-12: explaining the problem by rewording.	1	10	I: Well that's what you're ... well say you had a calculator where the 7 and 8 keys didn't work. OK. How would you figure out what the answer is?
				13	L: Oh! OK.
					CALCULATOR OPERATING (Nos. 1 to 4), (4 minutes).
		14;16: not certain about having the answer.	95		What's the answer?
		15: questioning certainty of subject.	7	15	I: You think you've got it?
				16	L: Maybe.
		17-22: clarifying organization of printout.	9	17	I: OK. Let's take a look at what you've got. You've got it all bunched together there. Let's see what you did (advancing and removing printout). Well I can see you got an answer. So what I'd like you to do ... OK, like, did you get different ideas in different parts of it? Or did you get one idea through the whole thing?
		23: illustrates deductive reasoning	40	23	L: Well ... I could have did [sic] it different than I did. Like you can put different numbers together.

No.1. 200. + 30. + 44. = 274. ***	25-28: reference to calculations.	10	25 I: OK. Alright. Let's take a look at what you did do. What did you do in this first part here? So you have two hundred plus 30 plus 44 equals 274. What did you do here in this part where the first stars are?
	29;No.1: strategy of re-writing addend as the sum of three other numbers without unwanted digits.	47 40	29 L: Figure out numbers that equal 274.
	30: probing reasoning.	5	30 I: OK. Why did you pick 230 and 44?
	31;No.1: explains ideas. Note rounded addends as part of strategy.	65	31 L: Well just took 200 and then split 74 up.
			32 I: Why did you split 74 up?
	33;35: refers to conditions of the problem.	55	33 L: ... ... cause the 7 ...
			34 I: What about the 7?
			35 L: Well ... it didn't work.
	36-37: repeating subject.	4	36 I: So you're not allowed to use it. You're trying to take 74 and split it into two parts?
			38 L: Uh hum.
	39-40: referring to printout.	10	39 I: OK. So that's the first one. And then what'd you do? You just pressed equals to ...
	41;No.1: makes routine check.	47 55	41 L: To see if it worked.
	42: extending idea of subject.	13	42 I: Like to see if you got 274?
			43 L: Yeah.
No.2. 400. + 400. + 42. + 40. = 882. ***	44-45: reference calculations.	10	44 I: Oh. An ... alright. That's the first part. What did you do in this second set of stars? What did you do in here, in there?
			46 L: Split the 800 up and then split the 82 up.

No.1. 200. + 30. + 44. = 274. **	46;No.2: applies same algorithm as in No.1 with rounded addends again. Routine check. 49-50: review of results.	48 55 11	47	I: OK. You've got 400 and 400, so that's ...
			48	L: 800.
			49	I: That's splitting the 800 up and then you split the 82. So you know that's 882.
			51	L: Uh hum.
No.3: 1000. + 23. + 5. = 1028. **	52-54: reference to calculations.	10	52	I: Ah ... and so what - the equal does the checking there? What you do in this part in here (i.e., the work between the second and third set of stars - No.3)?
	55;No.3: same algorithm and check used again. 56-57: probing reasoning.	48 55 5	55	L: Split the 28 up. Took the 1000 off.
			56	I: Well, the thousand ... you ... why do you punch in 1000? You don't change that. Why?
			58	L: I don't know I just ...
			59	I: Well you're satisfied with it.
			60	L: Yeah.
	61-62: probing reasoning but with unimportant related questions.	12	61	I: There's no 8's or 7's so you stick with that. You're only working like with the units in here, right? Why didn't you then ah ...
	63;65: repeats process on third addend.	48 55	63	L: Like down here I went ...
			64	I: 5.
			65	L: 1023 (i.e., the second to last addend in the printout).

No.4 200. + 30. + 44. + 400. + 442. + 40. + 1023. + 5. = 2184. **	66-73;75-6;No.4: review of printout No.4 to explain the modified algorithm used by subject but with extension from calculations.	11 13	66	I: Here you worked ... so in this last step here ... that's what I was asking you ... like ... you know - what you've here is very economical. In other words you're only working with the things you need to work with. You're quite prepared to accept 200 (i.e., the first addend) because there's no changes to make in that. But when it came to 800 you were ready to make changes in the hundreds. So what I was wondering is if the only thing that is causing trouble with the 1028 is the 8.
			74	L: Yeah.
			75	I: So I was wondering why you didn't ... but then down here (i.e., 1023 + 5) you did, heh?
			77	L: Yeah.
			78	I: Like you realized that ... yeah ... I see what you're talking about.
	80: explains sequence of reasoning.	65	80	L: Like I realized that after I ... punched the ...
	81;83-4: reference to calculations.	10	81	I: OK. So let's just do that instead. So then you work with the 1028?
			82	L: Yeah.
			83	I: OK. So now, in No.4, what are you doing? In this big piece here, what are you doing?
			85	L: Adding ...
	86: comparison of printouts.	6	86	I: 200, 30 and 44. That's essentially what you had in No.1.
			87	L: Yeah.
	88: clarification using printout.	9	88	I: Now you're working with the first 400 of No.2?
			89	L: Uh hum.

No.4.	200. + 30. + 44. + 400. + 442. + 40. + 1023. + 5. = 2184. **	90: clarification.	9	90 I: What are you doing next?
		91: simplifying aspect of algorithm described. Combines previous work.	49 33	91 L: Putting the one 400 and 42 together.
		92: clarification.	9	92 I: Because you can do that at one time, heh?
				93 L: Yeah.
		94–95: reference to printout.	10	94 I: OK. So that's 442 and then you stick your 40 on. And ... what happened here? With the 1023?
		96: describes mental component of work.	40	96 L: You just add them mentally, like.
3:58		97–98: general summation of last calculations.	11	97 I: So in other words you weren't working with 1000 and 23 separately. And then you added 5 and got that.